

Analytical Modeling with Computer Simulation Validation of an Inductive Heating System for Metals Melting Application

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Abstract -- Inductive heating is a fast and environmentally friendly heating method that employs eddy current resistance losses in heating of conducting materials. The eddy current being a product of magnetic induction process resulting from the electromagnetic fields generated by an induction coil powered by a high frequency voltage source. Due to the complexity of the system, the design of the induction heating systems often requires the use of mathematical and computer simulation tools which helps in shortening the development time and cost. Both analytical and computer modeling techniques were employed in this study. The analytical modeling was used in computing the electrical parameters of the induction coil that makes for optimal heating of the work-piece for any heating application while the computer modelling was used to investigate thermal dynamic characteristics of the inductively heated work piece. A 2-D axisymmetric computer model of a cylindrical graphite crucible work piece of dimensions (6 cm height, 2 cm external diameter and 1.5 cm internal diameter) was simulated in the study with 60 Amps excitation current at 100 kHz resonant frequency. A heating temperature of about 1386 oC was achieved around 600 seconds simulation time. The coil inductance of 0.03 mH was obtained for analysis using copper tube coil of radius 0.5 cm and coil diameter 7.5 cm. The derived parameters will then be used in the fabrication of the induction heating system for experimental validation of aluminum melting process.

Indexed Terms: Induction heating, Electromagnetic fields, 2-D axisymmetric, computer simulation, resonant frequency

I. INTRODUCTION

Material processing is one of most industrial manufacturing activity or product development, usually requires heating of raw material(s) before, during or after processing activities. The heating process serves as a means of providing the thermal

energy required to achieve the processing temperature that determines the product properties and quality. To achieve the desired product properties and quality most often requires efficient and effective heat management in form of heat addition or extraction, thermal flow control or phase change that will result in efficient temperature control which is an integral component of most material processing and manufacturing activities. Different heating methods such as flame, beam, element and electromagnetic are available as a means of providing the thermal energy requirements though with varying degrees of efficiency is, effectiveness and limitations. Among the different heat sources and heating methods available for material processing and product development, the electromagnetic (induction heating (IH)) method stands out due to the numerous advantages it offers. These include high process productivity, universality, cleanliness and possibility for heating automation, as well as environmental friendliness and reach desire temperature with a short time. Induction heating is a non-contact, flame free, precision heating method that combines electromagnetic, heat transfer and metallurgical phenomenon in the heating of electrically conductive materials. Other important properties include frequency dependency of the heated depth (skin depth effect), accurate location of the heated area by the induction coil-workpiece coupling design and high-power density. The ability to harness the above-mentioned properties of the IH system will require proper equipment and material selection.

II. SKIN DEPTH

The heated depth usually depends on the skin depth (δ_{ω}) of the heated material as well as the frequency of the induction power supply (Simpson. 1960). Skin

depth which is a product of the Skin effect phenomenon in electrically conductive materials that is responsible for the non-uniform distribution of current across the conductor while carrying alternating current. Because of this, the induced eddy current concentrates near the surface of workpiece and exponentially decreases towards the center of the material. The skin depth is given as:

$$\delta_{\omega} = 1/2\pi \sqrt{(\rho/(\mu_o \mu_r f))} \quad (1)$$

where ρ is the resistivity of the material, μ the magnetic permeability and f the frequency of the induction power supply.

III. MATERIALS AND METHOD

3.1 Design and Analysis Methods

In the application of IH, the electromagnetic-thermal coupling between the induction coil and the work piece makes its design a bit complex especially with use of mathematical analysis. This is as a result of the coupled effect of non-uniform heat generation through the workpiece, heat transfer as well as the strong temperature dependence of the metallurgical properties of most material. At the early stages of induction heating applications, the design and development of the IHS was based mainly on analytical, experience and experimental methods. However this has evolved over the years as advancements and technological developments has resulted in more efficient design procedures and methods in the development and applications of induction heating systems. This coupled with the development of high-speed personal computing systems that has further simplified the design process through computer modeling using computer simulation software. Computer simulation or modeling now therefore provides a powerful tool for development, optimization and deployment of induction heating systems for both industrial (such as heat treatment, joining, brazing, melting, welding, soldering), domestic (induction cooking) as well as medical applications.

In this study analytical method based on the electrical equivalent circuit of IHS coil-work piece and computer simulation based on finite element analysis (FEA) are exploited in the design of a simple inductive heating system for melting Aluminum material. The

basic setup of an induction heating system is shown in Figure 1.

