Contrast Driven Elastica for Image Segmentation

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Abstract -- Minimization of boundary curvature is a classic regularization technique for image segmentation in the presence of noisy image data. Gradient descent methods have been used for deriving the techniques for minimizing curvature which could be trapped by a local minimum and, therefore, required a good initialization. Recently, this barrier has been overcome by the combinatorial optimization techniques by providing solutions that can achieve a global optimum. However, when the true object has high curvature regularization methods can fail. In these circumstances, existing methods depend on a data term to overcome the high curvature of the object. Unfortunately, these methods fail when the data term is unambiguous in some images. So in order to eliminate these problems, we propose a contrast driven elastic model (including curvature), which can accommodate high curvature objects and an ambiguous data model. We demonstrate that we can accurately segment extremely challenging synthetic and real images with ambiguous data discrimination, poor boundary contrast, and sharp corners. By using this method we have provided a quantitative evaluation of our segmentation approach when these are applied to a standard image segmentation data set.

Index Terms- Euler elastic, weighted curvature, combinatorial optimization, primal formulation, image segmentation.

I. INTRODUCTION

A classic approach to image segmentation is done by formulating the problem as an energy minimization problem

$$E = E_{data} + E_{boundary}$$

Where the boundary of the object is modeled by $E_{boundary}$ and the object and background appearance (intensity, color, texture etc) is modeled by E_{data}.. The main principle of this model is that the data term may be noisy or ambiguous, and to overcome this noise or ambiguity the boundary model can be used as a regularization technique.

A classic prior model for the boundary of the objects which models the object boundary as having low curvature and short length is the elastic model. Mumford has proposed and theoretically justified this model and also appeared in the first active contour work by Kass et al.[16] who proposed an optimization of the boundary curvature. Subsequently, the optimization of boundary curvature became a common feature of variation methods for active contours and level sets [7],[12],[27] . However, as these methods are using descent based optimization it has been resulting in the solution to get stuck in a local minimum and is depending strongly on good initialization. Because of this problem of dependence on initialization (and speed of the curvature optimization) have caused many researchers to abandon the curvature term, particularly after combinatorial and convex optimization methods became popular for producing global optima of the data and boundary length terms. Beyond the elastica model, in the computer vision literature curvature regularization has appeared in several forms, of which Mumford's elastica model is just one example. In order to provide image segmentation which is curvature dependent some approaches use cycle ratios [15], [24]. Schoenemann et al. presented globally optimal image segmentation by minimizing the ratio of the flux over the weighted sum of length and curvature of the object of interest. However it became impractical for many vision applications because of the memory requirements and computational time of this approach.

In past a method has been used to address the unreliability of data term by using user defined seeds; whereas a foreground seed is a small subset of pixels that have been labeled as belonging to object, and background seed is a small subset of pixels that have been labeled as belonging to background . These seeds may be obtained interactively from a user who is specifying a particular object (eg.,[4],[22]) or automatically from a system trained to look for a particular object(eg.,[13],[32]). For example, Graph Cuts [4], Random Walker[14], geodesic segmentation[2] and power watersheds[9]. are the algorithms that has been employed which uses seeds and contrast-sensityive edge weighting However, since curvature is a function of triple- cliques it became much less clear in using the traditional contrast weighting for a curvature regularization model. Seeds may also be incorporated into our contrast driven elastic method to interactively specify regions with poor data term definition.

The advantage of the pseudo-elastica approach in is that its ability to tackle open contours which cannot be made that easily by using our region based weighted curvature regularization method. This was illustrated by the filament example in.

II. METHODS

Let's start this part with a short review of the elastica optimization method presented in before proceeding to our generalized contrast driven elastica formulation, optimization and results.

A) *Contrast Driven Elastica Energy:*

The continuous formulation of Mumford's elastica model is defined for curve C as

$$E(C)=\int(a+k^2)ds \quad a, b>0$$

Where ds denotes the arc length element and k represents the scalar curvature. When a = 0 (the arc length is ignored), the model reduces to the integral of the boundary squared curvature The combinatorial optimization of a discrete form of this model (in which boundary polygons are mapped to cuts) has been presented in [10]. We now review the main points of this formulation.

