Brinkman Model for Unsteady Flow of A Stratified Fluid With Heat Generation And Radiation

Dr Madhava Reddy Ch
Department of Mathematics, N B K R I S T, Vidyanagar, Andhra Pradesh, India

Abstract— In this investigation, the effects of magneto-hydrodynamics, heat generation and thermal radiation have been discussed on unsteady natural convection flow in a Brinkman porous medium saturated with a chemically reacting doubly stratified fluid. Initially, the flow is described through time-dependent differential equations. These are converted into a set of coupled non-dimensional differential equations. Crank-Nicolson type scheme is employed to analyze the solution of the model. In order to analyze the usefulness of the investigation the dimensionless velocity, temperature and concentration have been presented graphically for different combinations of flow governing parameters.

Index Terms—**Brinkman porous medium, Crank-Nicolson method, Double stratification, Heat generation and Chemical reaction, Radiation, Unsteady.**

I. INTRODUCTION

In heat transfer problems, radiation plays a significant role in both the free and forced convection processes [1]. The electrical power generation, nuclear power plants, gas turbines, solar power technology, missiles etc. are some examples where radiation effects are quite considerable. A chemical reaction which depends on whether it occurs as a single-phase volume reaction or at an interface by treating it as either a homogeneous or heterogeneous is also a situation worth citing in this context.

With the emerging applications of industrial and engineering problems, several researchers are working to explore the significance of chemical reaction in fluid flow problems. The steady MHD natural free convection flow has been discussed by Srinivasacharya and Upender [2] in the presence of first order chemical reaction and thermal radiation and Jain [3] presented these similar effects with unsteady double diffusive free convective flow. Further, an extensive research has been carried out on the effect of heat generation on free convection boundary layer problems with different surface geometries such as vertical and horizontal wavy surfaces, vertical and inclined plates, rotating and stationary cylinders etc. Initially, Martin [4] analyzed the free convection flow in a vertical cylinder with internal heat generation. Effect of internal heat generation on free convection flow along a wavy surface, by taking into account variable thermal conductivity and MHD, has been investigated by Alim *et al*. [5].

The fluid saturated porous medium with the free and mixed convective transport are of enormous curiosity (For more application of porous medium, one may refer Nield and Bejan[6]). Rout *et al*. [7] considered the radiation and chemical reaction whereas Chamkha *et al*. [8] analysed the effect of internal heat absorption or generation on free convective flow through a porous medium. Shateyi and Marewo [9] obtained numerical solution of unsteady flow over a porous body with MHD, thermal radiation and chemical reaction. Recently, the problem of natural convection in a nanofluid saturated non-Darcy porous medium with the effects of suction/injection and internal heat generation has been studied by Chamka *et al*.[10].

Several researchers explored the importance of convective flow in a doubly stratified porous medium using Brinkman and Darcy–Forchheimer models, since the stratification of fluid arises due to concentration differences, temperature variations or the presence of different fluids. The free convection within a porous medium in the presence of thermal stratification has been discussed by Nakayama and Koyama [11]. Then significance of stratifications (thermal and solutal) on natural convection in a Darcian and non-Darcy porous medium has been discussed noticeably by quite a few researchers (for example see Murthy et al. [12], Lakshmi Narayana and Murthy [13], Srinivasacharya and RamReddy [14]; Ibrahim and Makinde [15] etc).

The aim of this paper is to consider the combined effects of MHD, thermal radiation and internal heat generation on unsteady free convection flow in a chemically reacting doubly stratified fluid saturated Brinkman porous medium using the Crank-Nicolson scheme. The effects of governing parameters on velocity, temperature and concentration are analyzed and shown graphically.

II. MATHEMATICAL FORMULATION

Consider the physical model and the coordinate system such that the *x*-axis is along the vertical plate and the *y*-axis is normal to the plate. A two-dimensional unsteady laminar incompressible free convective flow past a vertical plate in an electrically conducting doubly stratified fluid saturated porous medium is considered. A magnetic field of uniform strength B_0 is introduced normal to the direction of the flow. The fluid and the plate are assumed to be at the constant temperature and constant concentration initially at $t'=0$ whereas the temperature and concentration of the plate are changed to T_w and C_w respectively for the time $t'>0$. The ambient medium is assumed to be vertically linearly stratified with respect to both temperature and concentration $T_\infty(x) = T_{\infty,0}+Ax$ and $C_\infty(x) = C_{\infty,0}+Bx$ respectively (See Ref [13]-[14]).

