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Prediction for Future Population Growth of Bangladesh by Using Exponential & Logistic Model

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Abstract- Bangladesh is an overpopulated country. It is the world's 8th most populous country in south Asia. Population problem in Bangladesh is one of the most serious issues in the recent years. So the increasing population is a great threat to the nation and for this reason, the projection of the population of Bangladesh is very much essential. The purpose of this paper is to model and design the population growth in Bangladesh to predict the future population size. The exponential and the logistic growth models are used to predict the population of Bangladesh during 2000 to 2050 using the actual data from 2000 to 2019. By using the exponential growth model, the predicted growth rate has been estimated approximately 2% and the population of Bangladesh has been predicted to be 340.3 million in 2050. We have determined the carrying capacity (K) and vital coefficients and b for the population prediction. Thus, the population growth rate of Bangladesh according to the logistic model has been estimated approximately 4% and the total Population of Bangladesh has been predicted to be 236.3 million in 2050.

I. INTRODUCTION

Population is the most vital element of world but population projection has become one of the most alarming problems in the world. Population size and growth in a country directly influence the situation of the economy, policy, culture, education and environment of that country and determine exploring the cost of natural resources [26]. Every government and collective sectors always require proper idea about the future size of various subsistence like population, resources, demands and consumptions for their future activities [11, 25]. To obtain this information, the behavior of the connected variables is analyzed based on the previous data by the statisticians and mathematicians and using the conclusions drawn from the analysis, they make future projections of the aimed at variable [2, 11, 4]. There are enormous concerns about the consequences of human population growth for social, environmental and economic development which Intensify all these problems in population growth. Mathematical modeling is a broad interdisciplinary science that uses mathematical and computational techniques to model and elucidate the phenomena arising in real life problems [25]. Thus, it is a process of mimicking reality by using the language of mathematics in terms of differential equations which describe the changing phenomena of the underlying systems. The population models determine the present state in terms of the past and the future state in terms of its present state which describes a typical human way of coping with the reality. The main reason for solving many differential equations is to learn the behavior about an underlying physical process that the equation is believed to model [3, 5]. The population models are used in forecasting of population growth, population decaying, maximum or minimum production, food preserving, capacity, environmental, and many other applications [1]. Mathematical models can take many forms including dynamical systems, statistical models and differential equations [12]. These and other types of models can overlap, with a given model involving a variety of abstract structures. A population model is a type of mathematical model that is applied to the study of population dynamics. Models allow a better understanding of how complex interactions and processes work [6]. Modeling of dynamic interactions in nature can provide a manageable way of understanding how numbers change over time or in relation to each other. We refer readers to [7, 20-22] for some recent studies on population models in analyzing biological systems.

In this paper, we discuss an overview of population growth models in terms of nonlinear differential equations in the form of mathematical modeling which have been applied to study the future prediction of human population in Bangladesh. The first order differential equations have been used to govern the growth of the human species. Two simple deterministic population models, namely, Malthusian growth model and logistic growth model have been studied and analyzed to discuss the dynamical behavior of the population viability analysis for both short-term and long-term prediction in Bangladesh. Our study shows that Malthusian model is more accurate for short-term prediction, but for long-term prediction, Malthusian model is not suitable. On the other hand, logistic model is more realistic than Malthusian model for long term prediction. Both the exponential and the logistic growth models have been applied to predict the population of Bangladesh during 2000 to 2050 using the actual data from 2000 to 2019 collected from the Bangladesh statistical bureau [28]. We have shown by the exponential growth model that the predicted growth rate is approximately 2% and the population of Bangladesh has been predicted to be 340.3 million by 2050. In similar fashion, we have determined the carrying capacity (K) and vital coefficients and for the population prediction in logistic growth model. Our analysis shows that the population growth rate of Bangladesh according to the logistic model has been estimated approximately 4% and the total population of Bangladesh has been predicted to be 236.3 million in 2050. Our results shows that the predicted populations of Bangladesh are very similar to the present trends of the population size.