1. *Curvature energy*:

To minimize the elastica model by using the comb inatorial optimization has prompted the discrete formulation of the curvature on a graph. A graph G = {V, E} consists of a set of vertices v ∈ V and a set of edges e ∈ E ⊆ V × V. An edge incident to vertices v_i and v_j is denoted e_{ij} . In our formulation, each pixel is identified with a node, vi . A weighted graph is a graph in which every edge e_{ij} is assigned a weight w_{ij} . Whereas an edge cut is any collection of edges that separates the graph into two sets, S ⊆ V and S, which may be represented by a binary indicator vector x,

$$X_i = \begin{cases} 1 \ if \ \ vi \in S \\ 0 \quad else \end{cases}$$

The cost of the cut represented by any x is given by

$$Cut(x)=\sum_{eij} w_{ij}|x_i-x_j|$$

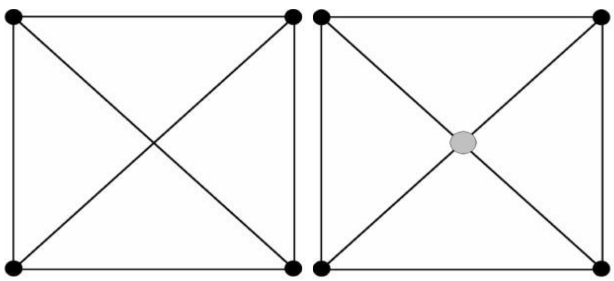
2. *Plannar Vs Non-planar Graphs*:

The primal graph formulation in [10] is exactly equivalent to the original formulation presented by Schoenemann in [25] when a planar graph is used (i.e. the dual exists). This holds only for 4-connected lattices that may present metrication artifacts. As 8-connected graphs are non-planar so they have no dual. A procedure to plenaries the 8- connected lattice has been proposed in this paper just by adding auxiliary nodes to the 8 -connected lattice. Any non-planar graph can be planarized as follows:

1) In the first step determine all the edge crossings in the non-planar graph.

2) In the second step the non-planar graph should be planarized by adding an auxiliary node at each edge crossing. Figure 2 shows the non-planar 8-connected lattice and the corresponding planarized lattice created by adding an auxiliary vertex at the crossing of the diagonal edges. Hence, the curvature formulation in can be used for the planarized lattice in Figure 2(b).

Weighted Curvature Energy: The formulation that has been presented places a strong disadvantage on boundaries which have a high curvature, even if that curvature is supported by the boundary contrast. In this each edge pair which is shown above is associated with an angle in the boundary polygon. Therefore, we propose to relax the penalty associated with a sharp angle when the angle is well-supported with boundary contrast by weighting the corner (edge pair) to reflect the boundary contrast. Traditional contrast edge weighting formulas have been constructed in terms of node



(a) (b)

Fig 1. 8 point non-planar graph may by inserting a auxillary node at each edge crossing of a nonplanar graph (a)8-connected lattice of non-polar. (b)planarized lattice with auxillary nodes (in gray) inserted at every edge crossing.

It is much less clear how to construct a contrast –weighting formula for the type of node triple –clique that forms the basis of the curvature formulation in [10].

B) Optimization

In our contrast driven formulation, the elastic problem was transformed into the problem of finding a min cut on a graph in which some of the edge weights were negative. Unfortunatley the third term of [6] violets the break sub modularity constraint and cases the min cut problem to be non submodular, i.e straight forward max-flow/min-cut algorithm will not yield a min cut. However, it was shown in [10] that the quadratic pseudo Boolean optimization (QPBO) and quadratic pseudo boolean optimization with probing (QPBOP) offered a solution to the optimization problem that frequently offered a complete, optimal solution.