Under the usual Boussinesq's and Brinkman porous medium approximation, under above assumptions, the governing boundary layer equations for the flow are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\varepsilon\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\varepsilon\nu\frac{\partial^2 u}{\partial y^2}+\varepsilon^2 g\left[\beta_T(T'-T_\infty(x))+\beta_C(C'-C_\infty(x))\right]-\frac{\varepsilon^2\mu}{k}u-\frac{\sigma B_0^{\,2}}{\rho}\varepsilon u \tag{2}$$

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\alpha\frac{\partial^2 T'}{\partial y^2}+\frac{Q_0}{\rho C_P}\left(T'-T_\infty(x)\right)+\frac{16\sigma^* T'^3_{\infty,0}}{3\rho C_P k_0}\frac{\partial^2 T'}{\partial y^2} \tag{3}$$

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=D\frac{\partial^2 C'}{\partial y^2}-R(C'-C_\infty(x)) \tag{4}$$

where u and v are Darcy velocity components along the x and y directions respectively, ρ is the density, g is the acceleration due to gravity, C_p is the specific heat, μ is the coefficient of viscosity, σ is the electrical conductivity, k is the permeability, ε is the porosity, T' is the temperature, C' is the concentration, β_T and β_C are the coefficients of thermal and solutal expansions, α is the thermal diffusivity, D is the mass diffusivity. The third term on RHS of Eq. (3) is obtained by using Rosseland approximation [1] and $T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty$.

The boundary conditions are

$$t'\le 0: u(x,y,t')=0,\ v(x,y,t')=0,\ T'(x,y,t')=T_\infty(x),\ C'(x,y,t')=C_\infty(x)$$

$$t'>0: u(x,0,t')=0,\ v(x,0,t')=0,\ T'(x,0,t')=T_w,\ C'(x,0,t')=C_w$$

$$u(0,y,t')=0,\ v(0,y,t')=0,\ T'(0,y,t')=T_{\infty,0},\ C'(0,y,t')=C_{\infty,0}$$

$$u(x,\infty,t')\to 0,\ T'(x,\infty,t')\to T_\infty(x),\ C'(x,\infty,t')\to C_\infty(x) \tag{5}$$

Introducing the following non-dimensional variables

$$X=\frac{x}{L},\ Y=\frac{y}{L}Gr^{1/4},\ U=\frac{uL}{\upsilon}Gr^{-1/2},\ V=\frac{vL}{\upsilon}Gr^{-1/4},\ t=\frac{t'\upsilon}{L^2}Gr^{1/2},\ T=\frac{T'-T_\infty(x)}{T_w-T_{\infty,0}},\ C=\frac{C'-C_\infty(x)}{C_w-C_{\infty,0}}$$

into equations (1)-(4), we obtain

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{6}$$

$$\varepsilon\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\varepsilon\frac{\partial^2 U}{\partial Y^2}+\varepsilon^2\left(T+NC\right)-\frac{\varepsilon^2}{DaGr^{1/2}}U-\frac{\varepsilon M}{Gr^{1/2}}U \tag{7}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{\Pr}(1+N_R)\frac{\partial^2 T}{\partial Y^2}+$$

$$QT-\varepsilon_1 U \qquad (8)$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2}-R^{'}C-\varepsilon_2 U \qquad (9)$$

