

Experimental and Numerical Study for the Representation of Standing Waves in a Kundt's Tube

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Abstract- *The study of the behavior of a standing wave inside a resonant cavity, such as a closed tube, has always been of great revelation in the manufacture of wind musical instruments, in addition to research carried out on the transfer and absorption of heat, energy and pressure for the acoustic characterization of materials by injecting sound waves at one end of the Kundt's tube. In this work, we did the implementation of two experiments for measure the speed of sound, using a Kundt's tube where also we obtained the bellies and nodes of the sound wave and we verified with a computational code that was written for us in Python, in which we made simulation of the temporal evolution was achieved of the wave that vibrated inside the tube. Also, we show the wave profile for the first four harmonics with computational simulations and we presented the experimental results by two different methods, these were compared with the obtained using the equations modeled by the undulatory theory of physics. Finally, we show in this article, the respective errors obtained from these two experimental configurations, which were made with the purpose of showing university students the importance of being able to visualize the wave phenomenon of sound inside a Kundt's tube. (Kundt's tube, resonant cavity, harmonics, stationary waves, fundamental frequency).*

I. INTRODUCTION

We have shown in other works the importance of the use of experimental and computational tools to improve the understanding of some physical phenomena of great importance in mechanical and electrical engineering [1-3].

Undulatory phenomena have been of great importance in the study of physics [4-11], for teachers, researchers

and engineers in many fields such as optics and acoustics, in which these phenomena have several relevant applications. Using the fundamental concepts, belonging to the branch of physics waves and particles, we present an experimental procedure, with which the definition of a standing wave can be explained, as the superposition of two waves propagating in opposite directions [9,10], besides being able to show the position of the nodes and bellies using small spheres of polystyrene inside the tube (See the Figure 3).

With the purpose of model the equations that govern the movement of the wave inside the tube, we realized a computational simulation, with which we show the change in the behaviour of the wave for different frequency values, in order to better observe the vibratory movement of the standing wave present in the tube, here it was possible to identify and verify the corresponding independence of the number of nodes and antinodes as a function of the oscillation fundamental frequency and the harmonics of the acoustic wave.

The remaining part of this paper is organized as follows: Section II, describes the theoretical background model corresponding to the calculates that we are needing for modelling the phenomenon to study. In Section III, shown the experimental procedure for the data-taking. In Section IV, presents the results of the numerical analysis for the four first harmonic that is developed with a series of simulations, after of this we showed the experimental results of calculation sound velocity in the Kundt's tube, using two different methods.

II. THEORETICAL BACKGROUND

The physical system to be studied is shown in the Figure 1.

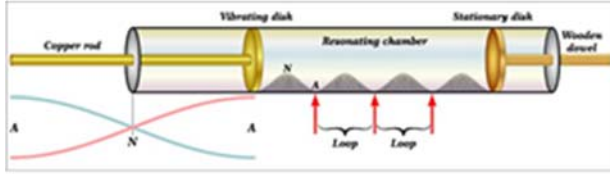


Figure 1. Physical scheme of system Kundt's tube, where A indicates the position of the antinodes and N the position of the nodes.

The theoretical basis for understanding the operation and being able to interpret the experimental results with the Kundt's tube focuses on the study of standing waves and their discretization in normal modes [11,14], stationary waves are a particular case of the phenomenon of wave interference, since it deals with the superposition of waves with equal amplitudes and wavelengths, which travel in the same direction but in the opposite directions [6,7].

In the standing waves in tubes, each gas molecule oscillates around its equilibrium position when the tube is excited at a certain frequency. In the case of the Kundt's tube, the standing waves are inside a tube that usually has one end closed.

When the tube is closed at both ends it is called a closed tube. In addition, it is of great importance to emphasize that the amplitude of the resulting stationary wave generated, despising the effects of absorption and damping, must be twice the amplitude of the waves that initially overlap to form it. The equation that models the oscillatory behavior of a standing wave is expressed in the Equation (1)

$$y(x,t) = 2A \sin(kx) \cos(\omega t), (1)$$

where k and ω , are the number of wave and the angular frequency respectively, their mathematical expression is given by:

$$k = \frac{2\pi}{\lambda}, \omega = 2\pi f. (2)$$

where λ is the wavelength, in order to find the position of the nodes and bellies inside the tube, we

must take into account Equation (1), where to find the nodal points of zero oscillation, we do:

$$\sin\left(\frac{2\pi}{\lambda}x\right) = 0. (3)$$

The condition for the sinusoidal function to be canceled is fulfilled only for n integer values of π .

