© SEP 2019 | IRE Journals | Volume 3 Issue 3 | ISSN: 2456-8880

On The Response of Non-Prismatic Rotating Timoshenko Beam Under the Actions of Concentrated Loads Travelling At Time Dependent Speeds

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Abstract- This paper is focused on the study of motions of non- prismatic rotating Timoshenko beams traversed by constant and harmonic variable magnitude moving loads. The versatile Galerkin's method and the integral transform techniques were employed to treat the coupled second order partial differential equations governing the motion of the vibrating system. Numerical analyses in plotted curves are presented. The analyses depict interesting results on the effect of some structural parameters such as foundation moduli, prestressed forces and circular frequency on the dynamic behaviour of non- prismatic rotating Timoshenko beams under the actions of moving loads at time dependent speed. The resonance condition of the dynamical systems is also established.

Indexed Terms- non- prismatic, resonance, foundation stiffness, prestressed, transverse response, Galerkin's method.

I. INTRODUCTION

The movement of loads (people, cars, trains etc) on structural members (beams and plates) has always been an important and fundamental component of human endeavours since creation. The vibration analysis of beams or beam-like structural elements has been and continues to be the subject of numerous researchers, since it embraces a wide class of problems with immense importance in Engineering Science. The work of Timoshenko [1] gave impetus to research work in this aspect by using energy methods to obtain solutions in series form for simply supported finite beams on elastic foundation subjected to time-dependent point loads moving with uniform velocity across the beam. Steele [2] studied the response of a finite, simply supported Bernoulli-Euler beam to a unit force moving at a uniform velocity. The effects of this moving force on beams with and without an elastic foundation were analyzed. Zibdeh and Hilal [3] investigated the vibration analysis of beams with generally boundary conditions traversed by a moving force. The moving load is assumed to move with accelerating, decelerating and constant velocity type of motions. They showed the effects of type of motion, boundary conditions and damping. Kargarnovin and Younesian [4] studied the response of a Timoshenko beam with uniform cross – section and infinite length supported by a generalized Pasternak –type viscoelastic foundation subjected to an arbitrary distributed harmonic moving load. However, studies on beam problems have largely been restricted to the case when the beam structure is uniform. In particular, both moment of inertia I and mass per unit length μ of the beam did not vary with spartial coordinate x along the span of the beam. In recent years, such important Engineering problems as the vibration of turbines, hulls of ships and bridge girders or variable depth and so on, involving the theory of vibration of structures of variable cross-section have intensified the need for the study of the response of non-uniform elastic systems under the action of moving loads.

Among the earliest researchers on the dynamic analysis of an elastic beam was Ayre et al [2] who studied the e®ect of the ratio of the weight of the load to the weight of a simply supported beam for a constantly moving mass load. They obtained the exact solution for the resulting partial differential equation by using the infinite series method

Recently, Taha and Abohadima [6] investigated Mathematical model for vibrations of non-uniform flexural beams. Very recently, Adedowole [7] worked on flexural motions under moving distributed masses of Beam- type structures on Vlasor foundation and having time dependent boundary conditions. The author [8] also consider the dynamic response under travelling loads of simply supported non prismatic beam resting on variable elastic foundation Method of Laplace Integral transforms is employed to solve this initial valued problem to obtain the desired approximate solutions of the reduced equations for the transverse displacement response of the beam dynamical problem. Analyses show that higher values of the axial force and foundation stiffness decrease the transverse displacement response of the non-prismatic beam under the action of travelling loads resting on variable elastic foundation. Adeoye and Awodola [9] worked on dynamic response to moving distributed masses of pre-stressed uniform rayleigh beam resting on variable elastic pasternak foundation.

In all the aforementioned works, investigations were limited to the analysis of beam flexure of Bernoulli-Euler beams models. Specifically, the effects of shear deformation and rotatory inertia were neglected in the governing partial differential equations. Wang [10] who studied the vibration of multi-span Timoshenko beams to a moving force and Oni [11] who studied the transverse vibrations under moving loads of deep beams on a variable elastic foundation. Omolofe and Ogunyebi [12] studied the dynamic behaviour of a rotating Timoshenko beam when under the actions of a variable magnitude load moving at non-uniform speed. The more practical cases of rotating Timoshenko beam moving load problems in which the beam under consideration is of non-uniform cross-section have received little attention in literature. Also the case whereby the prestress of rotating Timoshenko beam is non-uniform at which the load is travelling is time dependent has been neglected. In all their works, it is tacitly assumed that the beam has uniform cross sections.

The main purpose of this study is to obtain closed form solutions to this dynamical problem for the boundary conditions. The reason for this is simple. Solutions so obtained often shed light on vital information about the vibrating system. Subsequently, the closed form solutions are analysed.

This present case study therefore, is concerned with the problem of the non-prismatic rotating Timoshenko beam under the actions of constant and harmonic magnitude loads with time dependent speeds.

II. PROBLEM FORMULATION

This paper considers the dynamic behaviour of a non-prismatic rotating Timoshenko beam resting on a elastic foundation when it is under the action of a moving load. The beam's properties such as moment of inertia I and the mass per unit length *of* the beam vary along the span L of the beam. The beam is assumed to maintain contact with the subgrade reaction modulus E_f and that there is no friction forces at the interface. The deflection $w(x,t)$ from the equilibrium and the rotation $u(x,t)$ of the beam under the action of moving load is described by the system of partial differential equations

$$\mu(x)w_{tt}(x,t)-K^{*}GA\left(w_{xx}(x,t)-u_{x}(x,t)\right)$$
$$+E_f(x)w(x,t)=F(x,t)+N(x)w_{xx}(x,t) \qquad (1)$$

and

$$H(x)+K^{*}GA\left(w_{x}(x,t)-u(x,t)\right)-I(x)\rho u_{tt}(x,t)=0 \qquad (2)$$

Where K* is a constant dependent on the shape of the cross-section, G is the modulus of elasticity in the shear, A is the cross-sectional area, $P(x,t)$ is the moving concentrated forces acting on the beam, μ is the mass of the beam per unit length L, w is the vertical response of the beam, $I(x)$ is the moment of inertia of the beam cross-section, E_f is the constant elastic foundation, N(x) is non-uniform prestress

The flexural moment acting on the beam cross section is related to the vertical response to rotation as

$$H(x)=\frac{\partial}{\partial x}\left(D_x(x,t)u_x\right) \qquad (3)$$

$D_x(x,t)$ is the flexural stiffness of the beam given as

$$D_x(x,t)=\gamma I(x) \qquad (4)$$

III. THE BOUNDARY CONDITIONS

The boundary conditions depend on the constraints at the beam ends. For a beam whose length is L, the vertical displacement at the beam ends are given as

$$w(0,t) = u(0,t) = 0\,,\ w(L,t) = u(L,t) = 0 \quad (5)$$

It is assumed that the initial conditions are

$$w(x,0) = 0 = w_t(x,0) \quad \text{and}$$

$$u(x,0) = 0 = u_t(x,0) \quad (6)$$

IV. NON UNIFORM CHARACTERISTICS

The distribution of the non-prismatic characteristics may be assumed as power functions. The parameters γ and n, are used to approximate the actual non uniformity of the beam given as

$$I(x) = I_o(1+\alpha x)^{n+2}, \quad \mu(x) = \mu_o(1+\alpha x)^n,$$

$$N(x) = N_o(1+\alpha x)^n \quad (7)$$

Where $I(x)$ is the variable moment of inertia of the beam, I_o, μ_o and N_o are the beam characteristics at $x = 0$.