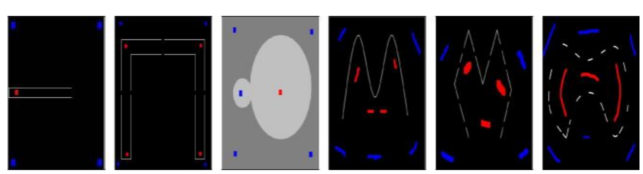
The QPBO is a technique which has the good ability in providing a partial labeling of the variables which is optimal for all the labeled variables. The output of QPBO is

$$X_i = \begin{cases} 1 \; if \; vi \in S \\ 0 \; if \; vi \in complement \; of \; s \\ \Phi \; otherwise \end{cases}$$

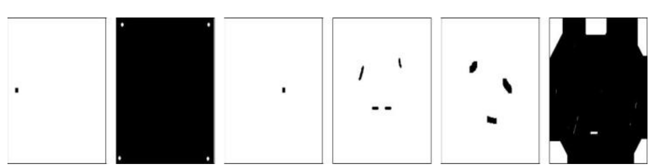
Recall that S represents the set of roads that minimizes our discrete elastica energy

III . EXPERIMENTAL RESULTS

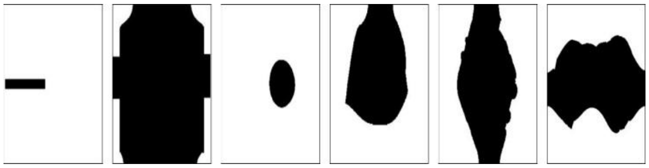
The contrast driven elastic algorithm was mainly aimed at employing higher-order regularization for objects which are having sharp corners, even in the presence of ambiguous or poorly –defined data terms. In order to do this first we have to check whether our contrast –weighting modification allows us to segment objects with high curvature and poor data term differentiation with respect to both a contrast weighted boundary length regularization and an unweight curvature regularization. In addition to this, we must verify that our modification of the elastica model does not cause it to behave poorly on images with a good data term differentiation. In order to verify it, we start by demonstrating the strength of our model on synthetic and real examples of images with poor data data term definition and images with sharp corners and complex boundaries. We show that our contrast driven elastic algorithm is far superior to the boundary length and unweight curvature models. In the later step we proceed to show a comparison quantitatively by applying our model to a standard image segmentation database and to show that our method performs well when compared with existing algorithms on this database, even though the data terms are relatively informative.



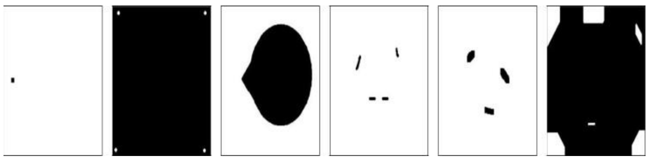
Input image with Seeds



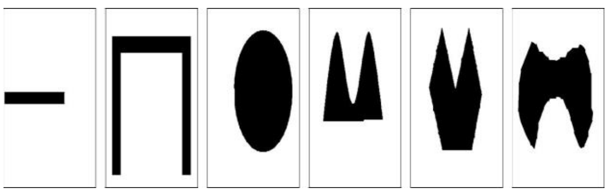
Segmentation outputs using Graph cuts



Segmentation output using Random walker



Segmentation output using Unweighted Curvature and Unary Data Term



Segmentation output using Contrast Driven Elastica

Fig2. Segmentation of synthetic images with identical foreground and background intensity profiles, weak object boundaries and irregular shapes. Comparison of segmentation results via weighted boundary length (Graph cuts) [4] umweighted curvature and our contrast driven elastic method model on synthetic