$$\frac{2\pi}{\lambda}x = n\pi, (4)$$

from the above equation it follows that the standing wave presents a node for the position x :

$$x = \frac{n\lambda}{2}. (5)$$

It is also very important to find the position of the bellies of the wave, which will be the values of maximum amplitude of the stationary wave, algebraically this is expressed as:

$$\sin\left(\frac{2\pi}{\lambda}x\right) = \pm 1, (6)$$

The condition for the function $\sin(kx)$ to be maximum is given for odd integer values $2n+1$, in the following way:

$$\frac{2\pi}{\lambda}x = \frac{(2n+1)\pi}{2}. (7)$$

From the above equation, we can deduce that the position for each belly is:

$$x = (2n+1)\frac{\lambda}{4}. (8)$$

From equations (5) and (8) it can be seen that the separation distance between two nodes and two consecutive bellies is equal to half a wavelength $\lambda/2$.

Continuing with the boundary conditions of a Kundt's tube, for which at the two ends of the tube, a node is presented in:

$$y(0,t) = 0, y(L,t) = 0. (9)$$

Where L is the length of the tube, applying these boundary conditions to equation (1), we obtain the n normal modes of system oscillation, which are:

$$2A \sin(kx) \cos(\omega t) = 0, (10)$$

continuing in the operation for the number wave k , we obtained:

$$\sin(kL) = 0, k_n = \frac{n\pi}{L}, (11)$$

where n are integers values, the above equation as a function of the wavelength, can be expressed as follows,

$$\lambda_n = \frac{2\pi}{k_n} = \frac{2L}{n}. \quad (12)$$

If we use the ratio of the speed of propagation of the wave, depending on the wavelength and frequency, we can obtain the frequency for the different harmonics n , in the following way:

$$v = \lambda_n f, f_n = \frac{nv}{2L}, \quad (13)$$

with this last expression, we can calculate the speed of sound inside the resonant chamber,

$$v_n = \frac{2L}{n} f. \quad (14)$$

III. EXPERIMENTAL STUDY

To perform the experimental study of standing waves in the Kundt's tubes, the assemblies shown in Figure 2 and Figure 3 were made, where in the experiment of Figure 2, it was necessary to use a digital oscilloscope to detect the position of the antinodal points and in Figure 3 this detection was used observing the organization of the small polystyrene spheres [3].

For both experiments, we used an signal generator and measured the separation distance between two consecutive bellies (antinodes) L_x , also we registered the wavelength value and period T , were recorded for six different frequencies values, these data are recorded in the Table 1 and Table 2.

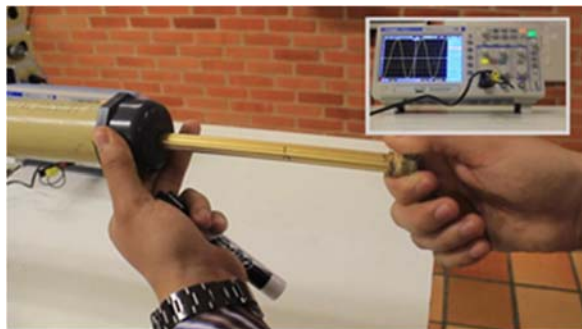


Figure 2. Experimental setup of a Kundt's tube, using a PVC tube and an oscilloscope.

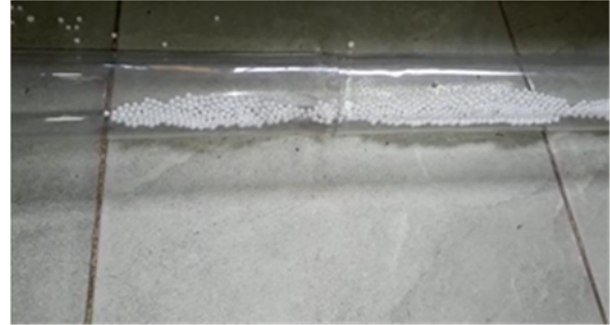


Figure 3. Experimental setup of a Kundt's tube, using an acrylic tube and small spheres of polystyrene inside the chamber.

| Frequency [hz] | L_x [m] | λ [m] | T [ms] |
|----------------|-----------|---------------|--------|
| 600 | 0.288 | 0.576 | 1.666 |
| 800 | 0.215 | 0.430 | 1.250 |
| 1000 | 0.173 | 0.346 | 1.000 |
| 1200 | 0.142 | 0.284 | 0.833 |
| 1400 | 0.127 | 0.254 | 0.714 |
| 1600 | 0.112 | 0.224 | 0.625 |

