Srinivasa Ramanujan's Achievements and Honours Got on Mathematics

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Abstract -- Ramanujan was born on 22 December 1887 in Erode, Tamil nadu at the residence of his maternal grandparents. His father, K. Srinivasa Iyengar, worked as a clerk in a sari shop and hailed from the district of Thanjavur. His mother, Komalatammal, was a housewife and also sang at a local temple. They lived in Sarangapani Street in a traditional home in the town of Kumbakonam. In December 1889, Ramanujan had smallpox and recovered. He moved with his mother to her parents' house in Kanchipuram, near Chennai. Since Ramanujan's father was at work most of the day, his mother took care of him as a child. From her, he learned about tradition and puranas. He learned to sing religious songs, to attend pujas at the temple and particular eating habits ? all of Brahmin culture. Just before the age of 10, in November 1897, he passed his primary examinations in English, Tamil, geography and arithmetic. With his scores, he stood first in the district. That year, Ramanujan entered Town Higher Secondary School where he encountered formal mathematics for the first time.

I. INTRODUCTION

Ramanujan was born on 22 December 1887 in Erode, Tamil nadu at the residence of his maternal grandparents. His father, K. Srinivasa Iyengar, worked as a clerk in a sari shop and hailed from the district of Thanjavur. His mother, Komalatammal, was a housewife and also sang at a local temple. They lived in Sarangapani Street in a traditional home in the town of Kumbakonam. In December 1889, Ramanujan had smallpox and recovered. He moved with his mother to her parents' house in Kanchipuram, near Chennai. Since Ramanujan's father was at work most of the day, his mother took care of him as a child. From her, he learned about tradition and puranas. He learned to sing religious songs, to attend pujas at the temple and particular eating habits – all of Brahmin culture. Just before the age of 10, in November 1897, he passed his primary examinations in English, Tamil, geography and arithmetic. With his scores, he stood first in the district. That year, Ramanujan entered Town Higher Secondary School where he encountered formal mathematics for the first time.

By age 11, he had exhausted the mathematical knowledge of two college students who were lodgers at his home. He was later lent a book on advanced trigonometry written by S. L. Loney. He completely mastered this book by the age of 13 and discovered sophisticated theorems on his own. By 14, he was receiving merit certificates and academic awards which continued throughout his school career and also assisted the school in the logistics of assigning its 1200 students (each with their own needs) to its 35-odd teachers. He completed mathematical exams in half the allotted time, and showed a familiarity with geometry and infinite series. Ramanujan was shown how to solve cubic equations in 1902 and he went on to find his own method.

In 1903 when he was 16, Ramanujan obtained from a friend a library-loaned copy of a book by G. S. Carr. The book was titled *A Synopsis of Elementary Results in Pure and Applied Mathematics* and was a collection of 5000 theorems. Ramanujan reportedly studied the contents of the book in detail. The book is generally acknowledged as a key element in awakening the genius of Ramanujan. The next year, he had independently developed and investigated the Bernoulli numbers and had calculated Euler's constant up to 15 decimal places.

When he graduated from Town Higher Secondary School in 1904, Ramanujan was awarded the K. Ranganatha Rao prize for mathematics by the school's headmaster, Krishnaswami Iyer who introduced Ramanujan as an outstanding student who deserved scores higher than the maximum possible marks. He received a scholarship to study at Government Arts College, Kumbakonam, However, Ramanujan was so intent on studying mathematics that he could not focus

on any other subjects and failed most of them, losing his scholarship in the process. He later enrolled at Pachaiyappa's College in Madras. He again excelled in mathematics but performed poorly in other subjects such as physiology. Ramanujan failed his Fine Arts degree exam in December 1906 and again a year later. Without a degree, he left college and continued to pursue independent research in mathematics. At this point in his life, he lived in extreme poverty and was suffering from starvation.

II. ADULTHOOD IN INDIA

On 14 July 1909, Ramanujan was married to a nine-year old bride, Janaki Ammal (21 March 1899 - 13 April 1994. After the marriage, Ramanujan developed a hydrocele testis. His family did not have the money for the operation, but in January 1910, a doctor volunteered to do the surgery for free. After his successful surgery, Ramanujan searched for a job. He stayed at friends' houses while he went door to door around the city of Chennai looking for a clerical position. To make some money, he tutored some students at Presidency College who were preparing for their exam.