The velocity of our moving force is non uniform

V. CASE I. DYNAMIC BEHAVIOR OF NON PRISMATIC ROTATING TIMOSHENKO BEAM TO CONSTANT MAGNITUDE LOADS.

The constant vertical excitation acting on the beam is chosen as

$$F_c(x,t) = P\delta(x - f(t)) \quad (8)$$

The concentrated load is assumed to be of mass M and the time t is assumed to be limited to that interval of time within the mass on the beam, that is;

$$0 \le f(t) \le L \quad (9)$$

The body moves with non-uniform velocity such that the motion of the contact of the moving load is given by

$$X_p = f(t) \quad (10)$$

The distance covered by the load on the same structure at any given instance of time t is given as

$$f(t) = x_o + x_1 \quad (11)$$

Where x_o is the equilibrium position of the longitudinal oscillating load, x_1 is the distance covered from equilibrium position x_o.

From equations of motion we have

$$x_1 = \frac{(v_i + v_f)t}{2} \quad (12a)$$

$$v_f = v_i + at \quad (12b)$$

Where v_i is initial speed, v_f is the final speed and a is the constant acceleration

Substituting equations (7), (8) and (11) into equation (1) and (2) taking n=1 for simplicity yield

$$\mu_o(1+\alpha x)w_{tt}(x,t) - K^*GA(w_{xx}(x,t) - u_x(x,t))$$

$$-N_o(1+\alpha x)w_{xx}(x,t) + E_f(x)w(x,t)$$

$$= P\delta\left[x - \left(x_o + \frac{(v_i + v_f)t}{2}\right)\right] \quad (13)$$

and

$$\frac{\gamma\partial}{\partial x}\left(I_0(1+\alpha x)^3 u_x(x,t)\right) + K^*GA(w_x(x,t) - u(x,t))$$

$$-I_0(1+\alpha x)^3 \rho u_{tt}(x,t) = 0 \quad (14)$$

Now we seek the closed form solution to the simultaneous second order partial differential equations (13) and (14). Consequently, an approximate analytical solution is desirable to obtain some vital information about the vibrating system.

VI. SOLUTION TECHNIQUE

In order to solve the beam problem above, we shall use the versatile solution technique called Galerkin's method often used in solving diverse problems involving mechanical vibrations [7]. This technique requires that the solutions equations of the form

The equation of the motion of an element of the beam is generally symbolically written in the form.

$$\Gamma w(x,t) - P(x,t) = 0 \qquad (15)a$$

where,
Γ is the differential operator, w is the structural displacement and P is the traverse load acting on the structure. To this effect, the solutions of the system of equations (13) and (14) are expressed as

$$w(x,t) = \sum_{i=1}^{n} e_i(t) z_i(x) \qquad (15)$$

and

$$u(x,t) = \sum_{i=1}^{n} y_i(t) r_i(x) \qquad (16)$$

where the functions $z_i(x)$ and $r_i(x)$ are chosen to satisfy the pertinent boundary conditions.

Thus, substituting equations (15) and (16) into the coupled simultaneous ordinary differential equations (13) and (14) we obtain

$$\sum_{i=1}^{n} \Big\{ \mu_o c_1(x) \ddot{e}_i(t) z_i(x) - K^* GA\big(e_i(t) z_i''(x) - y_i(t) r_i'(x)\big)$$
$$- N_o c_1(x) e_i(t) z_i''(x) + E_f(x) e_i(t) z_i(x) \Big\}$$
$$= P\delta\big[x - \big(x_o + \tfrac{1}{2}(v_i + v_f)t\big)\big] \qquad (17)$$

and

$$\sum_{i=1}^{n} \Big\{ \gamma I_o \Big(c_3(x) y_i(t) r_i''(x) + c_2(x) y_i(t) r_i'(x) \Big)$$
$$+ K^* GA\big(e_i(t) z_i'(x) - y_i(t) r_i(x)\big)$$
$$- I_0 c_3(x) \rho \ddot{y}_i(t) r_i(x) \Big\} = 0 \qquad (18)$$

where

$$c_1(x) = (1 + \alpha x)$$
$$c_2(x) = \left(3\alpha + 6\alpha^2 x + 3\alpha^3 x^2\right)$$
$$c_3(x) = \left(1 + 3\alpha x + 3\alpha^2 x^2 + \alpha^3 x^3\right) \qquad (19)$$

To determine $e_i(t)$ and $y_i(t)$, the expressions on the left hand sides of equations (17) and (18) are required to be orthogonal to the functions $e_k(t)$ and $y_k(t)$ respectively. Thus,

$$\int_0^L \Bigg[\sum_{i=1}^{n} \Big\{ \mu_o c_1(x) \ddot{e}_i(t) z_i(x) - K^* GA\big(e_i(t) z_i''(x) - y_i(t) r_i'(x)\big)$$
$$- N_o c_1(x) e_i(t) z_i''(x) + E_f(x) e_i(t) z_i(x) \Big\}$$
$$- P\delta\big[x - \big(x_o + \tfrac{1}{2}(v_i + v_f)t\big)\big]\Bigg] z_k(x) dx = 0 \qquad (20)$$

and

$$\int_0^L \Bigg[\sum_{i=1}^{n} \Big\{ \gamma I_o \Big(c_3(x) y_i(t) r_i''(x) + c_2(x) y_i(t) r_i'(x) \Big)$$
$$+ K^* GA\big(e_i(t) z_i'(x) - y_i(t) r_i(x)\big)$$
$$- I_0 c_3(x) \rho \ddot{y}_i(t) r_i(x) \Big\}\Bigg] r_k(x) dx = 0 \qquad (21)$$

Equation (20) and (21) after some rearrangements yield

$$\sum_{i=1}^{n} \left[q_a(i,k) \ddot{e}_i(t) + q_b(i,k) e_i(t) + q_c(i,k) y_i(t) \right]$$
$$= q_d \qquad (22)$$