Table 1. Experimental data, using the assembly of the Figure 2.

| Frequency [hz] | L_x [m] | λ [m] | T [ms] |
|----------------|-----------|---------------|--------|
| 600 | 0.275 | 0.550 | 1.666 |
| 800 | 0.221 | 0.442 | 1.250 |
| 1000 | 0.178 | 0.356 | 1.000 |
| 1200 | 0.143 | 0.286 | 0.833 |
| 1400 | 0.129 | 0.258 | 0.714 |
| 1600 | 0.114 | 0.248 | 0.625 |

Table 2: Experimental data, using the assembly of the Figure 3.

It is of great importance to highlight that for the experimental assembly shown in Figure 2, we vary the length of the tube and frequency of speaker, where measure the wavelength by detecting two maximum of the amplitude of wave with the oscilloscope as shown in Figure 2. In the assembly of Figure 3, we have detected the bellies and nodes of the standing wave inside the tube, through visual observation of the location of the polystyrene spheres and unlike the

previous experiment the length of the tube in this case It remains constant and we vary only the frequency of the speaker located at one end of the tube. Something very similar was made in the experiment shown in reference [15].

IV. NUMERICAL ANALYSIS

One of the relevant features of this work consists in the elaboration of a computer code written in Python[12,13], and executed using the Jupyter Notebook tool [4], with which it was possible to observe the different harmonics for the case of a tube of 1 meter in length.

We did the simulation of the standing wave in a kundt's tube, for the first five harmonics using the equations (13) and (14) as shown in Table 3, where we take the speed of sound equal to 243 m/s [14,15].

| Length of tube | 1 m | | |
|----------------|-----|---------------|--------|
| Frequency [hz] | n | λ [m] | T [ms] |
| 171 | 1 | 2 | 5.847 |
| 343 | 2 | 1 | 2.915 |
| 514 | 3 | 0.660 | 1.945 |
| 686 | 4 | 0.500 | 1.457 |
| 857 | 5 | 0.400 | 1.167 |

Table 3: Number of harmonics present in the tube as a function of frequency and period.

In the figures 4-7, we can observe the behavior of the standing wave inside the Kundt's tube for the first four harmonics.

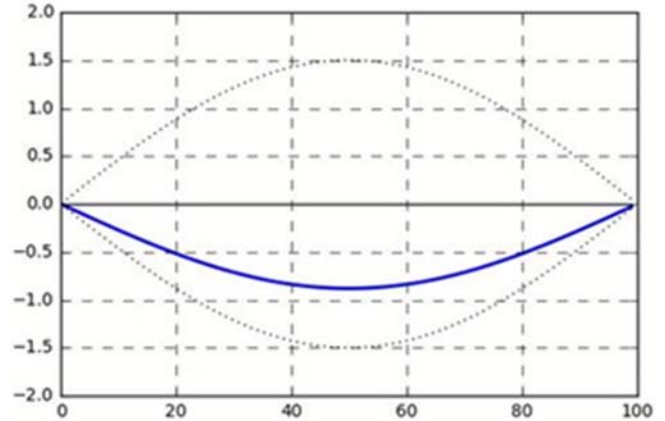


Figure 4. Behavior of the standing wave for the n first four harmonics, on the X axis is the length of the tube and on the Y axis the amplitude of the wave, n = 1.

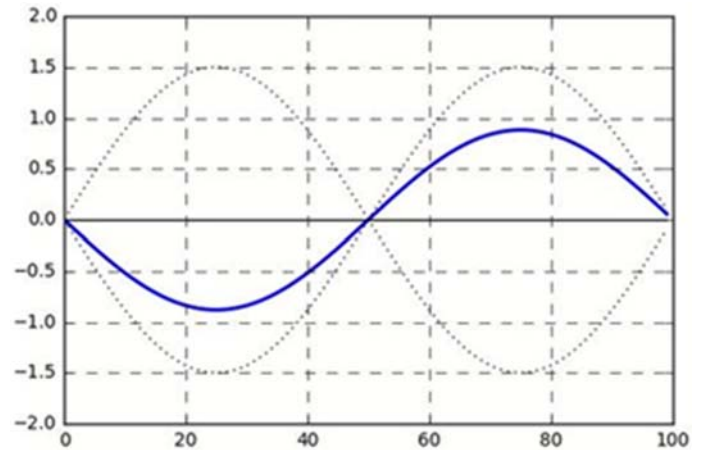


Figure 5. Behavior of the standing wave for the n first four harmonics, on the X axis is the length of the wave and on the Y axis the amplitude of the wave, n = 2.