The non-dimensional conditions associated with the reduced equations are

$$t\le 0: U=0,\ V=0,\ T=0,\ C=0$$

$$t>0: U(X,0,t)=0,\ V(X,0,t)=0,$$
$$T(X,0,t)-1+\varepsilon_1 X=0,$$
$$C(X,0,t)-1+\varepsilon_2 X=0$$

$$U(0,Y,t)=0,\quad V(0,Y,t)=0,$$
$$T(0,Y,t)=0,\quad C(0,Y,t)=0$$

$$U(X,\infty,t)\to 0,\ T(X,\infty,t)\to 0,$$

$$C(X,\infty,t)\to 0 \qquad (10)$$

where $Gr=g\beta_T L^3(T_w-T_{\infty,0})/\upsilon^2$ is the Grashof number, $N=\beta_C(C_w-C_{\infty,0})/(\beta_T(T_w-T_{\infty,0}))$ is the buoyancy ratio, $Da=k\upsilon/(\mu L^2)$ is the Darcy number, $M=\sigma B_0^2 L^2/(\rho\upsilon)$ is the magnetic parameter, $N_R=16\sigma^* T_\infty^{'3}/(3kk_0)$ is the thermal radiation parameter, $Q=Q_0L^2Gr^{-1/2}/(\rho C_p\upsilon)$ is heat generation parameter, $Pr=\upsilon/\alpha$ and $Sc=\upsilon/D$ are the Prandtl and Schmidt numbers, $R^{'}=RL^2Gr^{-1/2}/\upsilon$ is dimensionless chemical reaction parameter, $\varepsilon_1=AL/(T_w-T_{\infty,0})$ and $\varepsilon_2=BL/(C_w-C_{\infty,0})$ are the thermal and solutal stratification parameters.

The non-dimensional forms of physical parameters of interest such as local skin friction, Nusselt number and Sherwood number are obtained as

$$\tau_X=Gr^{3/4}\left(\frac{\partial U}{\partial Y}\right)_{Y=0},\quad Nu_X=-Gr^{1/4}(1+N_R)\frac{X\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{1-\varepsilon_1 X}$$

and $Sh_X=-Gr^{1/4}\dfrac{X\left(\frac{\partial C}{\partial Y}\right)_{Y=0}}{1-\varepsilon_2 X}$

(11)

The non-dimensional forms of average skin friction, Nusselt number and Sherwood number are obtained as

$$\bar{\tau}=Gr^{3/4}\int_0^1\left(\frac{\partial U}{\partial Y}\right)_{Y=0}dX,$$

$$\overline{Nu}=-Gr^{1/4}(1+N_R)\int_0^1\frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{1-\varepsilon_1 X}dX \quad \text{and}$$

$$\overline{Sh}=-Gr^{1/4}\int_0^1\frac{\left(\frac{\partial C}{\partial Y}\right)_{Y=0}}{1-\varepsilon_2 X}dX \qquad (12)$$

III. RESULTS AND DISCUSSION

The implicit finite difference scheme known as Crank-Nicolson type scheme (see Loganathan *et al.* [16] and citations therein) is used to solve the Eqs. (6)-(9) along with B.C.(10) and the results of non-dimensional velocity, temperature and concentration are analyzed. To test the accuracy of the present results, the velocity profiles for *Pr=0.71 Sc=0.94, N=1.0, Q = 0.0,* N_R = *0.0, R' = 0.0* and *M=0.0* of the present result are compared with the existing solution of Gebhart and Pera [17] in the absence of steady doubly stratified porous medium. In order to analyze the combined effects of physical parameters, the computations are taken for fixed values of *Pr=0.71, Sc=0.22, Gr=5.0, ε=0.6, M=1.0* and *N=1.0*. These computations are carried out at *t=2.0.*

The Fig. 3.1 show the variation of non-dimensional velocity, temperature and concentration in the presence and/or absence of stratification (both thermal and solutal) parameters for fixed values of other parameters. Numerical studies indicated that as the thermal stratification parameter increases, the time taken to reach the steady state is decreasing. Also as the solutal stratification parameter increases, the time taken to reach the steady state is increasing.

