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Heat Transfer of Combined Free and Forced Convection in a Cylinder whose Cross – Section Is A Sector

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Abstract- The purpose of this paper is to analyze the problem of the heat transfer of combined free and forced convection in a cylinder whose cross- section is a sector. The solutions are obtained in terms of Lommel, Bessel and associate functions.

I. INTRODUCTION

The problem of fully develop Laminar convection flow of incompressible viscous fluid under pressure gradient in a vertical circular cylinder with varying was solved by Tao (1) and Morton (2) and Dalip Singh (3) discussed the flow in incompressible viscous fluid under a pressure gradient in a vertical elliptical cylinder.

Here we have obtained solutions for fully developed laminar flow through a vertical cylinder whose cross section is a sector with boundaries θ= 0, θ = α, r=a of Linear axial wall temperature gradient with and without generation are treated. The solution are obtained in

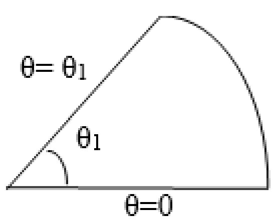

FIGURE: - SECTOR

terms of Lommel , Bessel and associated functions , corresponding values of Ra , E and Nu are tabulated for F= 0 and F=1 . The graphs have also been drawn in the above cases,

II. EQUATION OF MOTION

The flow is assumed to be fully developed study and in compressible and to have constant physical properties except density taking Z-axis along the axis of the cylinder the equation of continuity momentum for fully developed flow of in compressible viscous fluid in vertical cylinder of linearly varying wall temperature with heat sources are (Tao 1).

$$\frac{\partial u}{\partial z} = 0 \qquad \text{........ (1)}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\ell g \beta T}{\mu} = \frac{1}{\mu}\left[\frac{\partial p}{\partial z} + (\ell w - \ell_o)\, y\right] \qquad \text{......... (2)}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} - \frac{\ell C_p C_1 u}{K} = -\frac{Q}{K} + T_t \qquad \text{........... (3)}$$

Where p is the pressure,ℓ is the density ,μ the viscosity, g the acceleration due to gravity, β the expansivity , C_p the specific heat at constant pressure , K the thermal conductivity , Q the heat source intensity , and C_1 the wall temperature gradient , u is the axial velocity, T the difference of local and wall temperature.

Following tao the dimensionless forms of (2) and (3) are

$$\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + R_a \phi = E \qquad \text{........... (4)}$$

$$\frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} - U = -F \qquad \text{.......... (5)}$$

Where $X = \frac{x}{\sigma}$, $Y = \frac{y}{\sigma}$, $U = \frac{u}{u_m}$

$R_a = \frac{\ell^2 g C_p C_1 \beta\, \sigma^4}{K\mu}$, $\phi = \frac{KT}{\ell C_p C_1 u_m \sigma^2}$

$E = \frac{\sigma^2}{\mu u_m}\left[\frac{\partial p}{\partial z} + (\ell m - \ell_o) g\right]$

$F = \frac{Q}{\ell C_p C_1 u_m}$

R_a being Raileigh number u_m the average velocity

Equation (4) and (5) in polar coordination's transform to

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} + \frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2} + R_a\phi = E \quad \text{……….} (6)$$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} - U = -F \quad \text{……..} (7)$$

We shall consider that fluid is obtained in the cylinder whose boundaries are $r=0$, $\theta=0$, $\theta = \theta_1$

- BOUNDARY CONDITIONS

$U=0$, $\phi = 0$, when $\theta=0$, $\boldsymbol{\theta} = \alpha$

$U= 0$, $\phi = 0$, when $r=1$

- SOLUTION OF THE PROBLEM

For solving equation (6) multiply the equation by sin þθ and integrate within thelimits 0 to α.

$$\frac{\partial^2 \bar{U}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{U}}{\partial r} - \frac{þ^2}{r^2}\bar{U} + Ra\,\bar{\phi} = -\frac{2E}{þ} \quad \text{……….} (1.2)$$

$$\bar{U} = \int_0^\alpha U \sin þ\theta \, d\theta \quad \text{………..} (1.3)$$

$$\bar{\phi} = \int_0^\alpha \phi \sin þ\theta \, d\theta \quad \text{………..} (1.4)$$

And p's are the roots of the equation

$Þ\alpha = (2n + 1)\pi$ …..(1.5)

Now mulyiplying the whole equation (1.2) by rJp(qr) and integrate within the limits 0 to 1 we get

$$-q^2 \bar{U}_H + Ra\,\bar{\phi}_H + \frac{2E\lambda}{Þ} = 0 \quad \text{…..} (1.6)$$

Where $\bar{U}_H$ and $\bar{\phi}_H$ are Hankels transform of $\bar{U}$ and $\bar{\phi}$ respectively and q's are the root of

$J_p(q) = 0$ …..(1.7)