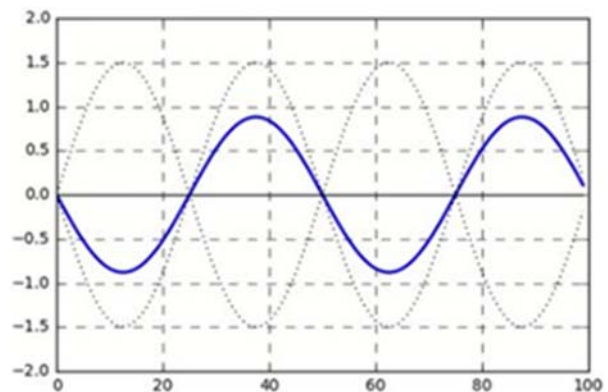


Figure 6. Behavior of the standing wave for the n first four harmonics, on the X axis is the length of the tube and on the Y axis the amplitude of the wave, n = 3.

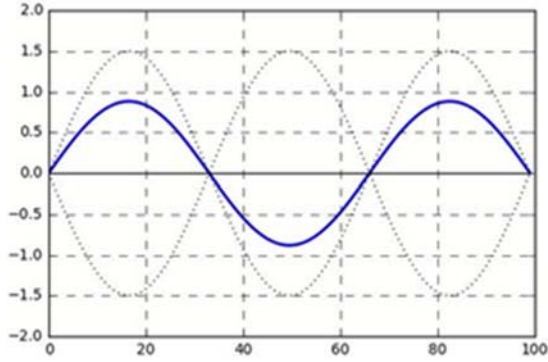


Figure 7. Behavior of the standing wave for the n first four harmonics, on the X axis is the length of the tube and on the Y axis the amplitude of the wave, $n = 4$.

V. EXPERIMENTAL ANALYSIS

In order to measure the speed of sound in the air [14,15], using the two experimental assemblies shown in the section 3, we look for an analytical model that can be adjusted to the experimental data presented in tables 1 and 2.

We used the values in Table 1 and performing a linear fit of the data with the Equation (15), we obtained the relationship shown in figures 4-7.

On the other hand, the linear fit for the data shown in Table 2 is shown in Figure 5, using the linear model shown in Equation (15), Table 4 was constructed, in which we can compare the experimental error obtained for both types of experimental assemblies, where the values obtained by these two experimental adjustments were compared with the speed of sound at 20 degrees Celsius ($v \approx 343.2$ m/s).

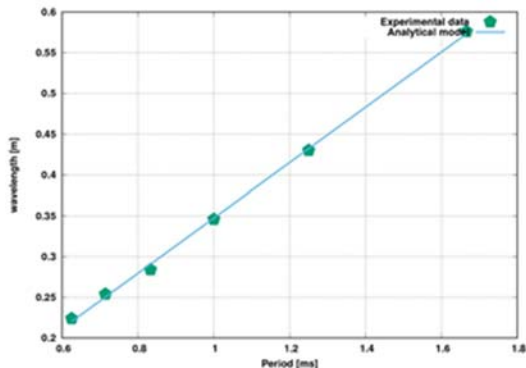


Figure 8. Linear fitting for the experiment in which we use the PVC tube.

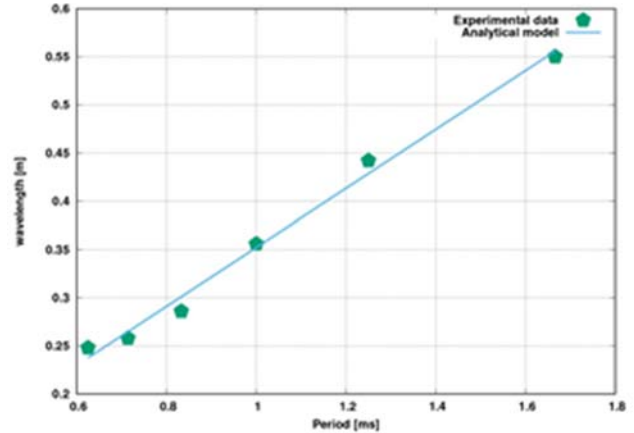


Figure 9. Linear fitting for the experiment in which we use the acrylic tube.

| Experimental setup | Speed of sound [m/s] | Absolute error [m/s] | % Relative error |
|------------------------|----------------------|----------------------|------------------|
| No.1 (tube of PVC) | 339.099 | 4.101 | 1.194 |
| No.2 (tube of acrylic) | 305.749 | 37.451 | 10.912 |

Table 4: Experimental errors obtained for the two experimental configurations implemented.