And

$$\sum_{i=1}^{n} \left[a_1(i,k) \ddot{y}_i(t) + a_2(i,k) e_i(t) + a_3(i,k) y_i(t) \right] = 0 \qquad (23)$$

Where

$$q_a(i,k) = \mu_0 \int_0^L (1 + \alpha x) z_i(x) z_k(x) dx$$

$$q_b(i,k) = \int_0^L \left(-K^* GA z_i''(x) - N_0 (1 + \alpha x) z_i''(x) + E_f z_i(x) \right) z_k(x) dx$$

$$q_c(i,k) = K^* GA \int_0^L r_i'(x) z_k(x) dx$$

$$q_d = \int_0^L P\delta\big[x - \big(x_o + \tfrac{1}{2}(v_i + v_f)t\big)\big] z_k(x) dx \qquad (24)a$$

$$a_1(i,k) = -I_0\rho\int_0^L \left(1+3\alpha x+3\alpha^2x^2+\alpha^3x^3\right)r_i(x)r_k(x)dx$$

$$a_2(i,k) = K^*GA\int_0^L z_i'(x)r_k(x)dx$$

$$a_3(i,k) = \int_0^L \left[\gamma I_0\left(\left(1+3\alpha x+3\alpha^2x^2+\alpha^3x^3\right)r_i''(x)\right.\right.$$
$$\left.\left.+\left(3\alpha+6\alpha^2x+3\alpha^3x^2\right)r_i'(x)\right)K^*GAr_i(x)\right]r_k(x)dx \quad (24)\text{b}$$

Since our beam has simple supports at both ends x = 0 and x = L, we therefore choose the functions $z_i(x)$ and $h_i(x)$ to be

$$z_i(x) = \sin\frac{i\pi x}{L}$$

and $h_i(x)\cos\frac{i\pi x}{L}$ (25)

Thus, in view of (25), integrals (24) are evaluated to yield

$$q_a(i,k) = \mu_o\left[I_1+\alpha I_2\right]$$

$$q_b(i,k) = \left(\frac{i\pi}{L}\right)^2\left(K^*GAI_1+N_0\left(I_1+\gamma I_2\right)\right)+E_fI_1$$

$$q_c(i,k) = -K^*GA\left(\frac{i\pi}{L}\right)I_1$$

$$q_d = P\sin\frac{k\pi\left(x_o+\tfrac{1}{2}\left(v_i+v_f\right)t\right)}{L} \quad (26)$$

$$a_1(i,k) = -I_0\rho\left[I_3+3\alpha I_4+3\alpha^2I_5+\alpha^3I_6\right]$$

$$a_2(i,k) = K^*GA\left(\frac{i\pi}{L}\right)I_1$$

$$a_3(i,k) = -\gamma I_0\left(\frac{i\pi}{L}\right)^2\left[I_3+3\alpha I_4+3\alpha^2I_5+\alpha^3I_6\right]$$

$$-\gamma I_0\left(\frac{i\pi}{L}\right)\left[3\alpha I_7+3\alpha^2I_8+3\alpha^3I_9\right]-K^*GAI_3$$

(27)

Where

$$I_1 = \int_0^L \sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx,$$

$$I_2 = \int_0^L x\sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx$$

$$I_3 = \int_0^L \cos\frac{i\pi x}{L}\cos\frac{k\pi x}{L}dx \quad ,$$

$$I_4 = \int_0^L x\cos\frac{i\pi x}{L}\cos\frac{k\pi x}{L}dx$$

$$I_5 = \int_0^L x^2\cos\frac{i\pi x}{L}\cos\frac{k\pi x}{L}dx,$$

$$I_6 = \int_0^L x^3\cos\frac{i\pi x}{L}\cos\frac{k\pi x}{L}dx$$

$$I_7 = \int_0^L \sin\frac{i\pi x}{L}\cos\frac{k\pi x}{L}dx,$$

$$I_8 = \int_0^L x\sin\frac{i\pi x}{L}\cos\frac{k\pi x}{L}dx$$

$$I_9 = \int_0^L x^2\sin\frac{i\pi x}{L}\cos\frac{k\pi x}{L}dx \quad (28)$$

Considering only ith concentrated moving force, equation (23) and (24) can be simplified further to give

$$q_a(i,k)\ddot{e}_i(t)+q_b(i,k)e_i(t)+q_c(i,k)y_i(t)$$
$$= P_0a_0\left(\cos\beta_1t\cos\beta_2t-\sin\beta_1t\sin\beta_2t\right)$$
$$+P_0b_0\left(\sin\beta_1t\cos\beta_2t-\cos\beta_1\sin\beta_2t\right) \quad (29)$$

and

$$a_1(i,k)\ddot{y}_i(t)+a_2(i,k)e_i(t)+a_3(i,k)y_i(t) = 0$$

(30)

Where

$$\beta_1 = \frac{k\pi v_f}{2L} \text{ and } \beta_2 = \frac{k\pi v_i}{2L} \quad (31)$$

which can further be simplified to take form

$$q_a(i,k)\ddot{e}_i(t)+q_b(i,k)e_i(t)+q_c(i,k)y_i(t) =$$
$$H_p\sin\phi t+H_a\cos\phi t \quad (32)$$

and

$$a_1(i,k)\ddot{y}_i(t)+a_2(i,k)e_i(t)+a_3(i,k)y_i(t) = 0$$

(33)

Where

$H_a = P_0 a_0, \qquad H_p = P_0 b_0$

and $\phi = \beta_1 + \beta_2$ (34)

In what follows we subject the system of ordinary differential equations (32) and (33) to a Laplace transform defned as

$(\sim) = \int_\circ^\infty (\cdot) e^{-st} dt$ (35)

where s is the Laplace parameter. In conjunction with the initial conditions define in (6), yields the following algebraic simultaneous equation

$$\left[q_a(i,k)s^2 + q_b(i,k)\right]e_i(s) + q_c(i,k)y_i(t) = H_p \frac{\phi}{S^2+\phi^2} + H_a \frac{S}{S^2+\phi^2} \quad (36)$$

And

$$\left[a_1(i,k)s^2 + a_3(i,k)\right]y_i(s) + a_2(i,k)e_i(t) = 0 \quad (37)$$

Further simplification and modification using Laplace transform.