Now by Erdelyl (4) page 333 where

$$\lambda = \int_0^1 r J_p(qr)\, dr$$

$$= \frac{1}{q^2}\left[þq\, J_p(q) S_{o,p-1}(q) - q\, J_{p-1}(q) S_{1,p}(q) + þ\right]$$

and $S_{\mu,\nu}(r)$ is a Lommels function having its solve

$$S_{\mu,\nu}(r) = \frac{ʒ^{\mu+1}}{(\mu-\nu+1)(\mu+\nu+1)}\, {}_1F_2\left(1, \frac{\mu+\nu+3}{2}, \frac{\mu-\nu+3}{2}, -\frac{1}{4ʒ^2}\right)$$

if $\mu \pm \nu$ is not an odd integer and

$S_{\mu,\nu}(r) = ʒ^{\mu-1}\{ 1-[(\mu-1)^2- \nu^2]\,ʒ^{-2} + [(\mu-1)^2- \nu^2][(\mu-3)^2- \nu^2]\,ʒ^{-4} + \ldots\ldots\}$

If $\mu \pm \nu$ is an odd integer.

Similarly from equation 7 on undergoing the same process we get

$$q^2 \bar{\phi}_H + \bar{U}_H + \frac{2F\lambda}{Þ} = 0 \quad \text{…………} (1.8)$$

On solving equation (1.6) and (1.8) we get

$$\bar{\phi}_H = -\frac{2}{Þ}\frac{(Fq^2 + E)\lambda}{R_a + q^4} \quad \text{…………..} (1.9)$$

$$\bar{U}_H = -\frac{2}{Þ}\frac{(F R_a - E q^2)\lambda}{R_a + q^4} \quad \text{…………..} (1.10)$$

Now by inversion formula 6.47 of Trainer (5)

$$\bar{\phi} = -\frac{4}{Þ}\sum \frac{J_p(qr)}{J^2_{þ+1}(q)}\frac{(Fq^2 + E)\lambda}{R_a + q^4} \quad \text{……………} (1.11)$$

By inversion theorem of Snedden (6)

$$\phi = -\frac{8}{\alpha}\sum_q\sum_þ \frac{J_p(qr)}{þ\, J^2_{þ+1}(q)}\frac{\lambda (Fq2 + E)\sin þ\theta}{R_a + q^4} \quad \text{……………} (1.12)$$

Similarly

$$U = -\frac{8}{\alpha}\sum_q\sum_þ \frac{J_p(qr)}{þ\, J^2_{þ+1}(q)}\frac{\lambda (F R_a - Eq^2)\sin þ'\theta}{R_a + q^4} \quad \text{……………} (1.13)$$

Now

$$T = \frac{\ell C_p C_1 u_m \sigma^2 \phi}{k}$$

Therefore

$$T = -\sum_q\sum_þ \frac{8 \ell C_p C_1 u\, m\, \sigma^2}{\alpha\; þK\alpha} \frac{Fq^2+E}{R_a+q^4} \frac{\lambda J_þ(qr) \sin þ\theta}{J^2_{þ+1}(q)} \quad \ldots\ldots (1.14)$$

$u = U u_m$

$$= -u \sum_q\sum_þ \frac{8}{\alpha þ} \frac{FR_a - Eq^2}{R_a+q^4} \frac{\lambda J_þ(qr)}{J^2_{þ+1}(q)} \quad \ldots\ldots (1.15)$$

In dimensionless form

$$T_M = \frac{\int_D \phi U\, d_A}{\int_D U\, d_A}$$

$$T_M = \frac{\sum_q\sum_þ \frac{8(Fq2+E)}{\alpha þ\; R_a+q^4} \frac{\lambda}{J^2_{þ+1}(q)} \sum_q\sum_{þ'} \frac{8}{\alpha þ} \frac{FR_a - Eq^2}{R_a+q^4} \frac{\lambda}{J^2_{þ+1}(q)} \int_0^1\int_0^{\alpha} r\, J_þ(qr) J_{þ'}(q.r) \sin þ\theta \sin þ'\theta\, dr\, d\theta}{\sum_q\sum_þ \frac{8}{\alpha þ} \frac{FR_a - Eq^2}{R_a+q^4} \frac{\lambda}{J^2_{þ+1}(q)} \int_0^1\int_0^{\alpha} r\, J_þ(q) J_þ \sin þ\theta\, d\theta\, dr}$$

Now if $þ = þ'$ $\quad \int_0^{\alpha} \sin þ\theta \sin þ'\theta\, d\theta = \frac{\alpha}{2}$

$þ \neq þ'$ $\quad = 0$

Further by equation (48) page 70 of Erdelyl (6)

$$\int_0^1 r\, J_p(q_m r) J_p(q_n r)\, dr = 0 \text{ if } m \neq n$$

$$= \frac{1}{2}[J_{þ+1}(q_n)]^2 \text{ if } m=n$$

So

$$T_M = \frac{\frac{\alpha}{2} \sum_q\sum_þ \frac{8(Fq2+E)}{\alpha þ\; R_a+q^4} \frac{\lambda}{J^2_{þ+1}(q)} \cdot \frac{4}{\alpha þ} \frac{FR_a - Eq^2\lambda}{R_a+q^4}}{\sum\sum \frac{8}{\alpha þ^2} \frac{FR_a - Eq^2}{R_a+q^4} \frac{\lambda^2}{J^2_{þ+1}}} \quad \ldots\ldots (1.16)$$

