

Investigation of the Binding Energy in Some Nuclides

KHIN ZAW¹, YI YI SOE²

¹Department of Physics, Mohnyin Degree College, Mohnyin, Myanmar

²Department of Engineering Physics, Technological University, Myitkyina, Myanmar

Abstract -- This paper focus on calculating the binding energy of some nuclide. Nuclear energy may be liberated by atomic fission, when heavy atomic nuclei are broken apart into lighter nuclei. It is also released during atomic fusion, when light nuclei are combined to form heavier nuclei. The nuclear force is effective only at very short distance (~fm). At greater distances, the electrostatic force dominates. The nucleus is stable against break-up into the individual nucleons, because its mass must be less than the mass of these constituents when separated by large distances. The missing mass serves as the binding energy of the nucleus that holds nucleons together. The curve of binding energy is a graph that plots the binding energy per nucleon against atomic mass (A). As the calculated results, in the periodic table of elements, the series of light elements from hydrogen up to sodium is observed to exhibit generally increasing binding energy per nucleon as the atomic mass increases. Over a large part of the periodic table the binding energy per nucleon is roughly constant. As nuclei get heavier than helium, their binding energy per nucleon grows more and more slowly, reaching its peak at iron(Fe). It is also found that the maximum binding energy is found in medium-sized nuclei. It is believed that the more higher binding energy the more tightly the nucleons are bound together. The curve has its main peaks at iron(Fe) and nickel(Ni) and then slowly decreases again, and also a narrow-isolated peak at helium(He), which as noted is very stable. So, we can conclude that the nuclides whose main peaks are prominent are said to be more bound than the neighboring nuclides.

power, nuclear weapons, nuclear medicine and magnetic-resonance imaging, industrial and agricultural isotopes, ion implantation in material engineering, and radiocarbon dating in geology and archaeology. Such applications are studied in the field of nuclear engineering.

Particle physics evolved out of nuclear physics and the two fields are typically taught in close association. Nuclear astrophysics, the application of nuclear physics to astrophysics, is crucial in explaining the inner workings of stars and the origin of the chemical elements.

On the other hand, atomic physics is the physics of the behavior of atoms and in particular of their electronic structure. That layer of the structure of matter depends for its existence on the nuclear atom but it is not markedly affected by the properties of the nucleus. Similarly the behavior of the nucleus is little affected by the electronic structure surrounding it so that nuclear physics can, as a subject, be almost completely isolated from atomic physics. However, all methods of detecting nuclear and particle radiation depend on atomic physics. From 1913 onwards atomic physics and nuclear physics advanced in parallel. The following of quantum mechanics brought order to atomic physics and is essential to the interpretation of nuclear physics.

I. INTRODUCTION

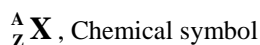
Nuclear physics is the field of physics that studies atomic nuclei and their constituents and interactions. Other forms of nuclear matter (nuclear size, nuclear shape, mass, nuclear matter density) are also studied. Nuclear physics should not be confused with atomic physics, which studies the atom as a whole, including its electrons.

Discoveries in nuclear physics have led to applications in many fields. This includes nuclear

Following the discovery of the radioactivity of uranium, there was a successful search for other naturally occurring radioactive elements and recognition by Rutherford and Soddy that radioactivity involved a change in the mass and chemical nature of an element. The new element produced in any radioactive change was itself frequently radioactive. In fact three radioactive series were recognized in which heavy elements lost mass and changed their atomic number in successive changes, the changes ending only when the element became an isotope of lead. These series may include thorium series, uranium series and neptunium series.

In all the spontaneous changes, three types of radiation were recognized. They are (i) α - rays, (ii) β - rays and (iii) γ -rays.