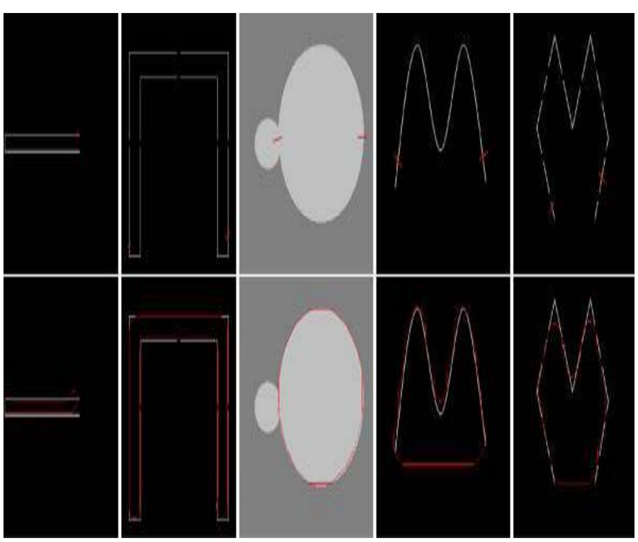


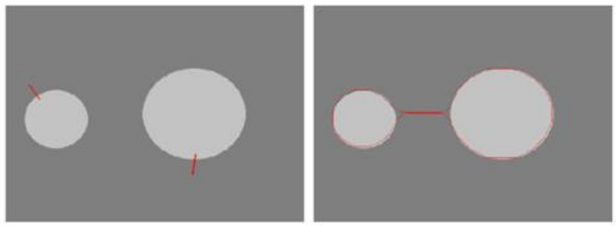
Fig3. Results of the pseudo elastic segmentation approach on the challenging synthetic images with no appearance discrimination between the object of interest and the background

A) Images With Poor Data Differentiation

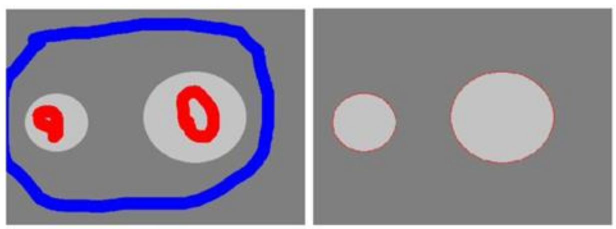
For explaining the difficulty associated with objects which are having high curvature and noisy data term differentiation, we first created some synthetic images which were designed to give importance to three main challenges: 1) The ambiguity which is present in data term 2) The boundaries which are incomplete, 3) Accurately segmenting the objects with high curvature features such as sharp corners and cusps.

Figure 2 shows six synthetic images in which an appearance term is not useful to define a segmentation and for which the object boundaries are incomplete and the shapes are irregular. Such images present a strong challenge to existing algorithms which rely on appearance descriptions and a model of short boundary length. For each image, we provide the results of our proposed approach and a comparison with other segmentation approaches typically used to segment such images. Segmentation results for weighted boundary length, random walker segmentation and unweighted curvature with unary data,μ_f and $\mu_B$are calculated as the mean values for the foreground and background seeds

Confusion or ambiguity of the data part has been explained in the synthetic examples which have been created in figure 2, the foreground and background in these images share the same intensity profile. Whereas the Weighted boundary length segmentation benefits the cut with the minimum number of edges which in turn results in a little solution around the seeds. The Random walker method works by starting at a certain pixel and then it will first reach one of the seeds. Therefore, it suffers from a proximity problem which in turn results in a premature stopping because



Segmentation of disconnected objects using the pseudo elastica.



Segmentation of disconnected objects using our approach.

Fig4. Comparing the weighted pseudo elastic with segmenting disconnected objects.

A random walk from an erroneously –background –labeled pixel would have a higher probability reaching the background seeds then the fore ground one producing an under segmented object as illustrated by the result of the first and third images. Whereas the Unweighted curvature regularization have failed to provide a segmentation in a proper way because the unary data term does not provide any distinction in the first image , by allocating all the image to one class except for the seeds of the other class the minimum curvature is obtained. Whereas our method contrast driven elastica approach allows the correct segmentation as it extends the seeds due to strong contrast at the boundary .Whenever the information of boundary information isnot present such as the gaps in the second image and last closed polygon, the elastica model fevers to bridge these gaps and produce a connected object because if in case they are disconnected then it will result in a boundary of higher curvature.

The initialization for the segmentation of each image has been showed in the top row of figure 3. The row in bottom indicates the segmentation results which have been obtained by using the weighted pseudo elastic. The outcome of second image illustrate that even in the existence of a sharp contrast the sharp corners cannot be preserved very well. It has been founded this kind of persistent behavior no matter how we choose the parameters. Until the corners are supported with high contrast our weighted curvature formulation preserves the corners very accurately. Moreover, when the images get more stimulating the algorithm face to provide proper completion to such boundaries as shown in the first forth images.