Thus

$$e_i(s) = \frac{\psi_1}{\psi_0} \quad (38)$$

and

$$y_i(s) = \frac{\psi_2}{\psi_0} \quad (39)$$

where

$$\psi_0 = \left\{q_a(i,k)s^2 + q_b(i,k)\right\} * \left\{a_1(i,k)s^2 + a_3(i,k)\right\} - a_2(i,k)q_c(i,k) \quad (40)$$

$$\psi_1 = \left(a_1(i,k)s^2 + a_3(i,k)\right) * \left[H_p \frac{\phi}{S^2+\phi^2} + H_a \frac{S}{S^2+\phi^2}\right] \quad (41)$$

$$\psi_2 = -a_2\left[H_p \frac{\phi}{S^2+\phi^2} + H_a \frac{S}{S^2+\phi^2}\right] \quad (42)$$

Where

Furthermore, equations (38) and (39) can be re-written in the form

$$e_i(s) = \frac{1}{(s^2+\eta_1^2)} * \frac{1}{(\eta_1^2-\eta_2^2)} - \frac{1}{(s^2+\eta_2^2)} * \frac{1}{(\eta_1^2-\eta_2^2)} * z_q\left[H_p \frac{\phi}{S^2+\phi^2} + H_a \frac{S}{S^2+\phi^2}\right] \quad (43)$$

and

$$y_i(s) = Q_k\left[\frac{1}{(s^2+\eta_1^2)} - \frac{1}{(s^2+\eta_2^2)}\right] * \left\{H_p \frac{\phi}{S^2+\phi^2} + H_a \frac{S}{S^2+\phi^2}\right\} \quad (44)$$

Where

$$\eta_1^2 = \frac{-B_1 + \sqrt{B_1 - 4B_2}}{2} \text{ and}$$

$$\eta_2^2 = \frac{-B_1 - \sqrt{B_1 - 4B_2}}{2} \quad (45)$$

$$B_1 = \frac{q_a(i,k)a_3(i,k) + q_b(i,k)a_1(i,k)}{q_a(i,k)a_1(i,k)} \text{ and}$$

$$B_2 = \frac{q_a(i,k)a_3(i,k) - q_c(i,k)a_2(i,k)}{q_a(i,k)a_1(i,k)} \quad (45)$$

Solving equations (43) and (44) further, one obtains

$$e_i(s) = \left[\frac{1}{(s^2+\eta_1^2)} - \frac{1}{(s^2+\eta_2^2)}\right] * z_q\left\{H_p \frac{\phi}{S^2+\phi^2} + H_a \frac{S}{S^2+\phi^2}\right\} \quad (46)$$

And

$$y_i(s) = Q_k\left[\frac{1}{(s^2+\eta_1^2)} - \frac{1}{(s^2+\eta_2^2)}\right] * \left\{H_p \frac{\phi}{S^2+\phi^2} + H_a \frac{S}{S^2+\phi^2}\right\} \quad (47)$$

Some simplifications and rearrangements of equations (46) and (47) yield

$$e_i(s)=\frac{H_p z_q}{(\eta_1^2-\eta_2^2)}\left(\frac{1}{(s^2+\eta_1^2)}*\frac{\phi}{S^2+\phi^2}\right.$$
$$\left.-\frac{1}{(s^2+\eta_2^2)}*\frac{\phi}{S^2+\phi^2}\right)$$
$$+\frac{H_a z_q}{(\eta_1^2-\eta_2^2)}\left(\frac{1}{(s^2+\eta_1^2)}*\frac{s}{S^2+\phi^2}\right.$$
$$\left.-\frac{1}{(s^2+\eta_2^2)}*\frac{s}{S^2+\phi^2}\right) \quad (48)$$

$$y_i(s)=H_p Q_k\left[\frac{1}{(s^2+\eta_1^2)}*\frac{\phi}{S^2+\phi^2}\right.$$
$$\left.-\frac{1}{(s^2+\eta_2^2)}*\frac{\phi}{S^2+\phi^2}\right]$$
$$+H_a Q_k\left[\frac{1}{(s^2+\eta_1^2)}*\frac{s}{S^2+\phi^2}\right.$$
$$\left.-\frac{1}{(s^2+\eta_2^2)}*\frac{s}{S^2+\phi^2}\right] \quad (49)$$

Where

$$Q_k=\frac{a_2(i,k)}{B_1(\eta_1^2-\eta_2^2)}$$
$$z_q=\frac{\left[a_1(i,k)\eta_1^2+a_3(i,k)\right]}{B_1(\eta_1^2-\eta_2^2)} \quad (50)$$

Subjecting equations (48) and (49) to Laplace transformation with the initial conditions

$$w(x,0)=0=w_t(x,0) \quad \text{and}$$
$$u(x,0)=0=u_t(x,0) \quad (51)$$

Thus

$$e_i(t)=z_q\left\{\frac{H_p}{\eta_2\eta_1}\left(\eta_2\int_0^L \sin\eta_1(t-u)\sin\phi u du\right.\right.$$
$$\left.-\eta_1\int_0^L \sin\eta_2(t-u)\sin\phi u du\right)$$
$$+\frac{H_a}{\eta_2\eta_1}\left(\eta_2\int_0^L \sin\eta_1(t-u)\cos\phi u du\right.$$
$$\left.\left.-\eta_1\int_0^L \sin\eta_2(t-u)\cos\phi u du\right)\right\}$$
(52)

and

$$y_i(t)=Q_k\left\{\frac{H_p}{\eta_2\eta_1}\left(\eta_2\int_0^L \sin\eta_1(t-u)\sin\phi u du\right.\right.$$
$$\left.-\eta_1\int_0^L \sin\eta_2(t-u)\sin\phi u du\right)$$
$$+\frac{H_a}{\eta_2\eta_1}\left(\eta_2\int_0^L \sin\eta_1(t-u)\cos\phi u du\right.$$
$$\left.\left.-\eta_1\int_0^L \sin\eta_2(t-u)\cos\phi u du\right)\right\}$$
(53)

Further modifications of equations (52) and (53) we have

$$e_i(t)=z_q H_p\left\{\frac{\sin\eta_1 t}{(\eta_1^2-\phi^2)}\left[(\eta_1\chi_1-\phi\kappa_1+\phi)\right.\right.$$
$$\left.+\cot\eta_1 t(\eta_1\tau_1-\phi\zeta_1)\right]$$
$$-\frac{\sin\eta_2 t}{(\eta_2^2-\phi^2)}\left[(\eta_2\chi_2-\phi\kappa_2+\phi)\right.$$
$$\left.\left.-\cot\eta_2 t(\eta_2\tau_2-\phi\zeta_2)\right]\right\}$$
$$+z_q H_a\left\{\frac{\sin\eta_1 t}{(\eta_1^2-\phi^2)}\left[(\phi\tau_1-\eta_1\kappa_1)\right.\right.$$
$$\left.+\cot\eta_1 t(\phi\chi_1-\eta_1\kappa_1+\eta_1)\right]$$
$$+\frac{\sin\eta_2 t}{(\eta_2^2-\phi^2)}\left[(\phi\tau_2-\eta_2\kappa_2)\right.$$
$$\left.\left.+\cot\eta_1 t(\phi\chi_1-\eta_1\kappa_1+\eta_1)\right]\right\} \quad (54)$$