It was through the investigation of the scattering of α particles that the most important advances were made. Rutherford proposed the nuclear model of atoms and showed that such a model can explain both small and deflections of a beam of α particles traversing a thin foil and large-angle deflections. Rutherford used his model to calculate a cross-section for the large-angle scattering that was in accord with observation from which he observed that the nucleus is a massive core of an atom. Here, the notation for representing a given nuclide is



where, Z is atomic number, A is mass number of the nuclide and X is the symbol that describes the name of the nuclide. The word 'nuclide' is used to describe the nucleus of an element. The nucleons reside in the nucleus of the atom stickly together.

There are around 94 naturally occurring elements on earth. The atom of an element has a nucleus containing a specific number of protons, and some number of neutrons, which is often roughly a similar number.

Two atoms of the same element having different numbers of neutrons are known as isotopes of the element. Different isotopes may have different properties - for example one might be stable and another might be unstable, and gradually undergo radioactive decay to become another element.

The nuclear mass, nuclear size, nuclear shape and the density of the nucleus and the binding energy of the nucleons inside the nucleus are a chief consideration in nuclear physics. In the next, the role of the binding energy of some nuclides will be studied.

II. THEORETICAL BACKGROUND

The theoretical backgrounds of calculating system are;

- (1) Nuclear energy
- (2) Nuclear force

(3) Nuclear binding energy

(4) Mass defect

2.1 Nuclear Energy

An absorption or release of nuclear energy occurs in nuclear reactions or radioactive decay; those that absorb energy are called endothermic reactions and those that release energy are exothermic reactions. Energy is consumed or liberated because of differences in energy between the incoming and outgoing products of the nuclear transmutation.

The best-known classes of exothermic nuclear transmutations are fission and fusion. Nuclear energy may be liberated by atomic fission, when heavy atomic nuclei (like uranium and plutonium) are broken apart into lighter nuclei. The energy from fission is used to generate electric power in hundreds of locations worldwide. Nuclear energy is also released during atomic fusion, when light nuclei like hydrogen are combined to form heavier nuclei such as helium. The Sun and other stars use nuclear fusion to generate thermal energy which is later radiated from the surface, a type of stellar nucleosynthesis.

In any exothermic nuclear process, nuclear mass might ultimately be converted to thermal energy, given off as heat. In order to quantify the energy released or absorbed in any nuclear transmutation, one must know the nuclear binding energies of the nuclear components involved in the transmutation.

2.2 Nuclear Force

Electrons and nuclei are kept together by electrostatic attraction (negative attracts positive). Furthermore, electrons are sometimes shared by neighboring atoms or transferred to them, and this link between atoms is referred to as a chemical bond, and is responsible for the formation of all chemical compounds.

The force of electric attraction does not hold nuclei together, because all protons carry a positive charge and repel each other. Thus, electric forces do not hold nuclei together, because they act in the opposite direction. It has been established that binding neutrons to nuclei clearly requires a non-electrical

attraction.

Therefore, another force, called the nuclear force (or residual strong interaction) holds the nucleons of nuclei together. It consists of two portions: attractive portion and repulsive one. In the attractive core, the nucleons tend to attract strongly and on the other hand, the repulsive core prevents the nucleons not to confuse given in Figure(1). This force is a residuum of the strong interaction, which binds quarks into nucleons at an even smaller level of distance.

The nuclear force must be stronger than the electric repulsion at short distances, but weaker far away, or else different nuclei might tend to clump together. Therefore, it has short-range characteristics. The nuclear force is a close-range force: it is strongly attractive at a distance of 1.0fm and becomes extremely small beyond a distance of 2.5fm, and virtually no effect of this force is observed outside the nucleus. The nuclear force also pulls neutrons together, or neutrons and protons. This property is called the saturation. An analogy to the nuclear force is the force between two small magnets: magnets are very difficult to separate when stuck together, but once pulled a short distance apart, the force between them drops almost to zero.