It is, to some extent relevant to the corner preservation issue. Had the edge in the bottom of the forth image not been missing, two corners would have formed. A rounded smooth corner generally leads to form a bottom edge being lower than it supposed to be. This kind of phenomenon happens whenever the corner has a missing edge. So that weighted pseudo elastic fails to properly preserve the corner.

Its ability to capture topology changes correctly is another feature of differentiating with the weighted pseudo elastica. The weighted pseudo elastic approach inherits the limitations of the edge based segmentation approaches such as failure of providing segmentation in a proper way in the presence of variations in it's topology.

This kind of set up way has provided the boundary in a single connected component. However, our formulation is reason based end intuitively handless disconnected components of a single object. Figure 4 explains the segmentation of an object that consists of two disconnect circles. From the above result we can say that algorithm which we used have provided the correct segmentation of the circle. Whereas, the two circles are connected falsely by the weighted pseudo elastica.

B) Real Images With Ambiguous Appearance

Although the previous set of images were synthetically created to highlight challenges in image segmentation, these kind of phenomenon are actually very common in the case of segmentation of real images, especially in medical images.

In many of medical imaging segmentation problems the common thing which we encounter is the ambiguity in data term. whereas for these kind of cases the segmentation in a trivial way has been produced by graph cut method. Whereas the random walker results exhibits over dependence on the proximity of the seeds in this kind of indeterminate case, leading both false positives and false negatives. The unweighted curvature algorithm produces a boundary of min curvature that respects the global data model derived from the seeds, but this unweighted curvature method was unable to provide a good segmentation because as it follows the global data model and the poor data. Objects which are having weak boundaries also appear very commonly in medical images. Due to the closeness in the seeds ,the random walker algorithm leaks through the gaps in the boundary which in turn results in an over segmented objected But by our contrast driven elastica method these leakages or prevented because any leakages in the boundary will produce a higher energy solution and a will have a higher curvature .

C. Assessment of Database with positive data Differentiation

In this section, we are going to test whether our modified elastic model can do well when there is a situation of substantial differentiation in the data term. To assess this issue, we provide a quantitative comparison between the performances of our contrast-weighted boundary length model (Graph Cuts), the p-brush approach in (with two different p values) the random walker and the power watershed. We applied the six algorithms to the set of images in the Microsoft Grab Cut database used .The database contains ground truth segmentation for 50 color images corresponding to indoor as well as outdoor scenes. We quantify the results using four segmentation error measures.