III. ATTENTION FROM MATHEMATICIANS

Ramanujan met deputy collector V. Ramaswamy Aiyer, who had recently founded the Indian Mathematical Society . Ramanujan, wishing for a job at the revenue department where Ramaswamy Aiyer worked, showed him his mathematics notebooks. As Ramaswamy Aiyer later recalled: I was struck by the extraordinary mathematical results contained in it [the notebooks]. I had no mind to smother his genius by an appointment in the lowest level as clerk in the revenue department. Ramaswamy Aiyer sent Ramanujan, with letters of introduction, to his mathematician friends in Madras. Some of these friends looked at his work and gave him letters of introduction to R. Ramachandra Rao, the district collector for Nellore and the secretary of the Indian Mathematical Society. Ramachandra Rao was impressed by Ramanujan's research but doubted that it was actually his own work. Ramanujan mentioned a correspondence he had with Professor Saldhana, a notable Bombay mathematician, in which Saldhana expressed a lack of understanding for his work but concluded that he was not a phony. Ramanujan's friend, C. V. Rajagopalachari, persisted with Ramachandra Rao and tried to clear any doubts over Ramanujan's academic integrity. Rao agreed to give him another chance, and he listened as Ramanujan discussed elliptic integrals, hypergeometric series, and his theory of divergent series, which Rao said ultimately "converted" him to a belief in Ramanujan's mathematical brilliance. When Rao asked him what he wanted, Ramanujan replied that he needed some work and financial support. Rao consented and sent him to Madras. He continued his mathematical research with Rao's financial aid taking care of his daily needs. Ramanujan, with the help of Ramaswamy Aiyer, had his work published in the *Journal of Indian Mathematical Society*.

One of the first problems he posed in the journal was:

$$\sqrt{1+2\sqrt{1+3\sqrt{1+\cdots}}}.$$

He waited for a solution to be offered in three issues, over six months, but failed to receive any. At the end, Ramanujan supplied the solution to the problem himself. On page 105 of his first notebook, he formulated an equation that could be used to solve the infinitely nested radicals problem.

$$x+n+a=\sqrt{ax+(n+a)^2+x\sqrt{a(x+n)+(n+a)^2+(x+n)\sqrt{\cdots}}}$$

Using this equation, the answer to the question posed in the *Journal* was simply 3. Ramanujan wrote his first formal paper for the *Journal* on the properties of Bernoulli numbers. One property he discovered was that the denominators of the fractions of Bernoulli numbers were always divisible by six. He also devised a method of calculating B_n based on previous Bernoulli numbers. One of these methods went as follows:

Mr. Ramanujan's methods were so novel and his presentation so lacking in clearness and precision, that the ordinary mathematical reader,

unaccustomed to such intellectual gymnastics, could hardly follow him. Ramanujan later wrote another paper and also continued to provide problems in the *Journal*. In early 1912, he got a temporary job in the Madras Accountant General's office, with a salary of 20 rupees per month. He lasted for only a few weeks. Toward the end of that assignment he applied for a position under the Chief Accountant of the Madras Port Trust. Attached to his application was a recommendation from E. W. Middlemast, a mathematics professor at the Presidency College, who wrote that Ramanujan was "a young man of quite exceptional capacity in Mathematics". Three weeks after he had applied, on 1 March, Ramanujan learned that he had been accepted as a Class III, Grade IV accounting clerk, making 30 rupees per month. At his office, Ramanujan easily and quickly completed the work he was given, so he spent his spare time doing mathematical research. Ramanujan's boss, Sir Francis Spring, and S. Narayana Iyer, a colleague who was also treasurer of the Indian Mathematical Society, encouraged Ramanujan in his mathematical pursuits.

IV. MATHEMATICAL ACHIEVEMENTS

In mathematics, there is a distinction between having an insight and having a proof. Ramanujan's talent suggested a group of formulae that could then be investigated in depth later. It is said that Ramanujan's discoveries are unusually rich . As a by-product, new directions of research were opened up. Examples of the most interesting of these formulae include the infinite series for π, one of which is given below

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{(k!)^4 396^{4k}}.$$

One of his remarkable capabilities was the rapid solution for problems. He was sharing a room with P. C. Mahalanobis who had a problem, "Imagine that you are on a street with houses marked 1 through n. There is a house in between (x) such that the sum of the house numbers to left of it equals the sum of the house numbers to its right. If n is between 50 and 500, what are n and x?" This is a bivariate problem with multiple solutions. Ramanujan thought about it and gave the answer with a twist: He gave a continued fraction. The unusual part was that it was the solution to the whole class of problems. Mahalanobis was astounded and asked how he did it. "It is simple. The minute I heard the problem, I knew that the answer was a continued fraction. Which continued fraction, I asked myself. Then the answer came to my mind," Ramanujan replied

In 1918, Hardy and Ramanujan studied the partition function $P(n)$ extensively and gave a non-convergent asymptotic series that permits exact computation of the number of partitions of an integer. He discovered mock theta functions in the last year of his life. For many years these functions were a mystery, but they are now known to be the holomorphic parts of harmonic weak Maass forms.

V. RAMANUJAN'S NOTEBOOKS

Ramanujan recorded the bulk of his results in four notebooks of loose leaf paper. These results were mostly written up without any derivations. This is probably the origin of the misperception that Ramanujan was unable to prove his results and simply thought up the final result directly. This style of working may have been for several reasons. Since paper was very expensive, Ramanujan would do most of his work and perhaps his proofs on slate, and then transfer just the results to paper. Using a slate was common for mathematics students in the Madras Presidency at the time. His first notebook has 351 pages with 16 somewhat organized chapters and some unorganized material. The second notebook has 256 pages in 21 chapters and 100 unorganised pages, with the third notebook containing 33 unorganised pages. The results in his notebooks inspired numerous papers by later mathematicians trying to prove what he had found. Hardy himself created papers exploring material from Ramanujan's work as did G. N. Watson, A fourth notebook with 87 unorganised pages, the so-called "lost notebook", was rediscovered in 1976 by George Andrews.