Unlike gravity or electrical forces, the nuclear force is effective only at very short distance (~fm). At greater distances, the electrostatic force dominates: the protons repel each other because they are positively charged, and like charges repel. They cannot get close enough for the nuclear force, which attracts them to each other, to become important. Only under conditions of extreme pressure and temperature for example, within the core of a star, can such a process take place.

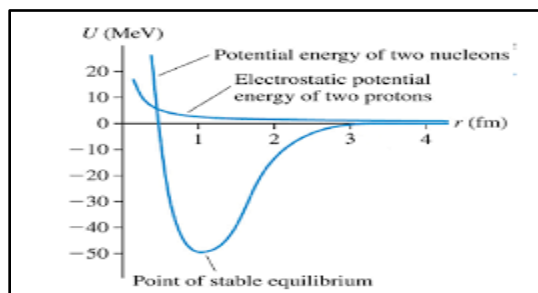


Figure 1. Nature of Nuclear Force Describing Attractive and Repulsive core

2.3 Nuclear Binding Energy

It has been known that the atomic mass is approximately AM_H where M_H is the hydrogen atomic mass. Let us consider only the nuclear mass for the moment, which constitutes almost the entire atomic mass and which we represent by $M(Z,A)$. Such a nucleus, in principle, can be pulled apart in Z protons and $N=A-Z$ neutrons. If the nucleus is stable against break-up into these individual nucleons, its mass must be less than the mass of these constituents when separated by large distances, that is

$$M(Z,A)c^2 = Z M_p c^2 + N M_n c^2 - B$$

The mass of proton and neutron are M_p and M_n respectively and B is the nuclear binding energy: this is the energy required to dismantle the nucleus into Z protons and N neutrons.

2.4 Mass Defect

Mass defect is the difference between the mass of a composite particle and the sum of the masses of its parts. For example, a helium atom containing 4 nucleons has a mass about 0.8% less than the total mass of four hydrogen nuclei (which contain one nucleon each).

The "mass defect" can be explained using Albert Einstein's formula $E = mc^2$, describing the equivalence of energy and mass. By this formula, adding energy also increases mass, whereas removing energy decreases mass. In the above example, the helium nucleus has four nucleons bound together, and the binding energy which holds them together is, in effect, the missing 0.8% of mass.

On the other hand, if one must inject energy to separate a system of particles into its components, then the initial mass is less than that of the components after they are separated. In the latter case, the energy injected is "stored" as potential energy, which shows as the increased mass of the components that store it. This is an example of the fact that energy of all types is seen in systems as mass, since mass and energy are equivalent, and each is a "property" of the other.

The latter scenario is the case with nuclei such as helium: to break them up into protons and neutrons, one must inject energy. On the other hand, if a process existed going in the opposite direction, by which hydrogen atoms could be combined to form helium, then energy would be released. The energy can be computed using $E = \Delta mc^2$ for each nucleus, where Δm is the difference between the mass of the helium nucleus and the mass of four protons (plus two electrons, absorbed to create the neutrons of helium).

Nuclear power is generated at present by breaking up uranium nuclei in nuclear power reactors, and capturing the released energy as heat, which is converted to electricity through Einstein's mass energy relation equation. A vast amount of energy can be obtained from a few gram of fuel nuclide.

III. THEORETICAL DEVELOPMENTS

(1) Determining nuclear binding energy

(2) Binding energy and nuclide masses

(3) Nuclear binding energy curve

3.1 Determining Nuclear Binding Energy

Mass defect is defined as the difference between the mass of a nucleus, and the sum of the masses of the nucleons of which it is composed. The mass defect is determined by calculating three quantities. These are: the actual mass of the nucleus, the composition of the nucleus (number of protons and of neutrons), and the masses of a proton and of a neutron. This is then followed by converting the mass defect into energy. This quantity is the nuclear binding energy; however, it must be expressed as energy per mole of atoms or as energy per nucleon.