and

$$y_i(t)=Q_k H_p\left\{\frac{\sin\eta_1 t}{(\eta_1^2-\phi^2)}\left[(\eta_1\chi_1-\phi\kappa_1+\phi)\right.\right.$$
$$\left.+\cot\eta_1 t(\eta_1\tau_1-\phi\zeta_1)\right]$$

$$-\frac{\sin\eta_2 t}{(\eta_2^2-\phi^2)}\left[(\eta_2\chi_2-\phi\kappa_2+\phi)\right.$$
$$\left.-\cot\eta_2 t(\eta_2\tau_2-\phi\zeta_2)\right]\Big\}$$
$$+Q_k H_a\left\{\frac{\sin\eta_1 t}{(\eta_1^2-\phi^2)}\left[(\phi\tau_1-\eta_1\kappa_1)\right.\right.$$
$$\left.+\cot\eta_1 t(\phi\chi_1-\eta_1\kappa_1+\eta_1)\right]$$
$$\frac{\sin\eta_2 t}{(\eta_2^2-\phi^2)}\left[(\phi\tau_2-\eta_2\kappa_2)\right.$$
$$\left.\left.+\cot\eta_1 t(\phi\chi_1-\eta_1\kappa_1+\eta_1)\right]\right\} \qquad (55)$$

Where

$$\chi_1=\sin\phi t\sin\eta_1 t, \qquad \chi_2=\sin\phi t\sin\eta_2 t$$
$$\kappa_1=\cos\phi t\cos\eta_1 t, \qquad \kappa_2=\cos\phi t\cos\eta_2 t$$
$$\tau_1=\sin\phi t\cos\eta_1 t, \qquad \tau_2=\sin\phi t\cos\eta_2 t$$
$$\zeta_1=\cos\phi t\sin\eta_1 t, \qquad \zeta_2=\cos\phi t\sin\eta_2 t$$
(56)

The transverse displacement response of the non-prismatic rotating Timoshenko beam under the action of constant magnitude load travelling at time dependent speed can be represented by

$$w(x,t)=\sum_{i=1}^{n}\left\langle z_q H_p\left\{\frac{\sin\eta_1 t}{(\eta_1^2-\phi^2)}\left[(\eta_1\chi_1-\phi\kappa_1+\phi)\right.\right.\right.$$
$$\left.+\cot\eta_1 t(\eta_1\tau_1-\phi\zeta_1)\right]$$
$$-\frac{\sin\eta_2 t}{(\eta_2^2-\phi^2)}\left[(\eta_2\chi_2-\phi\kappa_2+\phi)\right.$$
$$\left.-\cot\eta_2 t(\eta_2\tau_2-\phi\zeta_2)\right]\Big\}$$
$$+z_q H_a\left\{\frac{\sin\eta_1 t}{(\eta_1^2-\phi^2)}\left[(\phi\tau_1-\eta_1\kappa_1)\right.\right.$$
$$\left.+\cot\eta_1 t(\phi\chi_1-\eta_1\kappa_1+\eta_1)\right]$$
$$-\frac{\sin\eta_2 t}{(\eta_2^2-\phi^2)}\left[(\phi\tau_2-\eta_2\kappa_2)\right.$$
$$\left.\left.\cot\eta_1 t(\phi\chi_1-\eta_1\kappa_1+\eta_1)\right]\right\}\sin\frac{i\pi x}{L} \qquad (57)$$

Similarly, the rotation of the non-prismatic rotating Timoshenko beam under the action of constant magnitude load travelling at time dependent speed is represented by

$$u(x,t)=\sum_{i=1}^{n}\left\langle Q_k H_p\left\{\frac{\sin\eta_1 t}{(\eta_1^2-\phi^2)}\left[(\eta_1\chi_1-\phi\kappa_1+\phi)\right.\right.\right.$$
$$\left.+\cot\eta_1 t(\eta_1\tau_1-\phi\zeta_1)\right]$$
$$-\frac{\sin\eta_2 t}{(\eta_2^2-\phi^2)}\left[(\eta_2\chi_2-\phi\kappa_2+\phi)\right.$$
$$\left.-\cot\eta_2 t(\eta_2\tau_2-\phi\zeta_2)\right]\Big\}$$
$$+Q_k H_a\left\{\frac{\sin\eta_1 t}{(\eta_1^2-\phi^2)}\left[(\phi\tau_1-\eta_1\kappa_1)\right.\right.$$
$$\left.+\cot\eta_1 t(\phi\chi_1-\eta_1\kappa_1+\eta_1)\right]$$
$$-\frac{\sin\eta_2 t}{(\eta_2^2-\phi^2)}\left[(\phi\tau_2-\eta_2\kappa_2)\right.$$
$$\left.\left.\cot\eta_1 t(\phi\chi_1-\eta_1\kappa_1+\eta_1)\right]\right\}\cos\frac{i\pi x}{L} \qquad (58)$$

VII. CASE II RESPONSE OF NON-PRISMATIC ROTATING TIMOSHENKO BEAM TO HARMONIC VARIABLE MAGNITUDE MOVING LOADS.

The dynamic behavior of non-prismatic rotating Timoshenko beam when subjected to harmonic variable magnitude moving load is investigated in this section. Thus, the load $F_H(x,t)$ is given as

$$F_H(x,t)=P\cos\Omega t\,\delta(x-f(t)) \qquad (59)$$

where Ω is the circular frequency of the harmonic force and all parameters are as defined previously .
In view of (59) in equations (13) and (14), vibration of the beam is then described by

$$\mu_o(1+\alpha x)w_{tt}(x,t)-K^*GA\left(w_{xx}(x,t)-u_x(x,t)\right)$$
$$-N_o(1+\alpha x)w_{xx}(x,t)+E_f(x)w(x,t)$$
$$=P\cos\Omega t\,\delta\left[x-\left(x_o+1/2\left(v_i+v_f\right)t\right)\right]$$
(60)

and

$$\frac{\gamma\partial}{\partial x}\left(I_0(1+\alpha x)^3 u_x(x,t)\right)+K^*GA\left(w_x(x,t)-u(x,t)\right)$$
$$-I_0(1+\alpha x)^3\rho u_{tt}(x,t)=0 \qquad (61)$$

Using the property of Dirac delta, after some simplifications and rearrangements, the above equations can be rewritten as

$$q_a(i,k)\ddot{e}_i(t)+q_b(i,k)e_i(t)+q_c(i,k)y_i(t)$$
$$=H_p/2\,(\sin\Omega_2 t-\sin\Omega_3 t)$$
$$+H_a/2\,(\cos\Omega_2 t-\cos\Omega_3 t) \qquad (62)$$

