Propagation of Small Disturbances in a Visco Elastic Fluid Contain In an Infinite Cylinder Having Sector as Its Cross-Section Due To Slow Angular Motion of the Circular Plate Fixed At the Base

DR. SARVESH NIGAM
Assistant Professor, *Department of Mathematics, Pt. J. L. N. P. G. College Banda*

Abstract- In this paper we study the propagation of disturbances in the visco elastic fluid contained in an infinite cylinder having the cross section to be sector due to slow angular motion about the z-axis is a line passing through the center whose sector is a point. The base is a circular plate which is fixed to the cross-section and is given the rotatory motion about z-axis and the angular motion about the z-axis for t>0 is represented by W (t) where W (t) = W_0 sin nt, and W_0 is a constant.

I. INTRODUCTION

Lundquist (1) Bhatnagar and Kumar (2) and kumar (3, 4) have considered the propagation of disturbances in viscous incompressible fluids in the presence of magnetic field. P. N. Srivastava had discussed propagation of disturbances in an idealized viscoelastic fluid occupying the space z>0 due to the slow angular motion of a disc $x^2+y^2 = a^2$, z=0, P. N. Srivastava (5) considered the propagation of disturbances in the visco elastic fluid contained in an infinite cicular cylinder when only relaxation phenomenon of the fluid was considered.

In this paper we study the propagation of disturbances in the visco elastic fluid contained in a infinite cylinder having the cross section to be sector due to flow angular motion about the z-axis .The z-axis is a line passing through the center whose sector is a point. The base is a circular plate which is fixed to the cross section and is given the rotatory motion about z-axis and the angular motion about the z-axis fot t>0 is represented by W (t) where W (t) = W_0 sin nt, and W_0 is a constant.

II. FORMULATION OF THE PROBLEM

The stress strain rate relation for an idealized visco-elastic fluid which shows wessemberg rising effect and when only relaxation phenomenon is taken into consideration is of the form given by oldroyd(8)

$$S_{ik} + \lambda\left[\frac{\partial}{\partial t} S_{ik} + \nu_j S_{ik,} - \nu_{ij} S_{ik} - \nu_{kj} S_{ij} + \nu_{ij} S_{ik}\right] = 2\mu \ell_{ik} \qquad (1)$$

Where λ is the relaxation time constant and μ is the coefficient of viscosity.

Now as the movement is about z-axis hence we can assume $\nu_r = \nu_z = 0$ and Now since the disc is rotating about z-axis so $\left(\frac{\partial b}{\partial z}\right) = 0$

Under these assumptions on solving we get

$$\lambda\frac{\partial^2 \nu}{\partial t^2} + \frac{\partial \nu}{\partial t} = \nu\left[\frac{\partial^2 \bar{\nu}}{\partial r^2} + \frac{1}{r}\frac{\partial \nu}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \bar{\nu}}{\partial \theta^2} + \frac{\partial^2 \bar{\nu}}{\partial z^2} - \frac{\nu}{r^2}\right] \qquad (2)$$

Where $\nu = \frac{\mu}{r}$ is the kinematic coefficient of viscosity.

III. BOUNDARY CONDITIONS

$$\begin{array}{ll} \nu = 0 & t=0 \\ \frac{\partial}{\partial t}\nu = 0 & t=0 \\ \nu = rW0\sin nt & 0 \leq r < 1 \quad z=0 \\ \nu = 0 & \left\{\begin{array}{l} r=0,\ r=1 \\ \theta=0,\ \theta=\alpha \end{array}\right\} \quad z \geq 0 \\ \nu = 0 & \text{when } z \to \infty \end{array}$$

IV. SOLUTION OF THE PROBLEM

Multiply equation (2) by $e^{-þt}$ and integrate the whole equation within the limits t=0 to t=∞

$$\lambda\int_0^\infty \frac{\partial^2 v}{\partial t^2}. e^{-þt}dt + \int_0^\infty \frac{\partial v}{\partial t}. e^{-þt}dt$$
$$= \nu\int_0^\infty \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{\partial^2 v}{\partial ʒ^2} - \frac{v}{r^2}\right) e^{-þt}dt$$