II. MODEL

2.1 Malthusian growth model

A Malthusian growth model is an essentially exponential growth based on a constant rate. The model is named after Thomas Robert Malthus. The Malthusian growth model is also known as a simple exponential growth model [7]. Malthusian model is single species population model. Let t denotes the time and (t) denotes the number of individuals presents at a time t. In practice (t) is a non-negative integer.

We assume that x (t) is continuously differentiable [1].

The growth rate ofpopulation is the rate at which population changes [5]. If the population (t) at time t changes to $(t+\partial t)$ the average per capita growth rate at the time t is $\frac{x(t+\partial t)-x(t)}{x(t)t}$

Taking limit $\partial t \to 0$, we get the instantaneous growth rate at the time *t*

$$\lim_{\partial t \to 0} \frac{x(t+\partial t)-x(t)}{x(t)\partial t} = \frac{x'(t)}{x(t)}$$

Now let, b = Intrinsic birth rate

=The average number of off spring born per individuals per time. d =deathIntrinsicrate.

=The fraction of individuals of the population dies per unit time.

$r = b - d$

= Intrinsic growth rate of the population.

= Excess of birth over death per unit time individuals. Now, we consider a single species of population, the growth model is described by

$$\frac{x'(t)}{x(t)} = r$$

$x'(t) = rx(t)$ (1)

With the initial population $x(t_0) = x_0 > 0$. Thus we have the

Mathematical model described the growth of single species population as$x'(t) = rx(t)$, $x(t_0) = x_0 > 0$

General solution of this equation (1) is$x(t) = ce^{rt}$. For the initial condition $x(t_0) = x_0$

We have,$x(t_0) = ce^{rt_0} = x_0$

$$c = x_0 e^{-rt_0}$$

$$x(t) = x_0 e^{-rt_0} e^{rt} = x_0 e^{r(t-t_0)}$$

Therefore$x(t) = x_0 e^{r(t-t_0)}$ (2

Which is known as Malthusian law of growth [5]

2.2 Logistic model

Logistic model is a modification of Malthusian model. The Logistic model is a model of population growth first published by a Belgian mathematical biologist Pierre Verhulst [6]. He observed that the population growth not only depends on the population size but also on how far this size is from its carrying capacity [10]. He modified Malthusian model to make a population size proportional to both the previous population and a new term [13].

$$\frac{a-bp(t)}{a} \quad (3)$$

Where a and b are the vital coefficients of the population.

Now as the population value gets closer to $\frac{a}{b}$ this new term will become very small and tend to zero, which gives the right feedback to limit the population growth [15]. Thus the second term models the competition for available resources, which tends to unite the population growth [19]. So the modified equation using this new term is

$$\frac{dp}{dt} = \frac{ap(t)[a-bp(t)]}{a} \quad (4)$$

This equation is known as the logistic law of population growth model.

Solving (4) and applying the initial condition $p(t_0) = p_0$ then (4) become

$$\frac{dp}{dt} = ap - bb^2$$

$$\frac{dp}{p(a-bp)} = dt \quad (5)$$

By the application of separation of variables

$$\frac{1}{a}\left(\frac{1}{p} + \frac{b}{a-bp}\right) = dt \quad (6)$$

Now integrating (6), we obtain

$$\frac{1}{a}[lnp - \ln(a-bp)] = t + c \quad (7)$$

At t=0 we get $p(t_0) = p_0$

$$c = \frac{1}{a}[lnp - \ln(a-bp_0)] \quad (8)$$

Equation (7) becomes

$$\frac{1}{a}[lnp - \ln(a-bp)] = t + \frac{1}{a}[lnp_0 - \ln(a-bp_0)]$$

$$P = \frac{\frac{a}{b}}{1+(\frac{\frac{a}{b}}{p_0}-1)e^{-at}} \quad (9)$$

If we take the limit of equation (9) as $t \to \infty$, we get

$$p_\infty = \lim p = \frac{a}{b}$$

(10)

Taking t=0, t=1, t=2, the values of p_0, p_1, p_2 respectively.