And

$$a_1(i,k)\ddot{y}_i(t)+a_2(i,k)e_i(t)+a_3(i,k)y_i(t)=0$$
(63)

Where

$$\Omega_2=\Omega+\phi \quad and \quad \Omega_3=\Omega-\phi$$
(64)

Equations (62) and (63) are analogous to equations (29) and (30), thus subjecting equations (62) and (63) to Laplace transform in conjunction with the boundary conditions stated in (6) and using convolution theory we obtain

$$w(x,t)=\sum_{i=1}^{n}\left\langle z_q H_p\left\{\frac{\sin\eta_1 t}{(\eta_1^2-\Omega_2^2)}\left[(\eta_1\chi_3-\Omega_2\kappa_3+\Omega_2)\right.\right.\right.$$
$$\left.+\cot\eta_1 t(\eta_1\tau_3-\Omega_2\zeta_3)\right]$$
$$-\frac{\sin\eta_2 t}{(\eta_2^2-\Omega_2^2)}\left[(\eta_2\chi_3-\Omega_2\kappa_3+\Omega_2)\right.$$
$$\left.-\cot\eta_2 t(\eta_2\tau_4-\Omega_2\zeta_4)\right]$$
$$-\frac{\sin\eta_1 t}{(\eta_2^2-\Omega_3^2)}\left[(\eta_1\chi_5-\Omega_3\kappa_5+\Omega_3)\right.$$
$$\left.+\cot\eta_1 t(\eta_1\tau_5-\Omega_3\zeta_5)\right]$$
$$+\frac{\sin\eta_2 t}{(\eta_2^2-\Omega_3^2)}\left[(\eta_2\chi_6-\Omega_3\kappa_6+\Omega_3)\right.$$
$$\left.\left.+\cot\eta_1 t(\eta_1\tau_5-\Omega_3\zeta_5)\right]\right\}$$
$$+z_q H_a\left\{\frac{\sin\eta_1 t}{(\eta_1^2-\Omega_2^2)}\left[(\Omega_2\tau_3-\eta_1\kappa_3)\right.\right.$$
$$\left.+\cot\eta_1 t(\phi\chi_1-\eta_1\kappa_1+\eta_1)\right]$$
$$-\frac{\sin\eta_2 t}{(\eta_2^2-\Omega_2^2)}\left[(\Omega_2\tau_4-\eta_2\kappa_4)\right.$$
$$\left.-\cot\eta_2 t(\Omega_2\chi_4-\eta_2\kappa_4+\eta_2)\right]$$
$$\frac{\sin\eta_1 t}{(\eta_1^2-\Omega_3^2)}\left[(\Omega_3\tau_5-\eta_1\kappa_5)\right.$$
$$\left.-\cot\eta_1 t(\Omega_3\chi_5-\eta_1\kappa_5+\eta_1)\right]$$
$$-\frac{\sin\eta_2 t}{(\eta_2^2-\Omega_3^2)}\left[(\Omega_3\tau_6-\eta_2\kappa_6)\right.$$
$$\left.\left.-\cot\eta_2 t(\Omega_3\chi_6-\eta_2\kappa_6+\eta_2)\right]\right\}\sin\frac{i\pi x}{L}$$

(66)

where

$$\chi_3=\sin\Omega_2 t\sin\eta_1 t, \quad \chi_4=\sin\Omega_2 t\sin\eta_2 t$$
$$\kappa_3=\cos\Omega_2 t\cos\eta_1 t, \quad \kappa_4=\cos\Omega_2 t\cos\eta_2 t$$
$$\tau_3=\sin\Omega_2 t\cos\eta_1 t, \quad \tau_4=\sin\Omega_2 t\cos\eta_2 t$$
$$\zeta_3=\cos\Omega_2 t\sin\eta_1 t, \quad \zeta_4=\cos\Omega_2 t\sin\eta_2 t$$
$$\chi_5=\sin\Omega_3 t\sin\eta_1 t, \quad \chi_5=\sin\Omega_3 t\sin\eta_2 t$$
$$\kappa_5=\cos\Omega_3 t\cos\eta_1 t, \quad \kappa_6=\cos\Omega_3 t\cos\eta_2 t$$
$$\tau_5=\sin\Omega_3 t\cos\eta_1 t, \quad \tau_6=\sin\Omega_3 t\cos\eta_2 t$$
$$\zeta_5=\cos\Omega_3 t\sin\eta_1 t, \quad \zeta_6=\cos\Omega_3 t\sin\eta_2 t$$

(67)

Similarly, the rotation of the non-prismatic rotating Timoshenko beam under the action of variable magnitude load travelling at time dependent speed is represented by

$$u(x,t)=\sum_{i=1}^{n}\left\langle Q_k H_p\left\{\frac{\sin\eta_1 t}{(\eta_1^2-\Omega_2^2)}\left[(\eta_1\chi_3-\Omega_2\kappa_3+\Omega_2)\right.\right.\right.$$
$$\left.+\cot\eta_1 t(\eta_1\tau_3-\Omega_2\zeta_3)\right]$$
$$-\frac{\sin\eta_2 t}{(\eta_2^2-\Omega_2^2)}\left[(\eta_2\chi_3-\Omega_2\kappa_3+\Omega_2)\right.$$
$$\left.-\cot\eta_2 t(\eta_2\tau_4-\Omega_2\zeta_4)\right]$$
$$-\frac{\sin\eta_1 t}{(\eta_2^2-\Omega_3^2)}\left[(\eta_1\chi_5-\Omega_3\kappa_5+\Omega_3)\right.$$
$$\left.+\cot\eta_1 t(\eta_1\tau_5-\Omega_3\zeta_5)\right]$$

$$+\frac{\sin\eta_2 t}{(\eta_2^2-\Omega_3^2)}\left[(\eta_2\chi_6-\Omega_3\kappa_6+\Omega_3)\right.$$
$$\left.+\cot\eta_1 t(\eta_1\tau_5-\Omega_3\zeta_5)\right]\Big\}$$
$$+Q_k H_a\left\{\frac{\sin\eta_1 t}{(\eta_1^2-\Omega_2^2)}\left[(\Omega_2\tau_3-\eta_1\kappa_3)\right.\right.$$
$$\left.+\cot\eta_1 t(\phi\chi_1-\eta_1\kappa_1+\eta_1)\right]$$
$$-\frac{\sin\eta_2 t}{(\eta_2^2-\Omega_2^2)}\left[(\Omega_2\tau_4-\eta_2\kappa_4)\right.$$
$$\left.-\cot\eta_2 t(\Omega_2\chi_4-\eta_2\kappa_4+\eta_2)\right]$$
$$\frac{\sin\eta_1 t}{(\eta_1^2-\Omega_3^2)}\left[(\Omega_3\tau_5-\eta_1\kappa_5)\right.$$
$$\left.-\cot\eta_1 t(\Omega_3\chi_5-\eta_1\kappa_5+\eta_1)\right]$$
$$-\frac{\sin\eta_2 t}{(\eta_2^2-\Omega_3^2)}\left[(\Omega_3\tau_6-\eta_2\kappa_6)\right.$$
$$\left.\left.-\cot\eta_2 t(\Omega_3\chi_6-\eta_2\kappa_6+\eta_2)\right]\right\}\cos\frac{i\pi x}{L}$$