The major components include an induction coil connected to an alternating high frequency power supply and a conductive work piece to be heated placed inside the induction coil. The work piece inside the induction coil is heated by eddy current produced by electromagnetic field generated by the coil at a frequency defined by the application requirement. The generated eddy current within the work piece results joule heating (I^2R) of the work piece material in form of spatial and volumetric heating.

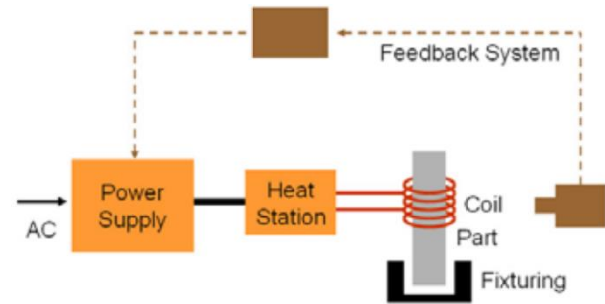


Figure 1: Basic Induction Heating System (HIS) Setup

3.2 The Electrical Equivalent Circuit Model

The induction coil-workpiece assembly of an induction heating system Figure 2(a) can be likened to an electrical step down transformer with multi-turns of primary winding (induction coil) and a single turn of secondary winding (workpiece). The electrical equivalent circuit which is represented in Figure 2 (b). For the mathematical model using the electrical equivalent circuit, the energy transfer between the coil and the workpiece is considered in deriving the total electrical impedance for the induction coil as well as the total power required for the heating process. This was achieved using formulas based on the geometrics of the induction coil and the workpiece. The derived electrical properties then provided the specifications for the computer simulation and fabrication of the IHS hardware. The derived work-coil and workpiece properties include: the induction coil resistance (R_c), workpiece resistance (R_w), coil reactance (X_c), workpiece reactance (X_w), coil-workpiece total impedance (ZT), coil voltage and current as well as the total power requirement and power factor.

3.2.1 The Series Equivalent circuit

In the series equivalent circuit (SEC) model Figure 3, the electrical reactance of the induction coil, air-gap and the work-piece are all considered to be in series.

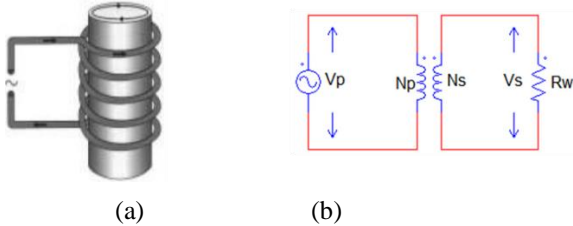


Figure 2: Induction Coil-Work-Piece and the Electrical Equivalent Circuit

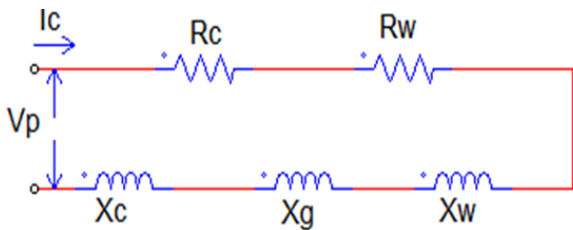


Figure 3: The Series Equivalent Circuit Model for an Induction Heating System

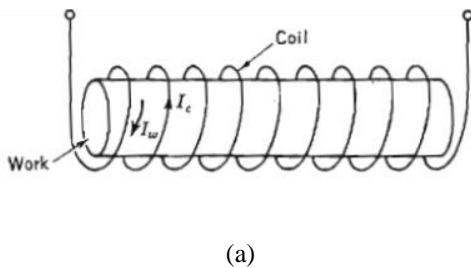
The effective series impedance of the series equivalent determines the coil terminal current, power and power factor for the applied voltage.

The effective circuit impedance (ZT) is given as:

$$Z_T = (R_c + R_w) + j(X_c + X_w + X_g) \text{ Ohms (1)}$$

where Xg is air-gap reactance. These values are obtained using the geometrical dimensions of the coil, the workpiece and expected distance between the coil and the workpiece.

