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On The Temperature Distribution of a Viscous Liquid under Oscillatory Rate of Heat Addition Superposed On the Steady Temperature of Incompressible Fluid between Curvillinear Quadilateral Cross Sectional Cylinder

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Abstract -- Solution for temperature distribution in a circular pipe have been given by various authors notably Gretz, Nusselts, Goldstein. All these are cited in (1) Krishna Lala (2) considered the temperature distribution in co-axial cylinders. (3) S. N. Bose considered temperature distribution in channel bounded by coaxial circular pipe for viscous incompressible fluid flowing through it by neglecting the dissipation due to friction when an oscillatory rate of heat addition is superposed on the steady temperature.

I. INTRODUCTION

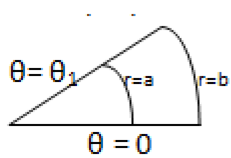

Fig. 1: Curvilinear Quadrilateral

In this paper expression for the temperature distribution in a channel bounded by curvilinear quadrilateral cross-sectional cylinder similarly situated for viscous incompressible fluid flowing through it neglecting the dissipation due to friction when an oscillatory rate of heat addition is superposed on the steady temperature.

II. ENERGY EQUATION AND ITS SOLUTION

The equation of energy in the present case is

$$\frac{\partial T}{\partial t} = \frac{1}{\varrho C_v}\frac{\partial Q}{\partial t} + K'\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial t} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right) \quad \text{..................(1.1)}$$

Where $K' = \dfrac{K}{\varrho C_v}$ is a constant and dissipation due to friction is neglected.

Now we assume that

$$\frac{1}{\varrho C_v}\frac{\partial Q}{\partial t} = \sum_{n=1}^{\infty} a_n e^{int} \quad \text{..........................(1.2)}$$

and

$$\frac{}{\varrho C_v} \quad \frac{T = T_o + \sum_{n=1}^{\infty} T_n(r)\, e^{int}}{\partial t} \quad \text{..........................(1.3)}$$

where a_n and T_n are real and T_n is a function of r only.

Substituting equation (1.2) and (1.3) and comparing the terms of the same family, the differential equations are

$$\frac{d^2 T_0}{dr^2} + \frac{1}{r}\frac{dT_0}{dr} \quad \text{..................(1.4)}$$

$$\frac{d^2 T_n}{dr^2} + \frac{1}{r}\frac{dT_n}{dr} + \frac{1}{r^2}\frac{d^2 T_n}{d\theta^2} - \frac{i_n T_n}{K'} + \frac{a_n}{K'} = 0 \quad \text{..................(1.5)}$$

Integrating 1.4 we have

$$T_o = A + B\log r \quad \text{.............. (1.6)}$$

Before supposing the oscillatory flow, we must have the fully developed steady motion with these conditions and with following boundary conditions.

$T_o = T_1$ where $r = r_1$
$T_o = T_2$ where $r = r_2$

And A and B in 1.6 are determinate. Thus

$$T_o = \frac{T_1 \log r_2/r + T_2 \log r/r_1}{\log r_2/r}$$

III. BOUNDARY CONDITION

$T = T_1 e^{int} + T_1$ $\quad t > 0$ when $r = r_1$

$T = T_2 e^{int} + T_2$ $\quad$ when $r = r_2$

$T = 0$ $\quad t > 0$ when $\theta = 0$

$\theta = \alpha$

$T = 0$ $\quad$ when $t = 0$

Now multiply equation 1.5 by $\sin þ\theta$ we get

$$\left[\frac{\partial^2 T_n}{\partial r^2} + \frac{1}{r}\frac{\partial T_n}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T_n}{\partial \theta^2}\right]\sin þ\theta \; -$$

$$\frac{i_n T_n}{K'}\sin þ\theta + \frac{a_n}{K'}\sin þ\theta = 0 \qquad \text{..................(1.7)}$$

Now since T_n is zero $\theta=0$ and $\theta=\alpha$ and if $þ\alpha = (n+1)\pi$

We have $\int_0^\alpha \sin þ\theta \frac{\partial^2 T_n}{\partial \theta^2} d\theta = -þ^2 \int_0^\alpha T_n \sin þ\theta \, d\theta = -þ^2 \overline{T}$