Now Nusselts number in this case is

$$N_u = \frac{\alpha/2}{2+\alpha} \cdot \frac{F-1}{T_M}$$

$$= \frac{\alpha}{4+2\alpha} \cdot \frac{(F-1)\sum_q\sum_þ (F R_a - Eq^2) \frac{\lambda^2}{\alpha^2 þ^2 J^2_{þ+1}(R_a+q^4)^2}}{\sum_q\sum_þ (Fq^2+E)\; \lambda^2 (F R_a - Eq^2)} \quad \ldots\ldots (1.17)$$

where the printed numerator also shows $R_a + q^4 \quad \alpha^2 þ^2\; J^2_{þ+1}(q)$.

If we consider only the first term we get

$$N_u = \frac{\alpha}{4+2\alpha} \frac{(F-1)(R_q+q^4)}{(Fq^2+E)} \quad \ldots\ldots (1.18)$$

Now to evaluate flow rate we here

$$\int_A U d_A = u_m \int d_A$$

$$\sum_q\sum_þ \frac{8}{\alpha þ} \frac{F R_a - Eq^2}{R_a+q^4} \frac{\lambda^2}{J^2_{þ+1}(q)} \cdot \frac{\alpha}{2} = \frac{\alpha}{2}$$

Or

$$\sum_q\sum_þ \frac{8}{\alpha þ} \frac{F R_a - Eq^2}{R_a+q^4} \frac{\lambda^2}{J^2_{þ+1}(q)} = 1$$

Or

$$\sum_q\sum_þ \frac{8}{\alpha þ} \frac{F R_a}{R_a+q^4} \frac{\lambda^2}{J^{2'}_{þ+1}(q)} - \sum_q\sum_þ \frac{8}{\alpha þ} \frac{Eq^2\lambda^2}{R_a+q^4 J^2_{þ+1}(q)} = 1$$

Or

$$E = F R_a - \frac{1}{\sum_q\sum_þ \frac{8}{\alpha þ} \frac{q^2\lambda^2}{J^2_{þ+1}(q)(R_a+q^4)}} \quad \ldots\ldots (1.19)$$

If we consider only first term, we get

$$E = F R_a - \frac{\alpha þ}{8\lambda^2 q^2}(R_a+q^4) J^2_{þ+1}(q) \quad \ldots\ldots (1.20)$$

To note the behavior how Nusselts number changes with change of E and R_a. Let us take the case of $\alpha = \pi/4$, with heat generation or no heat generation i.e. F=1 and F=0.

- PARTICULAR CASE I :

Let $\alpha = \pi/4$
Here p= 4, 10…………etc
And $\lambda 2 = .01$

The values of E and N_u corresponding to R_a for F= 0 i.e. there is no heat generations as follows.

R_a	E	N_u
0	-1286.144	47.626
10	-1289.284	48.725
10^2	-1317.544	49.327
10^3	1600.144	50.325
10^4	4426	51.279
10^5	32686	53.752

Now let α = π/2
p= 2, 6…………etc
$\lambda^2 = 4.41$

R_a	E	N_u
0	.375	99.275
10	.381	100.00
10^2	.435	101.275
10^3	.975	103.347
10^4	6.375	105.753
10^5	60.375	109.357

- PARTICULAR CASE II:

If F= 1 it is evident from (1.17) that N_u is zero throughout.

- PARTICULAR CASE III:

Let α = π/4
p= 4, 10…………etc
= 7.5883427
$\lambda^2 = (.01)$
The values of E and $N_u$corresponding to R_a are as follows:

R_a	E	N_u
0	-172.036	-11.8
5	-161.826	-15.2
10	-151.616	-19.8
15	-141.06	-23.9
20	-131.196	-27.2
25	-120.886	-32.5
30	-110.676	-37.2

Now let α = π/2
p= 2
q= 5.1356223
$\lambda^2 = 4.41$

R_a	E	N_u
0	-.025	3.247
5	9.975	3.001
10	20.975	2.738
15	30.975	2.327
20	40.975	2.001
25	50.976	1.732
30	51.967	1.431

III. REMARK AND DISCUSSION

In the present case the solution are in the form of double series. The rapidity of convergence is observed in these cases. It can be easily seen that first few terms are sufficient to give the shape of curve. Further it is observed that the result hold good for positive Raileigh number. By the curve it is seen that value of Nusselts number increases with the increase of angle. the value of Nusselts number is zero in case of F=1 and in case of small angle it becomes negative while is bigger angle it again become positive after touching zero at F= 1.

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