From the results shown in Table 4, we could notice that a lower percentage of error is obtained for the case of the first experimental assembly, in which we use the digital oscilloscope, unlike the second experimental setup, in which we observe the spatial distribution of the small spheres of polystyrene, for measured the distance between two consecutive bellies.

VI. CONCLUSIONS

In this article, we were able to show two experimental methods, in which we were able to measure the speed of sound, with an experimental percentage relative error below 11%, also we present the results obtained from the computational simulation; for the first four harmonics and with this, we can understand better the wave phenomenon that occurs inside a Kundt tube.

Finally, we show the obtained results of the speed of sound, using two different and independent experimental arrangements, in which we have been able to implement two kundt's tubes to show the wave

phenomenon (standing wave) produced inside a tube of these tubes, where the respective comparisons of the results were made, where it was possible to demonstrate that a very low experimental error was obtained for the first method.

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REFERENCES

- [1] Rubén Rodríguez, Jhon Amaya, Alex Estupiñán, Design and Experimental Study of Tesla's Thermomagnetic Engine, Teacher Education and Curriculum Studies. Vol. 4, No. 2, 2019, pp. 33-38. doi: 10.11648/j.tecs.20190402.11
- [2] K. L. Cristiano, A. Estupiñán and D. A. Triana. Python script used as a simulator for the teaching of the electric field in electromagnetism course. (2019, June). In Journal of Physics: Conference Series (Vol. 1247, No. 1, p. 012044). IOP Publishing.
- [3] Cristiano, K. L., & Triana, D. A. (2019, January). Google classroom as a tool-mediated for learning. In Journal of Physics: Conference Series (Vol. 1161, No. 1, p. 012020). IOP Publishing.
- [4] Elmore, W. C., Elmore, W. C., & Heald, M. A. (1985). Physics of waves. Courier Corporation.
- [5] Molina-Coronell, J., Celin Mancera, W., & Solano Mazo, C. (2017). Analizando ondas estacionarias en tubos abiertos y cerrados con el uso de smartphone. Revista mexicana de física E, 63(1), 76-82.
- [6] Patterson, H. S., & Cawood, W. (1931). Phenomena in a sounding tube. Nature, 127(3209), 667.
- [7] Vigran, T. E., Kelders, L., Lauriks, W., Leclaire, P., & Johansen, T. F. (1997). Prediction and measurements of the influence of boundary conditions in a standing wave tube. Acta Acustica United with Acustica, 83(3), 419-423.
- [8] Parolin, S. O., & Pezzi, G. (2015). Kundt's tube experiment using smartphones. Physics Education, 50(4), 443.
- [9] Alba, J., Marant, V., Aguilera, J. L., & Ramis, J. (2006). Criterios de selección de materiales acústicos absorbentes con técnicas basadas en tubo de Kundt. TecniAcústica, Gandía.
- [10] Bryant, P. A., & Morgan, M. E. (2004). Labworks and the Kundt's tube: A new way to determine the heat capacities of gases. Journal of chemical education, 81(1), 113.
- [11] Vanhuyse, J., Deckers, E., Jonckheere, S., Pluymers, B., & Desmet, W. (2016). Global optimisation methods for poroelastic material characterisation using a clamped sample in a kundt tube setup. Mechanical Systems and Signal Processing, 68, 462-478.
- [12] González Duque, R. (2014). Python para todos. Creative Commons Reconocimiento, 2.
- [13] Kluyver, T., Ragan-Kelley, B., Pérez, F., Granger, B. E., Bussonnier, M., Frederic, J., ... & Ivanov, P. (2016, May). Jupyter Notebooks-a publishing format for reproducible computational workflows. In ELPUB (pp. 87-90).
- [14] Da Costa Saab, S., Cássaro, F. A. M., & Brinatti, A. M. (2005). Laboratório caseiro: tubo de ensaio adaptado como tubo de kundt para medir a velocidade do som no ar. Caderno Brasileiro de Ensino de Física, 22(1), 112-120.
- [15] Nursulistiyo, E. (2018, March). Design and development of multipurpose Kundt's tube as physics learning media. In Journal of Physics: Conference Series (Vol. 983, No. 1, p. 012011). IOP Publishing.