3.2 Binding Energy and Nuclide Masses

Nuclear binding energy can be computed from the difference in mass of a nucleus [1], and the sum of the masses of the number of free neutrons and protons that make up the nucleus. Once this mass defect or mass deficiency, is known, Einstein's mass-energy equivalence formula $E = \Delta mc^2$ can be used to compute the binding energy of any nucleus. Here, the

following equation is used to calculate the binding energy B of the nuclides.

$$M(Z,A)c^2 = Z M_p c^2 + N M_n c^2 - B$$

$$B = Z M_p c^2 + N M_n c^2 - M(Z,A)c^2$$

where, B is the binding energy of the nuclide. $M(Z, A)$ is the mass of the nucleus obtained from spectrometer measurement of the masses of nuclei [1]. M_p is 1.007825 u and, M_n is 1.008665 u respectively [2]. $c^2 = 931.5$ MeV/u. Some of the naturally occurring and artificial nuclides are given in Table (1).

Dividing the binding energy of a nuclide with its respective number of nucleon gives rise the energy that a nucleon will accept to bind with its neighboring nucleon. These calculated results of the binding energy for some nuclides including their respective isotopes up to mass number of 240 are shown in the next section.

Table 1. Periodic Table of Some Nuclides with Their Respective Mass Number

3.3 Nuclear Binding Energy Curve

In the periodic table of elements, the series of light elements from hydrogen up to sodium is observed to exhibit generally increasing binding energy per nucleon as the atomic mass increases. This increase is generated by increasing forces per nucleon in the nucleus, as each additional nucleon is attracted by

other nearby nucleons, and thus more tightly bound to the whole.

The region of increasing binding energy is followed by a region of relative stability (saturation) in the sequence from magnesium ($A=24$) through xenon ($A=140$) in Figure(2). In this region, the nucleus has become large enough that nuclear forces no longer completely extend efficiently across its width. Attractive nuclear forces in this region, as atomic mass increases, are nearly balanced by repellent electromagnetic forces between protons, as the atomic number increases.

Finally, in elements heavier than xenon, there is a decrease in binding energy per nucleon as atomic number increases. In this region of nuclear size, electromagnetic repulsive forces are beginning to overcome the strong nuclear force attraction.

At the peak of binding energy, nickel ($A=62$) is the most tightly bound nucleus (per nucleon), followed by iron ($A=58$) and iron ($A=56$). This is the approximate basic reason why iron and nickel are very common metals in planetary cores.

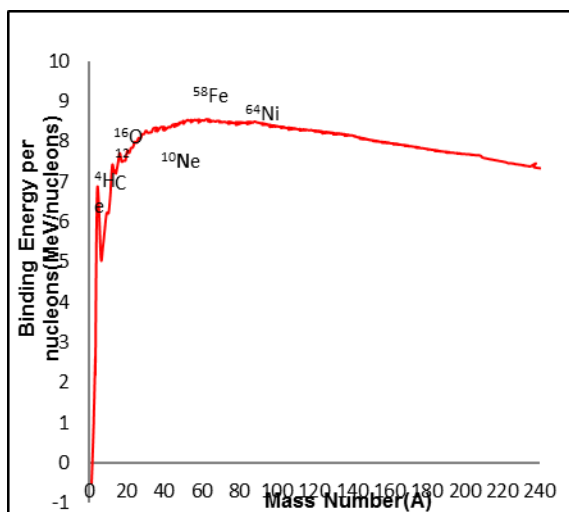


Figure 2. Binding Energy of Some Nuclides as a Function of the Number of Nucleons in the Nucleus