From Fig. 3.1 (a), it is observed that, the velocity decreases with an increase in the value of both thermal and solutal stratification parameters. The reason for this is that the presence of thermal stratification reduces the effective convective potential between the heated plate and the ambient fluid in the medium. Hence, the thermal stratification effect reduces the velocity in the boundary layer. It is seen from Fig. 3.1 (b) that, the temperature of the fluid in the medium increases slightly with an increase in the value of the solutal stratification parameter and decreases with an increase in the value of thermal stratification parameter. This is because of reduction in the effective temperature difference between the plate and the ambient fluid in the presence the thermal stratification effect. From Fig. 3.1 (c), it can be found that, the concentration of the fluid is increases slightly by increasing the thermal stratification parameter and decreases with an increase in the value of solutal stratification parameter.

The variations of non-dimensional velocity, temperature and concentration for different values of chemical reaction and internal heat generation parameters, are plotted in the Fig. 2.2 in the presence and/or absence of chemical reaction and internal heat generation parameters for fixed values of other parameters and observed that, in the case of absence of these parameters, the value of time to attain the steady state is low and in the case of presence of these parameters the time taken to attain the steady state decreases with an increase in the value of both chemical reaction and internal heat generation parameters.

Fig. 3.2 (a) presents the velocity profile and it is observed that, the velocity of the fluid is diminishing as heat generation parameter increases and is increasing with increase in the value of chemical reaction parameter. From Fig. 3.2 (b), it can be observed that, the temperature of the fluid in the medium is increased with an increase in the value of both heat generation and chemical reaction parameters. This can physically be explained because the internal heat generation results in an increase in the buoyancy forces, which in turn reduces flow along the plate. From Fig. 3.2 (c), it can be found that, the concentration of the fluid is decreased by increasing both the heat generation and chemical reaction parameters. An increase in chemical reaction will suppress species concentration. That the concentration of the species gradually changes from higher value to the lower value and matches the boundary condition on concentration as $\eta \to \infty$ in the solutal boundary layer.

The Fig. 3.3 display the variation of non-dimensional velocity, temperature and concentration for different values of Darcy and radiation parameters and fixed values of other parameters. Here in we notice that, the time to reach the steady state decreases with an increase in the value of both Darcy and radiation parameters.

Fig. 3.3 (a) presents the velocity profile and indicates that, the velocity of the fluid is expanding as both Darcy parameter and radiation parameter increases. With increasing permeability, the porous matrix structure becomes less and less prominent and in the point of accumulation, as $Da \to \infty$ $\left(i.e., -\frac{\varepsilon^2}{DaGr^{1/2}} U \to 0 \text{ and } \epsilon = 1 \right)$, the problem become an electrically conducting doubly stratified fluid in the absence of porous medium. From Fig. 3.3 (b), it can be observed that, the temperature of the fluid in the medium decreases with an increase in the value of Darcy parameter and increases with an increase in the value of the radiation parameter. From Fig. 3.3 (c), it can be found that, the concentration of the fluid decreases by increasing both the Darcy parameter and radiation parameter.

IV. CONCLUSION

In this paper, the combined effects of radiation, heat generation and chemical reaction on unsteady MHD free convective flow in a doubly porous medium are discussed. The governing equations are solved numerically by the implicit finite difference method of Crank-Nicolson type. The main findings are summarized as follows:

(a) As the thermal stratification parameter increases, the velocity and temperature decrease whereas the concentration increases. With an increase in the solutal stratification parameter, the velocity and concentration decrease whereas the temperature increases.

(b) As the chemical reaction parameter increases, the velocity and temperature increase whereas the concentration decreases. As the value of heat generation parameter increases, the velocity and concentration decrease whereas the temperature increases.

(c) As the Darcy parameter increases, the velocity increases, whereas the temperature and concentration decrease. And as the radiation parameter increases, the velocity and temperature increase, whereas the concentration decreases.