(68)

which is the rotation of the non-prismatic rotating Timoshenko beam under the action of variable magnitude load

VIII. RESONANCE CONDITION OF THE NON-PRISMATIC BEAM

At this juncture, in an undamped system such as this, it is pertinent to establish conditions under which resonance occurs. This occurs when the transverse displacement of elastic non-prismatic rotating Timoshenko beam grows without bound. Equation (57) clearly shows that the non-prismatic rotating Timoshenko beam resting on elastic foundation will experience resonance effects whenever

$$\tfrac{1}{2}\left(q_a(i,k)\cdot a_3(i,k)+\tfrac{1}{2}q_b(i,k)\cdot a_1(i,k)\right)^2$$
$$=q_a(i,k)\cdot a_1(i,k)\cdot\{q_b(i,k)\cdot a_3(i,k)-q_c(i,k)\cdot a_2(i,k)\}$$

$$\eta_1^{\ 2}=\phi^2,\qquad \eta_2^{\ 2}=\phi^2 \qquad (69)$$

while equation (66) shows that the same beam under the action of moving harmonic load experiences a state of resonance whenever

$$\eta_1^{\ 2}=\Omega_2^2,\ \eta_1^{\ 2}=\Omega_3^2,\quad \eta_2^{\ 2}=\Omega_2^2,$$
and $\eta_2^{\ 2}=\Omega_3^2$ (70)

It is also observed that as the foundation modulli and presstress increase the critical speed of the dynamical system increases thereby reducing the risk of resonant effects.

IX. FINDINGS AND COMMENTS ON THE OUTCOMES

The theory presented in this paper is illustrated numerically. For the purpose of Numerical analysis in this study, we consider the initial velocity v_i of the fast moving concentrated loads to be 8.128m/s and the span L of the beam to be 50m. The value of flexural rigidity EI is 6068242, the values of foundation moduli are varied between 0 N/m^3 and 4 x 10^4 N/ m^3, and the values of presstress N are varied between 0 N/ m^3 and 7 x10^3 N / m^3. The results are as shown on the various graphs below.

In figure 1 and figure 3 , the transverse displacement response of non- prismatic rotating Timoshenko beams beam under the actions of traveling concentrated forces when the travelling forces are of constant and variable magnitude respectively are displayed. It is clearly seen that when the value of prestress N is fixed, the displacements of a non-prismatic beam resting on elastic foundation and traversed by concentrated moving forces decreases as the values of foundation modulus E_f increases.

Figure 2 and Figure 4 display the deflection profile of a non- prismatic rotating Timoshenko beam resting on elastic foundation and under the actions of concentrated forces when the travelling loads are of constant and variable magnitude respectively. From the figure it is obvious that as the values of prestress N increases, for fixed value of foundation modulus E_f , the response amplitudes of the beam decreases.

Figure 5 depicts the deflection profile of the non-prismatic rotating Timoshenko beam resting on elastic foundation and subjected to fast traveling load. It is shown from the figure that for fixed values of foundation reaction E_f and prestress N, the deflection of the beam reduces as the values of the circular frequency Ω increases.

Figure 6 displays the comparison of the response amplitude of a non- prismatic rotating Timoshenko beam resting on elastic foundation and under the actions of concentrated forces when the travelling loads are of constant and variable magnitude respectively for fixed values of prestress N and foundation modulus. The response amplitude of constant magnitude moving load is higher than that of the variable magnitude moving load.

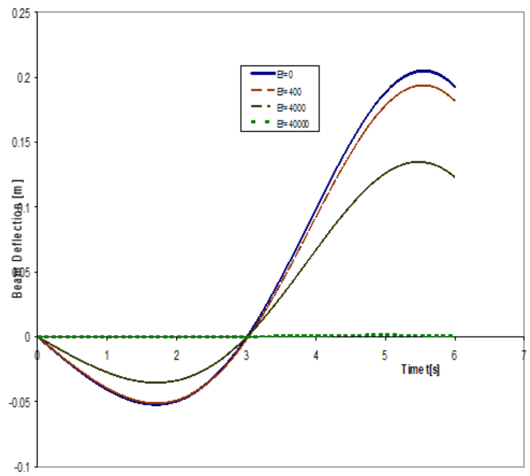

Figure 1: Transverse displacement of a non-prismatic rotating Timoshenko beam under the actions of constant moving force for various values of foundation moduli E_f .

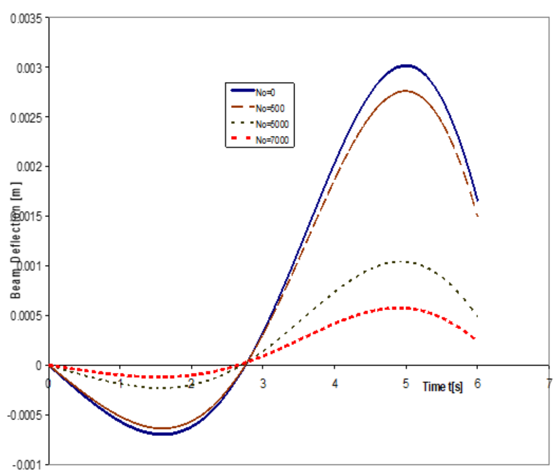

Figure 2: Displacement response of a non-prismatic rotating Timoshenko beam on elastic foundation and traversed by moving constant force for various values of N.