IV. RESULTS & DISCUSSION

The calculated results of binding energy for some nuclides in accord with atomic mass number are shown in Table (2). The curve of binding energy is a graph that plots the binding energy per nucleon

against atomic mass (A). Actually, the net binding energy of a nucleus is that of the nuclear attraction, minus the disruptive energy of the electric force. As nuclei get heavier than helium, their binding energy per nucleon grows more and more slowly, reaching its peak at iron (Fe). The fact that the maximum binding energy is found in medium-sized nuclei is a consequence of the trade-off in the effects of two opposing forces that have different range characteristics. As nucleons are added, the total nuclear binding energy always increases-but the total disruptive energy of electric forces also increases. Iron (^{58}Fe) is the most efficiently bound nucleus. However, nickel (^{64}Ni) is the most tightly bound nucleus in terms of energy of binding per nucleon because nickel (^{64}Ni) has a slightly higher ratio of neutrons/protons than iron (^{58}Fe).

Table 2. Calculated Numerical Values of the Binding Energy per Nucleon with the Respective Mass Number

Atomic number(Z)	Nuclide	Mass number(A)	Binding Energy/nucleon (BE/A)[MeV/nucleon]
1	1H	1	-0.511393
	2H	2	0.856514
	3H	3	2.656948
2	3He	3	2.231874
	4He	4	6.818347
3	6Li	6	5.07683
	7Li	7	5.387263
4	9Be	9	6.235564
	10B	10	6.219439
5	11B	11	6.695367
	12C	12	7.42452
	13C	13	7.233885
6	14C	14	7.30123
	14N	14	7.219989
7	15N	15	7.46088
	16O	16	7.720563
	17O	17	7.510136
8	18O	18	7.539871
	19F	19	7.536864
9	20Ne	20	7.776627
	21Ne	21	7.728256

	22Ne	22	7.8480565
11	23Na	23	7.866963
12	24Mg	24	8.005078
	25Mg	25	7.978111
	26Mg	26	8.097923
13	27Al	27	8.08542
14	28Si	28	8.19211
	29Si	29	8.201825
	30Si	30	8.28209
15	31P	31	8.233798
16	32S	32	8.237516
	33S	33	8.249787
	34S	34	8.342924
	36S	36	8.34818
17	35Cl	35	8.271959
	37Cl	37	8.335389
18	36Ar	36	8.264268
	38Ar	38	8.372125
	40Ar	40	8.365219
19	39K	39	8.307952
	40K	40	8.29524
	41K	41	8.339151
20	40Ca	40	8.295682
	42Ca	42	8.37312
	43Ca	43	8.362877
	44Ca	44	8.425819
	46Ca	46	8.446619
	48Ca	48	8.453478
21	45Sc	45	8.380312
22	46Ti	46	8.41191
	47Ti	47	8.42181
	48Ti	48	8.488584
	49Ti	49	8.481516
	50Ti	50	8.530677
23	50V	50	8.460628
	51V	51	8.511426
24	50Cr	50	8.455486
	52Cr	52	8.548174
	53Cr	53	8.528585
	54Cr	54	8.550635
25	55Mn	55	8.53254
26	54Fe	54	8.490122
	56Fe	56	8.552899
	57Fe	57	8.5369855

	58Fe	58	8.56299
27	59Co	59	8.533992
28	58Ni	58	8.485161
	60Ni	60	8.542118
	61Ni	61	8.530295
	62Ni	62	8.543625
	64Ni	64	8.55376
29	63Cu	63	8.516763
	65Cu	65	8.528957
30	64Zn	64	8.496169
	66Zn	66	8.527219
	67Zn	67	8.505206
	68Zn	68	8.530101
	70Zn	70	8.510649
31	69Ga	69	8.494807
	71Ga	71	8.497374
32	70Ge	70	8.487987
	72Ge	72	8.504543
	73Ge	73	8.480962
	74Ge	74	8.504141
	76Ge	76	8.489997
33	75As	75	8.475929
34	74Se	74	8.452821
	76Se	76	8.482778
	77Se	77	8.468956
	78Se	78	8.494969
	80Se	80	8.493556
	82Se	82	8.481239
35	79Br	79	8.461109
	81Br	81	8.475028
36	78Kr	78	8.425059
	80Kr	80	8.462887
	82Kr	82	8.486203
	83Kr	83	8.4739
	84Kr	84	8.498263
	86Kr	86	8.498052
37	85Rb	85	8.474929
	87Rb	87	8.493556
38	84Sr	84	8.446187
	86Sr	86	8.482563
	87Sr	87	8.481939
	88Sr	88	8.511835
39	89Y	89	8.4899
40	90Zr	90	8.482715