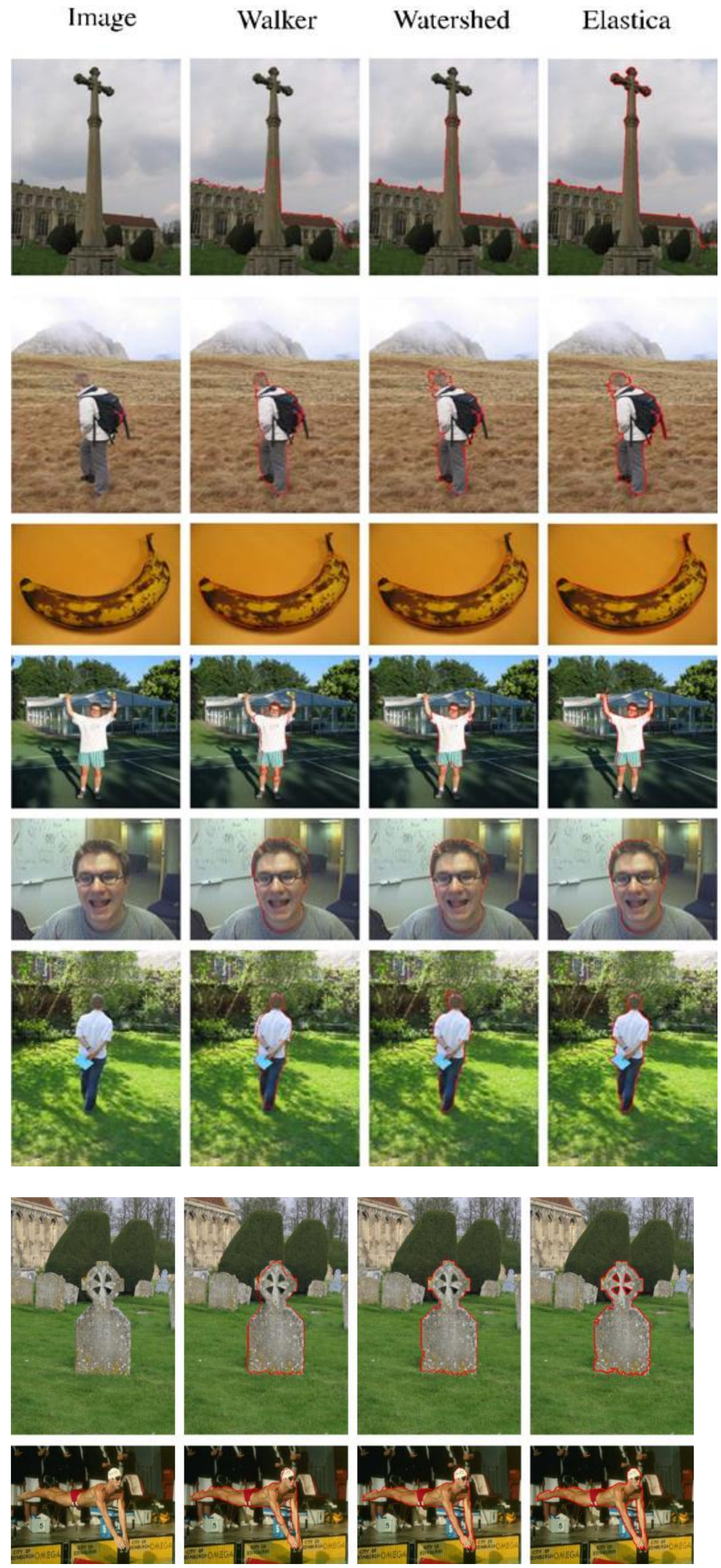


Fig5. Sample of the segmentation results of the grab cut data set with the different segmentation methods

TABLE1

We compare our contrast driven elastica segmentation algorithm to five other algorithms on the grabcut database of [3]. The data terms in this database are relatively informative and therefore. All six algorithms produce quality image segmentation results. Our contrast driven elastica algorithm provides segmentation results which are as good as or better than these algorithms on this database. Additionally sections iii-a and iii-b show that our algorithm still maintains its quality for images with ambiguous data terms .even with the other algorithms are unable to produce quality segmentation. We did not include a comparison to the unweighted elastica because all of the six algorithms listed above do not include unary data term but the unweighted elastica in [10] does so the comparison will not be fair

	BE	GCE	VOI	RI
Graph Cuts	3.276	0.028	0.196	0.970
p-brush ($p = 1.25$)	3.241	0.028	0.193	0.971
p-brush ($p = 1.75$)	3.206	0.027	0.187	0.972
Random Walker	3.206	0.026	0.185	0.972
Power Watershed	2.888	0.025	0.210	0.970
Contrast Driven Elastica	2.683	0.024	0.205	0.971

Table 1 shows the mean results of these segmentation algorithm for the 50 images in the data set. It can be seen that the segmentation obtained by the contrast driven elastica are as good or better than the other algorithms on these images , even though they generally have a strong data term differentiation between the background and the object .

IV. CONCLUSION

In the past curvature minimization method for segmentation of image has received a little attention during the difficulty of optimizing curvature models. Whereas the recent steps has introduced good formulation and optimization of curvature which have revived the research efforts in applying the curvature regularization to vision problems such as image segmentation, denoising and inpainting. However, the main problem associated with minimum boundary curvature models is segmenting the structures with high curvature such as sharp corners and cusps. Whereas the existing models are associated the curvature regularization with a global data model to help segment such structures. Unfortunately back ground and foreground seeds have sharing same data profile. For these kind of cases the segmentation of the curvature methods will fail in providing a correct solution. An image segmentation model that weights the curvature locally by the contrast information has been proposed by us in this paper. In this paper we also added the length term to curvature to complete the formulation of elasctica model which has been actually proposed by mum ford in. The inclusion of the length term allows us to find a global optimum of the model using QPBO/QPBOP in a few seconds.

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