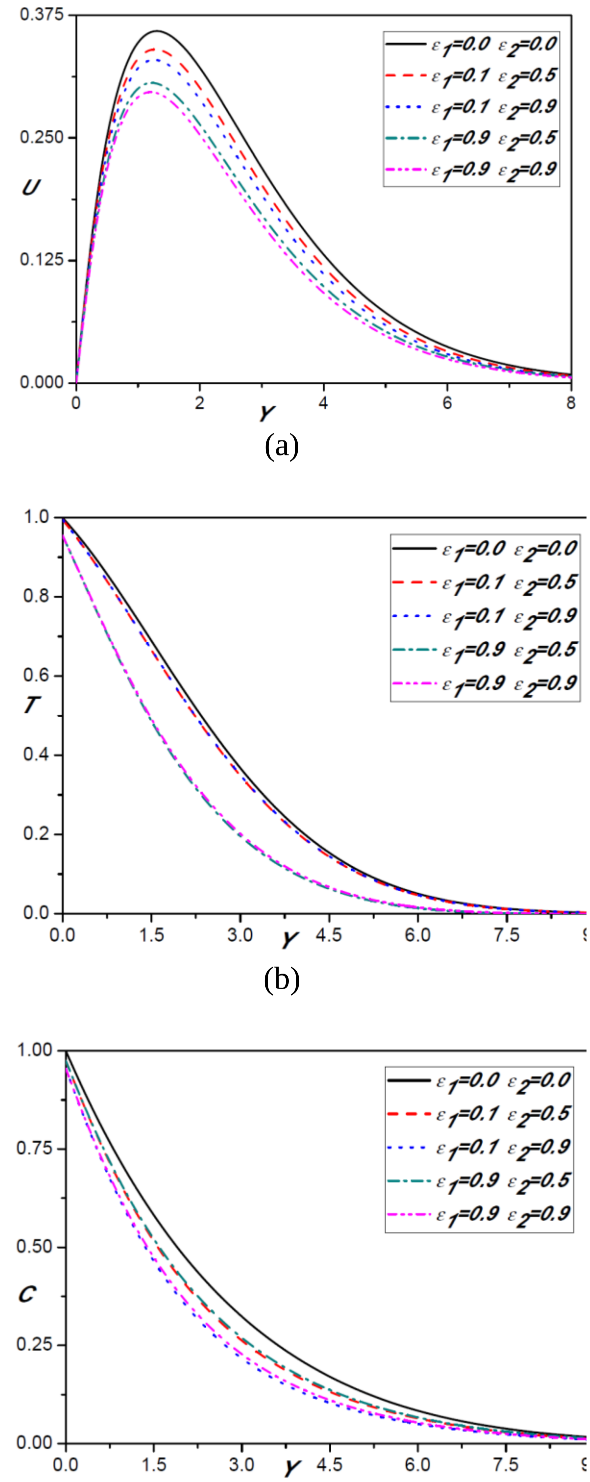
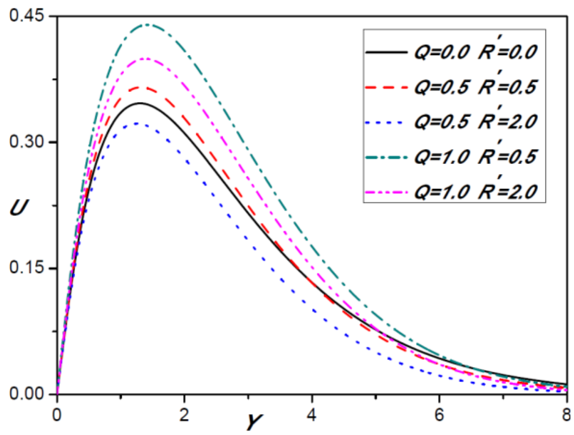
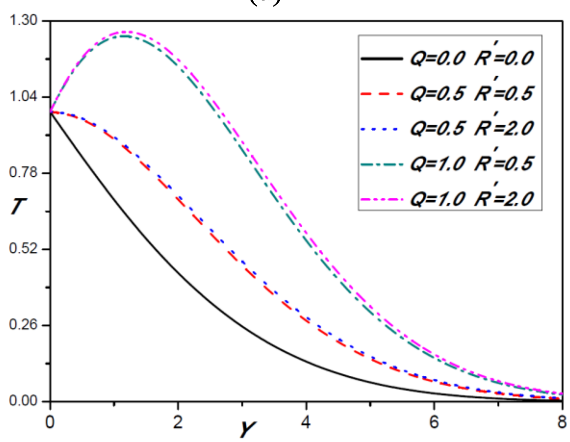


Fig-3.1: Effect of thermal and solutal stratification parameters on (a) Velocity, (b) Temperature and (c) Concentration profiles at $X=1$.