OR

$$\lambda \left(e^{-þt}\frac{\partial v}{\partial t}\right)_0^\infty + \lambda þ\int_0^\infty e^{-þt}\frac{\partial v}{\partial t}dt + \int_0^\infty \frac{\partial v}{\partial t}. e^{-þt}dt$$
$$= \nu\int_0^\infty \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{\partial^2 v}{\partial ʒ^2} - \frac{v}{r^2}\right) e^{-þt}dt$$

$$(\lambda þ+1)\, þ\, \bar{v} = \upsilon \left[\frac{\partial^2 \bar{v}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{v}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \bar{v}}{\partial \theta^2} + \frac{\partial^2 \bar{v}}{\partial ʒ^2} - \frac{\bar{v}}{r^2}\right] \quad (3)$$

Where $\bar{v} = \int_0^\infty v e^{-þt}\, dt$

i.e. ν is the Laplace transform of ν

OR
$$\frac{\partial^2 \bar{v}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{v}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \bar{v}}{\partial \theta^2} + \frac{\partial^2 \bar{v}}{\partial ʒ} - \frac{\bar{v}}{r^2} = \frac{(\lambda þ+1)}{\upsilon}\, þ\, \bar{v} \quad (4)$$

Let $\sqrt{\frac{(\lambda þ+1)\, þ}{\upsilon}} = k$

So equation (4) transform to

$$\frac{\partial^2 \bar{v}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{v}}{\partial r} - \frac{\bar{v}}{r^2} + \frac{\partial^2 \bar{v}}{\partial ʒ^2} + \frac{1}{r^2}\frac{\partial^2 \bar{v}}{\partial r} - k^2 v = 0 \quad (5)$$

Multiplying equation (5) by sinþθ and integrate within limits 0 to α

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} - \frac{(p^2+1)\, V}{r^2} + \frac{\partial^2 V}{\partial ʒ^2} - k^2 V = 0 \quad (6)$$

Where

$$V = \int_0^\alpha v \sin þ_1\theta d\theta$$
$$\& \; þ_1\alpha = (2n+1)\, \pi \quad (7)$$

Further let $(þ^2_1+1) = P^2$

Now let us introduce the finite Hankel's transform defined by

$$\bar{V} = \int_0^1 V r\, J_þ\, (qr) dr \quad (8)$$

Where q's are the roots of the equation Jþ (q) = 0

Where $J_þ$ (z) is the Bessel function of first kind.

Now multiplying equation (6) by rJþ (qr) and integrating within the limits 0 to 1 and using v=0 and r=1, we get

$$\frac{d^2\bar{V}}{d ʒ^2} = \overline{(k^2+q^2)\, V} \quad (9)$$

Now $\bar{V}$=0 and Z= ∞

$$\bar{V} = \int 1 r\, J_þ\, (qr)\, dr \left[\int_0^\infty W_0 \sin nt \int_0^\alpha þ_1\theta. d\theta e^{-þt}dt\right] dr$$
$$= \frac{2\, W_0 n}{þ_1\, (þ^2+n^2)} \int_0^1 r^2\, J_þ\, (qr)\, dr$$

Where $S_{\mu, \upsilon}$ (Z) is the Lommel function.
The solution of the equation (10) is

$$\bar{V} = \frac{n\, W_0\, N}{(þ^2+ n^2)}\; e^{-(q2+k^{\wedge}2)\,^{\wedge}1/2}\, ʒ$$

Hence know using Honkel's inversion theorem .Snedden (4) we have

$$V = \frac{2nW_0\, N}{(þ^2+ n^2)} \sum_q \frac{Jþ\, (qr).}{J^2{}_{þ+1}(q)}\, e^{-(q2+k^{\wedge}2)\,^{\wedge}1/2}\, ʒ \quad (11)$$

Where the summation is to be extended on the positive roots of $J_þ$ (q) = 0

By sine inversion
$$\bar{v} = \frac{4\, nW_0}{\alpha\, (þ^2+ n^2)} \sum_q\sum_þ \frac{N \sin þ_1\theta\, J_þ\, (qr)}{J^2{}_{þ+1}(q)} . e^{-(q2+k^{\wedge}2)\,^{\wedge}1/2}\, ʒ \quad (12)$$