	91Zr	91	8.4685563
	92Zr	92	8.470372
	94Zr	94	8.44924
	96Zr	96	8.422351
41	93Nb	93	8.43877
42	92Mo	92	8.424324
	94Mo	94	8.43388
	95Mo	95	8.4226781
	96Mo	96	8.430298
	97Mo	97	8.413721
	98Mo	98	8.416045
	100Mo	100	8.389881
43	98Tc	98	8.385657
44	96Ru	96	8.375019
	98Ru	98	8.390856
	99Ru	99	8.38143
	100Ru	100	8.394342
	101Ru	101	8.378584
	102Ru	102	8.386824
	104Ru	104	8.371085
45	103Rh	103	8.360764
46	102Pb	102	8.349966
	104Pb	104	8.358698
	105Pb	105	8.346656
	106Pb	106	8.358129
	108Pb	108	8.349276
	110Pb	110	8.333571
47	107Ag	107	8.329342
	109Ag	109	8.32749
48	106Cd	106	8.307573
	108Cd	108	8.322823
	110Cd	110	8.328219
	111Cd	111	8.316037
	112Cd	112	8.325705
	113Cd	113	8.309903
	114Cd	114	8.316334
	116Cd	116	8.300894
49	113In	113	8.301256
	115In	115	8.298741
50	112Sn	112	8.285443
	114Sn	114	8.298341
	115Sn	115	8.291799
	116Sn	116	8.302765
	117Sn	117	8.291154

	118Sn	118	8.299933
	119Sn	119	8.284683
	120Sn	120	8.281889
	122Sn	122	8.278439
	124Sn	124	8.261315
51	121Sb	121	8.266546
	123Sb	123	8.26036
52	120Te	120	8.255822
	122Te	122	8.260228
	123Te	123	8.249409
	124Te	124	8.258897
	125Te	125	8.245429
	126Te	126	8.252321
	128Te	128	8.241089
	130Te	130	8.225854
53	127I	127	8.232186
54	124Xe	124	8.214913
	126Xe	126	8.224664
	128Xe	128	8.227648
	129Xe	129	8.217418
	130Xe	130	8.225403
	131Xe	131	8.213035
	132Xe	132	8.218512
	134Xe	134	8.207689
	136Xe	136	8.193186
55	133Cs	133	8.198621
56	130Ba	130	8.18542
	132Ba	132	8.192542
	134Ba	134	8.194579
	135Ba	135	8.185525
	136Ba	136	8.192309
	137Ba	137	8.182921
	138Ba	138	8.186028
57	138La	138	8.164057
	139La	139	8.168477
58	136Ce	136	8.155652
	138Ce	138	8.162248
	140Ce	140	8.164624
	142Ce	142	8.138345
59	141Pr	141	8.14016
60	142Nd	142	8.130099
	143Nd	143	8.116068
	144Nd	144	8.113992
	145Nd	145	8.097728