Consider a hollow cylindrical work piece in an induction coil Figures 4(a-c)



(a)

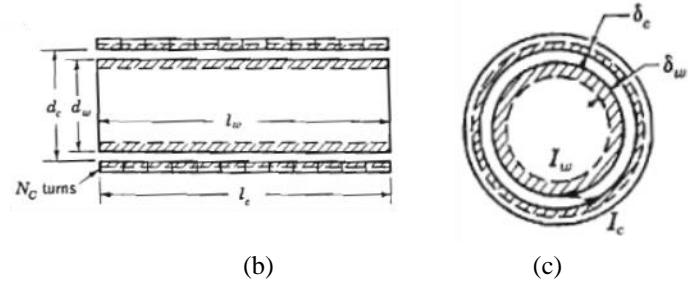


Figure 4: Induction Coil – Work-Piece Geometry

where Ic is coil current (Amps), Iw is workpiece current (Amps), δ_c is coil skin depth (m), δ_w is workpiece skin depth (m), Dc is coil diameter (m) is Dw is workpiece diameter (m), and lw is workpiece length (m)

3.2.2 The coil Resistance and Reactance

The following mathematical equation based on coil workpiece geometry of Figure 4 is used in computing the values of the coil and workpiece parameters. For a copper tube coil, with a skin depth of less than 2 times the thickness (m) of the tube, the coil resistance is assumed to be equals the coil reactance.

This is given as:

$$R_c = X_c = \rho_c \frac{(D_c + \delta_c) \pi N_c^2}{\delta_c l_c k_r} \text{ Ohms,}$$

for $\delta_c < 2t_c$ (2)

where t_c is coil tube thickness(m), l_c is coil length(m), Dc is coil diameter(m), ρ_c is coil Resistivity (Ω m), δ_c is coil skin depth and kr is coil spacing correction.

The coil spacing correction factor given as:

$$k_r = \frac{d_c N_c}{l_c} \text{ (3)}$$

kr lies between 1 and 1.5 with a typical value of 1.15

The workpiece resistance (Rw) and reactance (Xw) is given respectively as

$$R_w = \frac{\rho_w \pi (D_w - d_w)}{\delta_w l_w} \text{ (4)}$$

$$X_w = K(\mu_r q A_w) \text{ (5)}$$

where is given as:

$$K = \frac{2\pi f \mu_o N_c^2}{l_c} \text{ (6)}$$

D_w is workpiece external diameter(m), d_w is workpiece internal diameter, ρ_w is workpiece resistivity (Ωm), l_w is workpiece length(m), δ_w is workpiece skin depth (m), A_w workpiece cross sectional area(m²) and μ_r is the relative permeability of workpiece material.

The air gap reactance between the coil and the workpiece given as

$$X_g = \frac{K_n \mu_0 N_c^2 f (D_c^2 - D_w^2)}{l_c} \text{ Ohms} \quad (7)$$

where k_n the coil short correction factor is

$$K_n = \frac{1}{\left(1 + \frac{0.4502(D_c + \delta_c)}{l_c}\right)} \quad (8)$$

The effective impedance for the system is then obtained by substituting the values of R_c, R_w and X_c, X_w, X_g in equation 1.

For computation of the power dissipated by the induction coil and power induced on workpiece, the transformer equivalent circuit model of the IHS Figure 2(b) is used. From the TEC model, the general transformer equation is:

$$I_p N_p = I_s N_s \quad (9)$$

Where I_p, I_s, N_p and N_s represents the induction coil-workpiece current and turns respectively. Whereby

$$I_c N_c = I_w N_w \quad (10)$$

But for IHS the turns winding for the workpiece is taken as one (1).

hence the induced workpiece current is;

$$I_w = I_c N_c \text{ (Amps)} \quad (11)$$

The power induced on the workpiece by the coil is therefore given as

$$P_w = \frac{I_c^2 N_c^2 \rho_w \pi D}{\delta l} \text{ Watts} \quad (12)$$

While the power dissipated by the induction coil is given respectively is given as

$$P_c = \frac{I_c^2 N_c^2 \pi \rho_c D_c}{\delta_c l_c} \text{ Watts} \quad (13)$$

The total power requirement for the induction heating system for heating the workpiece to be supplied by the power source for of melting the aluminum material is

$$P_T = (P_w + P_c + P_R) \text{ Watts} \quad (14)$$

P_R is due to radiation losses due to the air-gap between induction coil and work piece

The radiation power is calculation using the expression

$$P_R = A_w e \sigma (T_w^4 - T_a^4) \text{ Watts} \quad (15)$$

Where A_w is the surface area of workpiece, e, is the emissivity of the workpiece surface, T_w and T_a are the workpiece and ambient temperatures (in K) respectively and σ, the Stefan Boltzmann constant. The induction coil and workpiece dimensions are given in Tables 1 and 2 respectively.