Then we get from equation (9)

$$\frac{b}{a}(1-e^{-a}) = \frac{1}{p_1} - \frac{e^{-a}}{p_0}$$

$$\frac{b}{a}(1-e^{-2a}) = \frac{1}{p_2} - \frac{e^{-2a}}{p_0} \quad (11)$$

Eliminating $\frac{b}{a}$ we get

$$e^{-a} = \frac{p_0(p_2-p_1)}{p_2(p_1-p_0)} \quad (12)$$

Substituting the values of e^{-a} in to the equation (9), we obtain

$$\frac{b}{a} = \frac{{p_1}^2 - p_0p_2}{p_1(p_0p_1-2p_0p_1+p_1p_2)} \quad (13)$$

Thus, limiting the value of P, we get

$$p_{max} = \lim_{t\to\infty} P = \frac{a}{b} = \frac{p_1(p_0p_1-2p_0p_2+p_1p_2)}{{p_1}^2 - p_0p_2} \quad (14)$$

III. RESULTS AND DISCUSSIONS

To predict the future population of Bangladesh, we need to determine growth rate of Bangladesh using the exponential growth model in (2). Using the actual population of Bangladesh in Table 1 below with t = 0 i.e. $t0$ corresponding to the year 2000, we have P (0) = 131.6, Again t = 1 i.e. $t1$ corresponding to the year 2001, we have (1) = 134.1 . Now we get from the equation (2),

$$p_1 = p_0 e^{k(t_1-t_0)}$$

$$134.1 = 131.6e^{k(1-0)}$$

K = 0.02

Hence, the general solution is

P (t) =131.6$e^{0.019t}$ (15)

This suggest that the prediction rate of population growth is 2% in Bangladesh with this we project the population of Bangladesh from 2010 to 2050 .Again based on table 1, let t=0.1 and 2 correspond to the year 2000, 2001 and 2003 respectively. Then p_0 ,p_1 and p_2 also corresponds 131.6, 134.1 and 136.6 (in millions) substituting the value of p_0 ,p_1 and p_2 into (14), we get

$$p_{max} = \frac{a}{b} = 268.2(millions) \quad (16)$$

This is the predicted carrying capacity of the population of Bangladesh. e^{-a}= 0.96

Hence $a = -ln$ (0.96). Therefore value of a=0.04

This is also implies that the predicted rate of population growth of Bangladesh is approximately 4% with the logistic growth model[13]. Now from equation (16) we obtain b= $1.49*10^{-10}$
Substituting the values of p_0, $a and b$ into the equation (9) then we get

$$p = \frac{268.2}{1+1.04(.96)^t} \qquad (17)$$

This equation is used to compute the predicted values of the population [17]. The predicted population of Bangladesh using both models is presented in the Table 1 from 2000 to 2050 with actual data [25].We have calculated the predicted population of Bangladesh from the equations (15) and (17), In Table 1 we get that the predicted populations of Bangladesh are expected to be 188.8 million (approximately) and 181.4 million (approximately) in 2019 by using Exponential model and the Logistic model respectively.

Let us now present the numerical simulation results of the actual and projected populations of Bangladesh from the year 2000 to 2019 using Malthusian model (15) and Logistic model (17) in Figures 1 and 2 respectively.