Now by using inversion theorem of Laplace transform

$$v = \sum_q\sum_{þ1} \frac{4\, nW_0 N}{2\pi i\alpha\, (þ^2+ n^2)} \frac{\sin þ_1\theta\, J_þ\, (qr)}{J^2{}_{þ+1}(q)}$$
$$\int_{c-i\infty}^{c-i\infty} e^{-[q2+\lambda/\upsilon þ(þ+1/\lambda)]\,^{\wedge}1/2}\, ʒ.\; e^{-þt}dþ \quad (13)$$

$$= \sum_q \sum_{þ1} \frac{8W_0}{þ_1 q^2} \frac{\sin þ_1\theta\, J_þ(qr)\, [(p+1)\, J_þ(q)\, S_{1,þ-1}(q)}{J^2{}_{þ+1}(q)}$$

$$- J_{þ-1}(q)\, S_{2,þ}(q)\,.\int_{c-i\infty}^{c+i\infty} \frac{n}{p^2+n^2} e^{þt}\, e^{-z[\lambda/\upsilon þ(þ+1/\lambda)+q2]\,\hat{}\,1/2}]\, dþ \quad (14)$$

By applying residue theorem, we get

$$v = \sum_{Þ1} \sum_{q} \frac{8W_0 \sin þ_1\theta\, J_þ\,(qr)}{\alpha þ 1q2\, J^2{}_{þ+1}(q)} [(p+1)\, J_þ\,(q)\, S_{1,\,þ-1}(q)$$

$$- J_{þ-1}(q)\, S_{2,\,þ}\,(q)].\, e^{-(\lambda z\hat{}2/\upsilon)\,1/2}\, \theta \sin [nt- \varpi\, (\lambda z^2/\upsilon)^{1/2}]$$

$$+\sum_{Þ1} \sum_{q} \frac{nb8W_0 \sin þ_1\theta\, J_þ(qr)}{\pi\alpha þ_1 q^2\, J^2{}_{þ+1}(q)} [(p+1)\, J_þ(q)\, S_{1,þ-1}(q)$$

$$- J_{þ-1}(q)\, S_{2,þ}(q)\,]\left(\int_{-1}^{1} \frac{e^{-(t/2\lambda+bxt]} \sin(\lambda z^2 b^2/\, e^{-(t/2\lambda)})^{1/2}(1-x^2)^{1/2} dx}{n^2+(bx+1/2\lambda)^2}\right. \quad (15)$$

Where $b=\frac{(1-4\lambda\upsilon q^2)^{1/2}}{4\lambda^2}$ (16)

$$\theta= \frac{1}{\sqrt{2}}[\{(\frac{\upsilon q^2}{\lambda} - r^2)^2 + \frac{n^2}{\lambda^2}\}^{1/2}+ (\frac{\upsilon q2}{\lambda} - n^2)]^{1/2} \quad (17)$$

$$\phi= \frac{1}{\sqrt{2}} [\{(\frac{\upsilon q^2}{\lambda} - n^2)^2 + \frac{n^2}{\lambda^2}\}^{1/2}+ (\frac{\upsilon q2}{\lambda} - n^2)]^{1/2} \quad (18)$$

If we take limit as $\lambda = 0$ i.e. the relaxation phenomenon of the fluid is not considered then on putting $y=1/x$ it becomes the ordinary viscous fluid and the result then becomes

$$v_{vis} = \lim_{\lambda\to 0} v$$

$$= \sum_{Þ1} \sum_{q} \frac{8W_0 \sin þ_1\theta\, J_þ\,(qr)}{\alpha þ_1 q^2\, J^2{}_{þ+1}(q)} [(p+1)\, J_þ\,(q)\, S_{1,\,þ-1}(q)$$

$$- J_{þ-1}(q)\, S_{2,\,þ}\,(q)].\, e^{-(z\hat{}2/2\upsilon)\,\hat{}1/2} \theta_1 \sin [nt-((z^2/2\upsilon)^{1/2}\, \varpi_1]$$

$$+ \sum_{Þ1} \sum_{q\alpha 1/2(\lambda v)\,1/2} \frac{nb8W_0 \sin þ_1\theta\, J_þ(qr)}{\pi\alpha þ_1 q^2\, J^2{}_{þ+1}(q)} [(p+1)\, J_þ(q)\, S_{1,þ-1}(q)\,)$$

$$- J_{þ-1}(q)\, S_{2,þ}(q)] \int_0^\infty \frac{e^{-(vq\hat{}2+y]t} \sin(z^2 y/\upsilon)^{1/2} dy}{n^2+(y+ vq^2)^2} \quad (19)$$

Where

$$\theta_1= \{(\upsilon\, ^2q^2 + n^2)^{1/2} + \upsilon q2\ \}^{1/2}$$

$$\&\ \ \phi= \{(\upsilon\, ^2q^4 + n^2)^{1/2} - \upsilon q^2\ \}^{1/2}$$

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