Table 1: Induction Coil Parameters

Material	Copper
Inside Diameter	50mm
Outer Diameter	5mm
Height	
Coil tube diameter	5mm
Coil tube thickness	0.5mm
Number of turns	6

Table 2: Workpiece Parameters

Material	Graphite
Outer diameter	30mm
Height	60mm
Thickness	5mm

3.3 Computer Modeling of the Electromagnetic and Thermal Field Distribution for the IHS

This is accomplished by the solution of the governing equations of the induction heating process which is the Maxwell's and Fourier equations for electromagnetic and temperature fields. The behavior of the electromagnetic field generated by the induction coil can be described using the Maxwell equations listed below.

$$\nabla \times \vec{H} = J + \frac{\partial \vec{D}}{\partial t} \quad (16)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (17)$$

$$\nabla \cdot \vec{B} = 0 \quad (18)$$

$$\nabla \cdot \vec{E} = 0 \quad (19)$$

where H(A/m) is the magnetic field strength, B (W/m²) is the magnetic field intensity, D(C/m²) is the electric displacement field, E(V /m) is the electric field intensity and J (A /m²), is the current density. A is magnetic vector potential.

The constitutive equation provides the material medium for the fields and are given as:

$$D = \epsilon E \quad (20)$$

$$B = \mu H \quad (21)$$

$$J = \sigma E \quad (22)$$

Where ϵ is the permittivity (F/m), σ is electrical conductivity (mhos/m, and μ is magnetic permeability (H/m).

$$\vec{B} = \nabla \times \vec{A} \quad (23)$$

Equations (16) - (19) are reduced a second-order equation by introducing the magnetic vector potential formulation equation (23), the constitutive equations, equations (16) - equation (19) and the vector identities.

$$\nabla^2 A - j\mu\sigma\omega A = \sigma\mu\nabla V \quad (24)$$

Equation (24) is numerical using computer simulation for 2-D axisymmetric geometry Figure 5 with the following assumptions:

- (1) The system is rotationally symmetric about the z-axis using axisymmetric model of the workpiece, i.e., all quantities are independent of the azimuthal coordinate ϕ ,
- (2) The materials are isotropic, non-magnetic and have no net electric charge,
- (3) The displacement current is negligible and distribution of ac current (voltage) in the work coil is uniform,
- (4) The self-inductance effect in the work-coil is considered and
- (5) The currents (impressed and induced)

have a steady state quality and as a result the electromagnetic field quantity are harmonically oscillating functions with a fixed single Frequency.

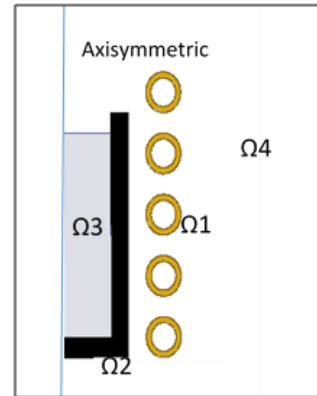


Figure 5: 2-D Axisymmetric model of the induction coil workpiece

Three distinct regions of the geometry are considered for the simulation namely; the work coil (Ω1), the work piece (Ω2, Ω3), and, the surrounding air (Ω4)

For the work coil connected to an external source we have,

$$\Omega 1: \quad \nabla^2 A - j\sigma\mu\omega A = -\mu J_s \quad (25)$$

Where J_s which a time varying current density source is given as:

$$J_s = -\sigma\nabla V$$

For the work-piece with induced current density J_{eddy} , we have:

$$\Omega 2, \Omega 3: \quad \nabla^2 A - j\sigma\mu\omega A = 0 \quad (26)$$

Where

$$J_{eddy} = -j\sigma\omega A$$

for the surrounding air ($\sigma = 0$)

$$\Omega 4: \quad \nabla^2 A = 0 \quad (27)$$

3.4 Thermal field modeling

The modeling of the temperature field distribution of the workpiece due to the eddy current induced by the induction coil on it is accomplished using the Fourier heat equation. For the thermal field solution, it is assumed that there is no interaction between the workpiece and the induction coil and that radiative and convective heat transfer do not

affect the coil because of the cooling of the coil. The thermal analysis is with the 2-D axisymmetric geometry of Figure 5, for the three regions of induction coil, workpiece and surrounding air.