Notebooks 1, 2 and 3 were published as a two volume set in 1957 by the Tata Institute of Fundamental Research (TIFR), Mumbai, India. This was a photocopy edition of the original manuscripts, in his own handwriting.

In December 2011, as part of Ramanujan's 125th birth centenary celebrations, TIFR republished the notebooks in a colored two volume collector's edition. These were produced from scanned and microfilmed images of the original manuscripts by expert archivists of Roja Muthiah Research Library, Chennai.

VI. OTHER MATHEMATICIANS' VIEWS OF RAMANUJAN

Hardy said : " Here was a man who could work out modular equations and theorems... to orders unheard of, whose mastery of continued fractions was... beyond that of any mathematician in the world, who had found for himself the functional equation of the zeta function and the dominant terms of many of the most famous problems in the analytic theory of numbers; and yet he had never heard of a doubly periodic function or of Cauchy's theorem, and had indeed but the vaguest idea of what a function of a complex variable was.". He also stated that he had "never met his equal, and can compare him only with Euler or Jacobi."

Quoting K. Srinivasa Rao , "As for his place in the world of Mathematics, we rate mathematicians on the basis of pure talent on a scale from 0 to 100, Hardy gave himself a score of 25, J.E. Littlewood 30, David Hilbert 80 and Ramanujan 100.'"

Professor Bruce C. Berndt of the University of Illinois, during a lecture at IIT Madras in May 2011, stated that over the last 40 years, as nearly all of Ramanujan's theorems have been proven right, there had been a greater appreciation of Ramanujan's work and brilliance. Further, he stated Ramanujan's work was now pervading many areas of modern mathematics and physics.

In his book *Scientific Edge*, noted physicist Jayant Narlikar spoke of "Srinivasa Ramanujan, discovered by the Cambridge mathematician Hardy, whose great mathematical findings were beginning to be appreciated from 1915 to 1919. His achievements were to be fully understood much later, well after his untimely death in 1920. "

During his lifelong mission in educating and propagating mathematics among the school children in India, Nigeria and elsewhere, P.K. Srinivasan has continually introduced Ramanujan's mathematical works.

VII. RECOGNITION

Ramanujan's home state of Tamil Nadu celebrates 22 December (Ramanujan's birthday) as 'State IT Day', memorializing both the man and his achievements, as a native of Tamil Nadu. A stamp picturing Ramanujan was released by the Government of India in 1962 – the 75th anniversary of Ramanujan's birth – commemorating his achievements in the field of number theory and a new design was issued on December 26, 2011, by the India Post.

Since the Centennial year of Ramanujan, every year 22 Dec, is celebrated as Ramanujan Day by the Government Arts College, Kumbakonam where he had studied and later dropped out.

Ramanujan's work and life are celebrated on 22 December at The Indian Institute of Technology (IIT), Madras in Chennai.

A prize for young mathematicians from developing countries has been created in the name of Ramanujan by the International Centre for Theoretical Physics (ICTP), in cooperation with the International Mathematical Union, who nominate members of the prize committee. The Shanmugha Arts, Science, Technology & Research Academy (SASTRA), based in the state of Tamil Nadu in South India, has instituted the SASTRA Ramanujan Prize of $10,000 to be given annually to a mathematician not exceeding the age of 32 for outstanding contributions in an area of mathematics influenced by Ramanujan. The age limit refers to the years Ramanujan lived, having nevertheless still achieved many accomplishments. This prize has been awarded annually since 2005, at an international conference conducted by SASTRA in Kumbakonam, Ramanujan's hometown, around Ramanujan's birthday, 22 December.

On the 125th anniversary of his birth, India declared the birthday of Ramanujan, December 22, as 'National

Mathematics Day.' The declaration was made by Dr. Manmohan Singh in Chennai on December 26, 2011. Dr Manmohan Singh also declared that the year 2012 would be celebrated as the National Mathematics YearA Disappearing Number

A film, based on the book The Man Who Knew Infinity: A Life of the Genius Ramanujan by Robert Kanigel, is being made by Edward Pressman and Matthew Brown with R. Madhavan playing Ramanujan.

Another international feature film on Ramanujan's life was announced in 2006 as due to begin shooting in 2007. It was to be shot in Tamil Nadu state and Cambridge and be produced by an Indo-British collaboration and co-directed by Stephen Fry and Dev Benegal.

A play, First Class Man by Alter Ego Productions, was based on David Freeman's First Class Man. The play is centered on Ramanujan and his complex and dysfunctional relationship with Hardy. Like the book and play it is also titled The First Class Man; the film's scripting has been completed and shooting is being planned from 2012.

A Disappearing Number is a recent British stage production that explores the relationship between Hardy and Ramanujan.

The novel The Indian Clerk by David Leavitt explores in fiction the events following Ramanujan's letter to Hardy.

On 22 March 1988, the PBS Series Nova aired a documentary about Ramanujan, "The Man Who Loved Numbers" (Season 15, Episode 9).

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