If we write $\overline{T} = \int_0^\alpha T_n \sin þ\theta \, d\theta$ .... (1.8)

We have an integrating equation 1.7 within the limits 0 to α we get

$$\left[\frac{\partial^2 \overline{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{T}}{\partial r} - \frac{þ^2 \overline{T}}{r^2}\right] - \frac{i_n \overline{T}}{K'} + \frac{2 a_n}{þ K'} = 0 \qquad \text{........ (1.9)}$$

If p's are the roots of the equation

$þ\alpha = (2n+1)\pi$ .....(1.10)

Now multiply the whole equation 1.9 by $r\beta_þ(qr)$ where $\beta_þ(qr)$ is given by

$\beta_þ(qr) = J_þ(qr)\,\gamma_þ(qr_1) - \gamma_þ(qr)\, J_þ(qr_1)$ (1.11)

where q's are the roots of the equation

$\beta_þ(q) = J_þ(qr_2)\,\gamma_þ(qr_1) - \gamma_þ(qr_2)\, J_þ(qr_1)$ .... (1.12)

where $J_þ(z)$ and $\gamma_þ(z)$ are Bessel functions of first and second kind and q's are the positive roots of the equation.

$\beta_þ(q) = J_þ(qr_2)\,\gamma_þ(qr_1) - \gamma_þ(r_1 q) - \gamma_þ(r_1 q)\, J_þ(r_2 q)$

$$\text{we get } r\beta_þ(qr)\left[\frac{\partial^2 \overline{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{T}}{\partial r} - \frac{þ^2 \overline{T}}{r^2}\right] -$$

$$\frac{i_n \overline{T}}{K'} r\beta_þ(qr) + \frac{2 \overline{a_n}}{þ K'} r\beta_þ(qr) = 0 \qquad \text{.... (1.13)}$$

Now my equation 6.58 of Trainter – (4)

$$\int_{r_1}^{r_2}\left[\frac{\partial^2 \overline{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{T}}{\partial r} - \frac{þ^2 \overline{T}}{r^2}\right] r\beta_þ(qr) =$$

$$\frac{4}{þ\pi}\left[(T_1 e^{int} + T_1)\frac{J_þ(r_1 q)}{J_þ(r_2 q)} - (T_2 e^{int} + T_2)\right] N - q^2 \overline{T}_H \qquad \text{......(1.14)}$$

since $\quad \frac{2}{þ}(T_1 e^{int} + T_1)\int_{r_1}^{r_2} r\beta_þ(qr)\, dr$

&

$\frac{2}{þ}(T_2 e^{int} + T_2)\int_{r_1}^{r_2} r\beta_þ(qr)\, dr$

Are the values of T_H at $r=r_1$ and $r=r_2$ respectively and T_H is the Hankel transform of T and

$$N = \int_{r_1}^{r_2} r\beta_þ(qr)\, dr$$

$$= r_2 s_{1,\text{þ}}(r_2 q)\left\{ J_{\text{þ}}(r_1 q)\gamma_{\text{þ}-1}(r_2 q) - J_{\text{þ}-1}(r_2 q)\gamma_{\text{þ}}(r_1 q) \right\} +$$

$$r_1 s_{1,\text{þ}}(r_1 q)\left\{ \gamma_{\text{þ}}(r_1 q) J_{\text{þ}-1}(r_1 q) - J_{\text{þ}}(r_1 q)\gamma_{\text{þ}-1}(r_1 q) \right\}$$

..(1.15)

On integrating equation 1.13 within the limits r_1 to r_2 , we get

$$\frac{4}{\text{þ}\pi}\left[(T_1 e^{int} + T_1)\frac{J_{\text{þ}}(r_1 q)}{J_{\text{þ}}(r_2 q)} - (T_2 e^{int} + T_2)\right] N -$$

$$q^2 \bar{T}_H - \frac{i_n \bar{T}_H}{K'} + \frac{2 a_n}{\text{þ} K'} N = 0$$

which gives

$$\bar{T}_H = \frac{\left\{\frac{4}{\text{þ}\pi}\left[(T_1 e^{int} + T_1)\frac{J_{\text{þ}}(r_1 q)}{J_{\text{þ}}(r_2 q)} - (T_2 e^{int} + T_2)\right] + \frac{2 a_n}{\text{þ} K'}\right\} N}{(q^2 + in/K')} \quad \text{.....(1.16)}$$