The governing equation for the thermal modeling is given as;

$$K(T).(\nabla^2 T) + Q = \rho C_p(T) \frac{\partial T}{t} \quad (28)$$

where ρ is the mass density of the workpiece, C_p the specific heat capacity, K the thermal conductivity of the material and Q the heat source density by the eddy current.

For the simulation, the ambient temperature is set at 30oC.

The thermal boundary conditions are as follows;

For heat transfer by convection we have;

$$Q_{(conv)} = h(T - T_{amb}) /m2 \quad (29)$$

For Radiation heat transfer,

$$Q_{(rad)} = \epsilon\sigma_b.(T^4 - T_{amb}^4)/m2 \quad (30)$$

h is the convective heat (film) heat transfer coefficient for the material medium that determines the rate at which the heated material losses heat to the surrounding by convection.

3.5 Computer Simulation

The computer simulation involves the solution of the governing equations for the electromagnetic and thermal problems using numerical method. Finite element method was for this solution with Multiphysics simulation software ANSYS Multiphysics. For the electromagnetic field distribution simulations Ansys Maxwell was used to solve the governing equation for the electromagnetic field analysis using 2-D axisymmetric geometry. The result of the electromagnetic simulation is then coupled to the thermal solution using Ansys Workbench.

The procedure for the numerical solution is as shown in Figure 6. The value are engineering constant used in simulation was on Table 3 for the workpiece.

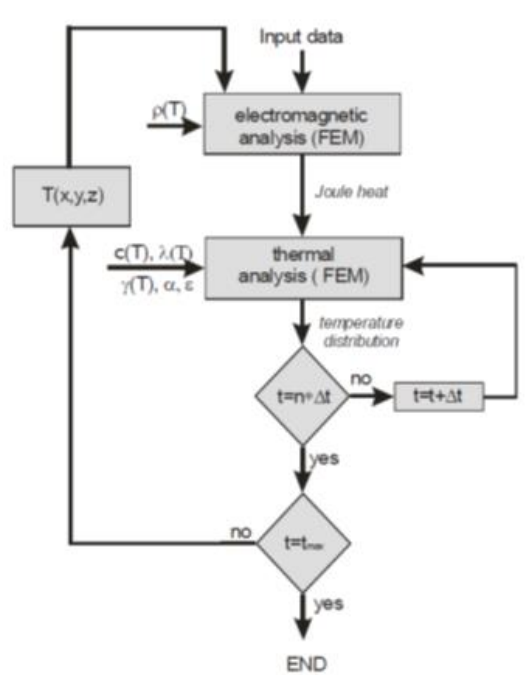


Figure 6: Procedure for The Computer Simulation.

The different computer simulation geometries are shown in Figures (7a)-(7b)

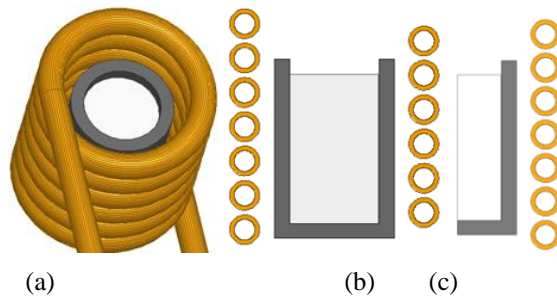


Figure 7: Computer Simulation Geometries; a. 3-D Model, 2-D Model and c. 2-D Axisymmetric

Table 3: Coil-Workpiece material properties

Materials	Graphite	Copper	Aluminum
Dimension	As in Table 2	As in Table 1	23mm (D) 45mm(H)
Electrical conductivity	3.3×10^5	5.8×10^7	3.703×10^7
Relative permeability(μ_r)	63000	1	1
Relative permittivity (ϵ_r)	10-15	1	1
Density	2.26g/cm^3		
Isotropic thermal conductivity (K)	24	400	237.5
Specific heat capacity (C_p)	774	378.34	933.33

IV. CONCLUSION

All the parameter in Table 1 and Table 2 were presuming dimension base on the size of propose design IHS and Table 3 contain standard engineering constant for the metallic materials selected. The current of 60 A and frequency 100 kHz was used in simulation result yield were coil inductance value, coil resistance and time to reach 1350 oC are 3 μH , 2 Ω and 600 s respectively. The capacitance requires to achieve the resonant 100 kHz is 845 nF.

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