Year	Actual population (in millions)	Projected population (in millions)			
		Exponential model	Absolute Percentage Error	Logistic model	Absolute Percentage Error
2000	131.6	131.6	0.0%	131.5	-0.1%
2001	134.1	134.1	0.0%	134.2	0.1%
2002	136.6	136.7	0.1%	136.9	0.2%
2003	139.0	139.3	0.2%	139.7	0.5%
2004	141.3	142	0.5%	142.4	0.8%
2005	143.4	144.7	0.9%	145.1	1.2%
2006	145.4	147.5	1.5%	147.8	1.7%
2007	147.1	150.3	2.1%	150.5	2.3%
2008	148.8	153.2	3.0%	153.2	3.0%
2009	150.5	156.1	3.8%	155.9	3.6%
2010	152.2	159.1	4.6%	158.6	4.2%
2011	153.9	162.2	5.4%	161.2	4.7%
2012	155.7	165.3	6.1%	163.8	5.2%
2013	157.6	168.5	6.9%	166.4	5.6%
2014	159.4	171.7	7.7%	169.0	6.0%
2015	161.2	175	8.6%	171.5	6.4%
2016	163.0	178.4	9.5%	174.0	6.8%
2017	164.7	181.8	10.4%	176.5	7.2%
2018	166.4	185.3	11.4%	178.9	7.5%
2019	168.1	188.8	12.3%	181.4	7.9%
2020		192.4		183.7	
2021		196.1		186.1	
2022		199.9		188.4	

2023		203.7		190.7	
2024		207.6		192.9	
2025		211.6		195.1	
2026		215.7		197.2	
2027		219.8		199.3	
2028		224		201.4	
2029		228.3		203.4	
2030		232.7		205.4	
2031		237.2		207.4	
2032		241.7		209.3	
2033		246.4		211.1	
2034		251.1		212.9	
2035		255.9		214.7	
2036		260.8		216.4	
2037		265.8		218.1	
2038		270.9		219.8	
2039		276.1		221.4	
2040		281.4		222.9	
2041		286.8		224.4	
2042		292.3		225.9	
2043		297.9		227.3	
2044		303.6		228.7	
2045		309.4		230.1	
2046		315.4		231.4	
2047		321.4		232.7	
2048		327.6		233.9	
2049		333.9		235.1	
2050		340.3		236.3	
Mean Absolute			4.7%		3.7%

Graph

Figure 1. The actual (blue) and predicted (read) populations of Bangladesh using Malthusian model (15) from 2000 to 2019

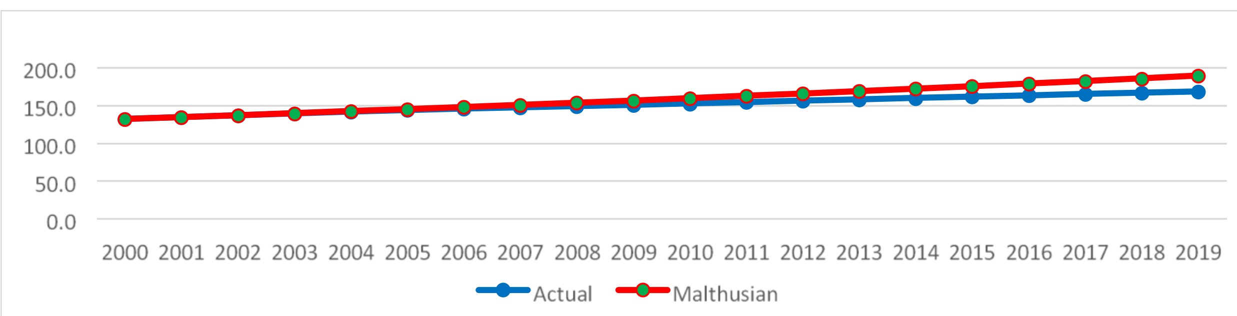


Figure 2. The actual (blue) and predicted (red) populations of Bangladesh using Logistic model (17) from 2000 to 2019