Now by inversion formula equation 6.53 of Trainter (4)

$$\bar{T} = \frac{\pi^2}{2}\sum_q \frac{q^2 J^2_{\text{þ}}(r_2 q)\, \beta_{\text{þ}}(\bar{qr})\, \bar{T}_H}{J^2_{\text{þ}}(r_1 q) - J^2_{\text{þ}}(r_2 q)} \quad \text{........(1.17)}$$

Now by sine inversion theorem we get

$$T_n = \frac{2\pi^2}{\alpha}\sum_q \sum_{\text{þ}} \frac{q^2 J^2_{\text{þ}}(r_2 q)\, \beta_{\text{þ}}(qr)}{J^2_{\text{þ}}(r_1 q) - J^2_{\text{þ}}(r_2 q)}\, \frac{\sin \text{þ}\theta\ \bar{T}_H}{\text{þ}} \quad \text{.... (1.18)}$$

on substituting the value of T_H in equation 1.18 we get

$$T_n = \frac{2\pi^2}{\alpha}\sum_q \sum_{\text{þ}} \frac{q^2 J^2_{\text{þ}}(r_2 q)\, \beta_{\text{þ}}(qr)}{J^2_{\text{þ}}(r_1 q) - J^2_{\text{þ}}(r_2 q)}\, \frac{\sin \text{þ}\theta}{\text{þ}}$$

$$\frac{X \left\{\frac{4}{\text{þ}\pi}\left[(T_1 e^{int} + T_1)\frac{J_{\text{þ}}(r_1 q)}{J_{\text{þ}}(r_2 q)} - (T_2 e^{int} + T_2)\right] + \frac{2 a_n}{\text{þ} K'}\right\}}{(q^2 + in/K')}$$

$$X\left(r_2 s_{1,\text{þ}}(r_2 q)\left\{ J_{\text{þ}}(r_1 q)\gamma_{\text{þ}-1}(r_2 q) - J_{\text{þ}-1}(r_2 q)\gamma_{\text{þ}}(r_1 q) \right\} + \right.$$

$$\left. r_1 s_{1,\text{þ}}(r_1 q)\left\{ \gamma_{\text{þ}}(r_1 q) J_{\text{þ}-1}(r_1 q) - J_{\text{þ}}(r_1 q)\gamma_{\text{þ}-1}(r_1 q) \right\}\right)$$

.... (1.19)

So T is given by

$$T = \frac{T_1 \log r_2/r + T_2 \log r/r_1}{\text{Log } r_2/r_1}$$

$$+ R\, e^{int}\left\{ \sum_{n=1}^{\infty} \frac{2\pi^2}{\alpha} \sum_q \sum_{\text{þ}} \frac{q^2 J^2_{\text{þ}}(r_2 q)\, \beta_{\text{þ}}(qr)}{J^2_{\text{þ}}(r_1 q) - J^2_{\text{þ}}(r_2 q)}\, \frac{\sin \text{þ}\theta}{\text{þ}} \right.$$

$$\frac{X\left\{\frac{4}{\text{þ}\pi}\left((T_1 e^{int} + T_1)\frac{J_{\text{þ}}(r_1 q)}{J_{\text{þ}}(r_2 q)} - (T_2 e^{int} + T_2)\right) + \frac{2 a_n}{\text{þ} K'}\right\}}{(q^2 + in/K')}$$

$$X\left(r_2 s_{1,\text{þ}}(r_2 q)\left\{ J_{\text{þ}}(r_1 q)\gamma_{\text{þ}-1}(r_2 q) - J_{\text{þ}-1}(r_2 q)\gamma_{\text{þ}}(r_1 q) \right\} + \right.$$

$$\left.\left. r_1 s_{1,\text{þ}}(r_1 q)\left\{ \gamma_{\text{þ}}(r_1 q) J_{\text{þ}-1}(r_1 q) - J_{\text{þ}}(r_1 q)\gamma_{\text{þ}-1}(r_1 q) \right\}\right)\right\} \quad \text{..............(1.20)}$$

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