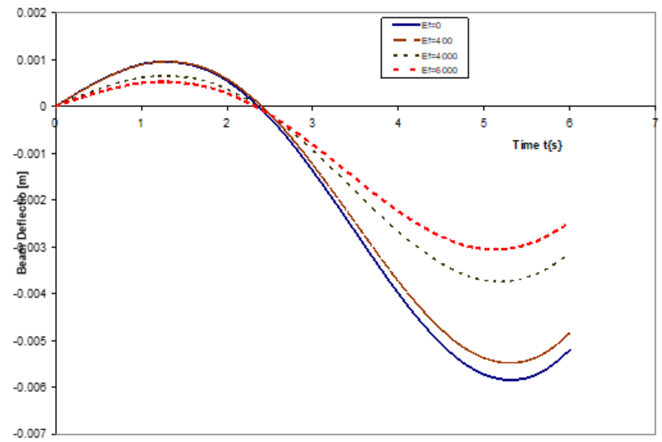

Figure 3: Deflection profile of a non-prismatic rotating Timoshenko beam subjected to Harmonic variable magnitude moving loads for various values of foundation moduli E_f and for fixed values of presstress N

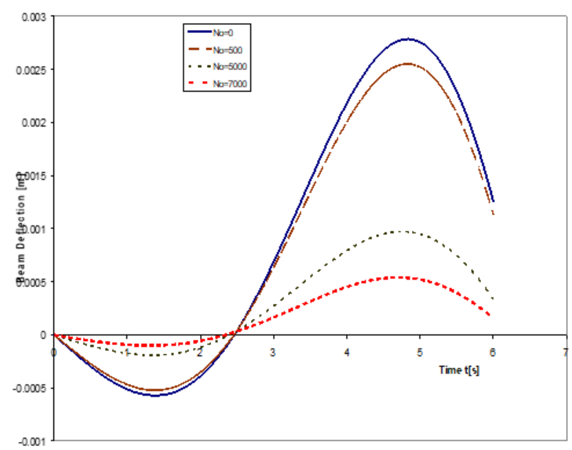

Figure 4: Transverse displacement response of a non-prismatic rotating Timoshenko beam resting on elastic foundation and subjected to Harmonic variable magnitude moving loads for various values of presstress N and for fixed value of foundation modulus

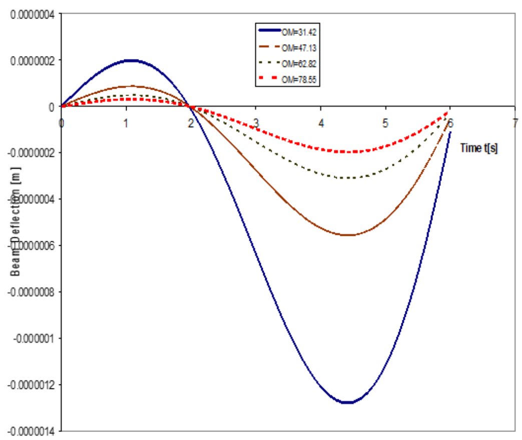

FIGURE 5. The response amplitude of a non-prismatic rotating Timoshenko beam resting on elastic foundation and under the actions of moving load for various values of circular frequency ω.

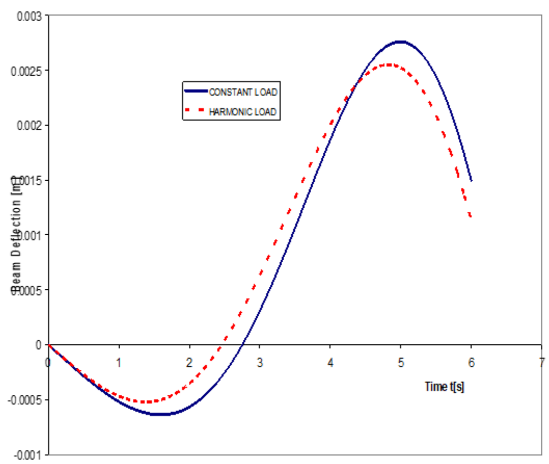

Figure 6: Comparison of the response of the non-prismatic rotating Timoshenko beam to constant and variable magnitude moving load for foundation modulus E_f and presstress N.

X. CONCLUDING REMARKS

This study investigated the behavior of a non-prismatic rotating Timoshenko beam resting on elastic foundation In particular, analytical solution in series form is obtained for the deflection and the rotation of the rotating Timoshenko beamand the effects of foundation stiffness E_f, the natural frequency ω and the presstress N on the vibrating system are investigated. Analytical solution and Numerical result in plotted curves show that, as the value of foundation stiffness E_f increases the deflection profile of the non-prismatic rotating Timoshenko beam resting on elastic foundation decreases. It is also observed that the response amplitudes of the dynamical systems decrease with an increase in the values of presstress N. Thus, in general, higher values of presstress N reduce the risk of resonance in a dynamical system involving non-prismatic rotating Timoshenko beam resting on elastic foundation

REFERENCES

[1] S.Timoshenko, “On the correction for shear of the differential equation for transverse vibration of prismatic bars.” Phil. Mag. S, 6(41).pp. 744-776.1921.

[2] C. R. Steel. “Beams and Shells with moving loads. Int. Journal of Solids and Structures.” Vol. 7, pp 1171-1198. 1971.

[3] H.S. Zibdeh, M.A. Hilal, “Vibration analyses of beams with general boundary conditions traversed by a moving force.” Journal of Sound and Vibration 229(2), pp 377-388. 2000.

[4] M. H. Kargarnovin, D, Younesian, “Dynamics of Timoshenko beams on Pasternak foundation under moving load.” Mechanics Research Communications (31) pp 713-723. 2004.

[5] R.S. Ayre, L.S. Jacobsen, and C.S. Hsu, editors, “Transverse vibrationof one-and of two-span beams under the action of a moving mass load,” Proceedings of the first U. S. National Congress of Applied Mechanics, pp.81-90, 1951.

[6] M.H Taha, S.Abohadima. **“**Mathematical model for vibrations of non-uniform beams.” Engineering MECHANICS, Vol. 15, (1), pp. 3–11. 2008

[7] Adedowole A. “Flexural motions under moving distributed masses of Beam- type structures on Vlasor foundation and having time dependent boundary conditions.” Ph.D thesis. Federal University of Technology, Akure, Nigeria. 2016

[8] A.Adedowole, K.S Famuwagun. “Simply supported non prismatic beam resting on variable elastic foundation carrying travelling harmonic loads.” Journal of Emerging Trends in

Engineering and Applied Sciences (JETEAS) 8(2): pp 98-103. 2017.

[9] A.S. Adeoye, T O Awodola. "Dynamic Response to Moving Distributed Masses of Pre-stressed Uniform Rayleigh Beam Resting on Variable Elastic Pasternak Foundation." Edelweiss Applied Science and Technology, 2, pp 1-9, 2018

[10] R.T. Wang. "Vibration of multi-span Timoshenko beams to a moving force." Journal of sound and vibration, 5(207), pp 731-742, 1997.

[11] S.T. Oni. "On the transverse vibrations under moving loads of deep beams on variable elastic foundations." Journal of Applied Sciences, 7(2), pp 4241-4256, 2004.

[12] B. Omolofe,S.N. Ogunyebi "Dynamic characteristics of a rotating Timoshenko beam subjected to a variable magnitude load travelling at varying speed." J. KSIAM. 20(1), pp. 17–35. 2016