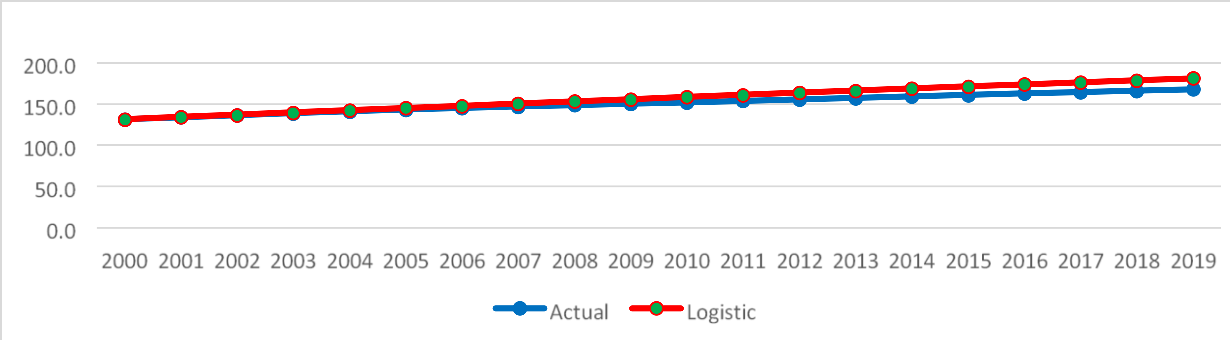
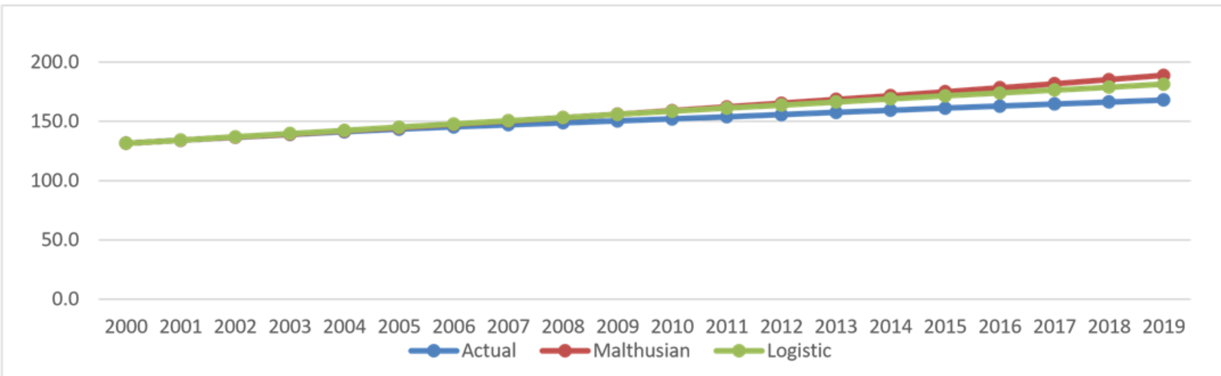


Figure 3. Comparison of population projection between Malthusian and Logistic model with actual data from 2000 to 2019



Now, a combined graph of the Malthusian and Logistic models showing a comparative population size in Bangladesh from 2000 to 2019 is shown in Figure 3.

Again we have calculated the predicted population of Bangladesh from 2000 to 2050 using the equations (15) and (17). As shown in Table 1, we get that the predicted populations of Bangladesh are expected to be 340.3 million (approximately) and 236.3 million (approximately) in 2050 by using Exponential model and the Logistic model respectively. In this situation, population growth is the pressing problem of Bangladesh like every developing country. For our limited resource, it will too difficult to cope with this over population but Bangladesh tries to reduce population growth rate, increase Literacy rate and create mass awareness to overcome this problem.

Now we present the numerical simulation results of the actual and projected populations of Bangladesh from the year 2000 to 2050 using Malthusian model (15) and Logistic model (17) in Figures 4 and 5 respectively.