	146Nd	146	8.094084
	148Nd	148	8.070006
	150Nd	150	8.045197
61	145Pm	145	8.087681
62	144Sm	144	8.083654
	147Sm	147	8.06504106
	148Sm	148	8.0663683
	149Sm	149	8.05082947
	150Sm	150	8.05040181
	152Sm	152	8.0352583
	154Sm	154	8.021104159
63	151Eu	151	8.0260877
	153Eu	153	8.01789538
64	152Gd	152	8.017837224
	154Gd	154	8.012037156
	155Gd	155	8.00224606
	156Gd	156	8.0056692
	157Gd	157	7.995189
	158Gd	158	7.99482278
	160Gd	160	7.9786059
65	159Tb	159	7.97989
66	156Dy	156	7.9761957
	158Dy	158	7.9766526
	160Dy	160	7.97324411
	161Dy	161	7.96381007
	162Dy	162	7.96525
	163Dy	163	7.9548557
	164Dy	164	7.95305044
67	165Ho	165	7.93944548
68	162Er	162	7.9378922
	164Er	164	7.93712406
	166Er	166	7.9326091
	167Er	167	7.92365135
	168Er	168	7.92274017
	170Er	170	7.907547335
69	169Tm	169	7.90590585
70	168Yb	168	7.898876029
	170Yb	170	7.89616659
	171Yb	171	7.88867739
	172Yb	172	7.8894367
	173Yb	173	7.8806407
	174Yb	174	7.8782522
	176Yb	176	7.86081206
71	175Lu	175	7.8618025

	176Lu	176	7.852857
72	174Hf	174	7.8570526
	176Hf	176	7.85228036
	177Hf	177	7.84395099
	178Hf	178	7.84272761
	179Hf	179	7.832988
	180Hf	180	7.830515025
73	181Ta	180	7.818581475
	181Ta	181	7.8172509
74	180W	180	7.8153264
	182W	182	7.81046372
	183W	183	7.80161791
	184W	184	7.799495065
	186W	186	7.783954548
75	185Re	185	7.783785
	187Re	187	7.7729109
76	184Os	184	7.777544
	186Os	186	7.77395845
	187Os	187	7.766035
	188Os	188	7.76722356
	189Os	189	7.75745314
	190Os	190	7.75763495
	192Os	192	7.7461841
77	191Lr	191	7.742037
	193Lr	193	7.7341914
78	190Pt	190	7.736735
	192Pt	192	7.73486
	194Pt	194	7.730508
	195Pt	195	7.722177
	196Pt	196	7.723194
	198Pt	198	7.712876
79	197Au	197	7.710748
80	196Hg	196	7.705809
	198Hg	198	7.705095
	199Hg	199	7.699867
	200Hg	200	7.701506
	201Hg	201	7.69419
	202Hg	202	7.694485
	204Hg	204	7.685167
81	203Tl	203	7.682153
	205Tl	205	7.676486
82	204Pb	204	7.674523
	206Pb	206	7.671956
	207Pb	207	7.667563

	208Pb	208	7.666003
83	209Bi	209	7.645052
84	209Po	209	7.630757
85	210At	210	7.604828
86	222Rn	222	7.496502
87	223Fr	223	7.473437
88	226Ra	226	7.462943
89	227Ac	227	7.450305
90	232Th	232	7.416747
91	231Pa	231	7.417068
92	234U	234	7.399756
	235U	235	7.390814
	238U	238	7.466662
93	237Np	237	7.374418
94	244Pu	244	7.327911

V. CONCLUSION

It is shown in Figure. (2) that the nuclear binding energy per nucleon (B/A) varies as a function of A . We note some peaks at ^4He , ^8Be , ^{12}C , ^{16}O , and ^{40}Ca but thereafter it is a moderately smooth function reaching a maximum of about 8.7 MeV at $A \approx 60$ thereafter declining slowly to about 7.6 MeV at uranium. The important fact is that over a large part of the periodic table the binding energy per nucleon is roughly constant.

It is believed that the more higher binding energy the more tightly the nucleons are bound together. The curve has its main peaks at iron(Fe) and nickel(Ni) and then slowly decreases again, and also a narrow isolated peak at helium(He), which as noted is very stable. So, we can conclude that the nuclides whose main peaks of the graph shown in Figure.(2) are said to be more bound than the neighboring nuclides. The heaviest nuclei in nature, uranium ^{238}U , are unstable.

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