(a)



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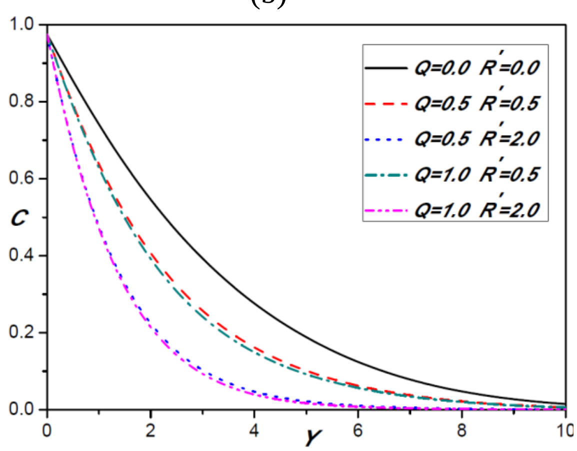
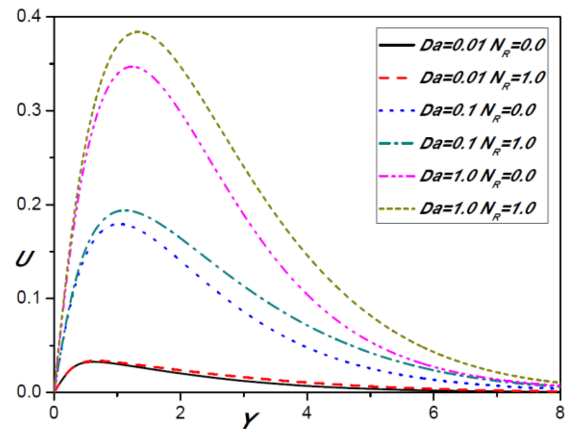
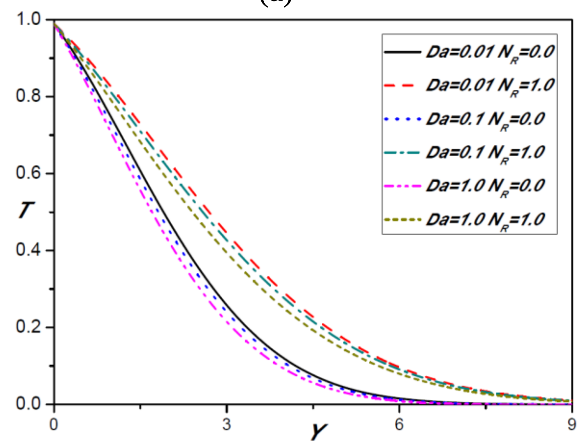


(c)

Fig-3.2: Effect of heat generation and chemical reaction parameters on (a) Velocity, (b) Temperature and (c) Concentration profiles at $X=1$.



(a)



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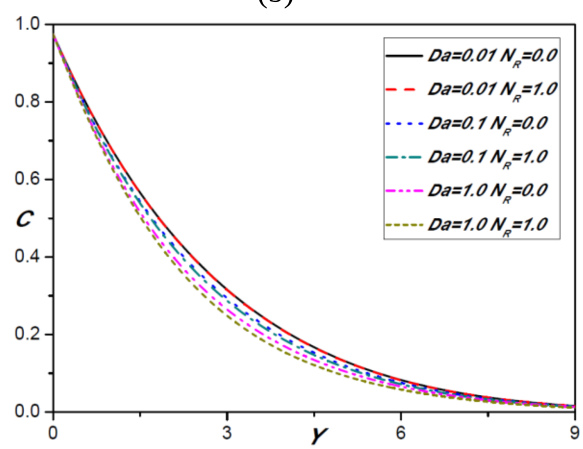

(c)

Fig-3.3: Effect of Darcy and radiation parameters on (a) Velocity, (b) Temperature and (c) Concentration profiles at $X=1$.

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