Figure 4. The actual (blue) and predicted (green) populations of Bangladesh using Malthusian model (15) from 2000 to 2050

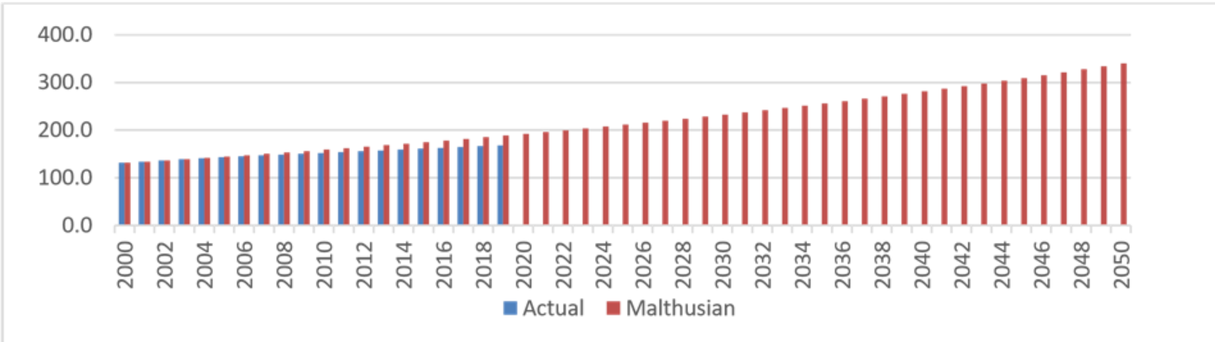
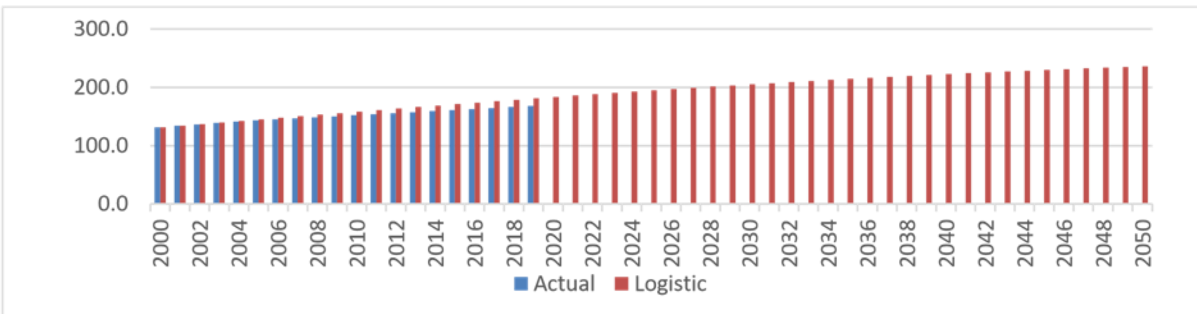


Figure 5. The actual (blue) and predicted (red) populations of Bangladesh using Logistic model (17) from 2000 to 2050



IV. CONCLUSIONS

In this paper, we consider two simple deterministic population growth models: the exponential and the logistic and make a comparison to predict the future population of Bangladesh. Our analysis gives good approximation for long term prediction of the population growth trends in comparison to present scenario of human population in Bangladesh. We analyze and investigate to predict the populations for 50 years, which gives the carrying capacity of around 268.2 million and this leads to an estimated population of around 268 million in 2050. According to the Bangladesh statistical bureau, the population of Bangladesh was around 161.2 million and 163 million in the years 2015 and 2016 respectively which are very close to our predicted population in comparison to the same years. So, our study provides a better prediction for the future population size of Bangladesh.

Finally, we find that according to the exponential model the predicted growth rate is approximately 2% and predicted population of Bangladesh is 340.3 million in 2050 with a Mean Absolute Percentage Error (MAPE) of 4.7%. On the other hand, the population growth rate of Bangladesh is approximately 4% according to logistic model and the carrying capacity for the population of Bangladesh is calculated as 268.2 million. By the Logistic model, the population of Bangladesh is calculated to be 236.3 million in 2050 with a Mean Absolute Percentage Error (MAPE) of 3.7%. As we know that the vital coefficients are responsible for population growth of any country. So we have calculated the vital coefficients a and b are 0.04 and $1.49*10{-10}$ respectively. It is also shown that Mean Absolute Percentage Error (MAPE) is very low in logistic model. From this point of view, we can finally conclude that the logistic model gives a good forecasting result as compared to the Malthusian model for a long term prediction.

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