Innovation for Solving the Problems in Social Life with Double Integrals

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Abstract- There are many different rules for mathematics field. Among them, we express the solving problems for practical life with integration methods. There are integrals, double integrals, triple integrals, etc. Among these method, we use the double integrals method to find the volume and area of a region between two curves and between line and curves that design of engineering's. We define and evaluate double integrals over bounded regions in the plane which are more general than rectangles. These double integrals are also evaluated as iterated integrals, with the main practical problem being that of determining the limits of integration. We can find the volume and area of the various shape by using the integration methods. For example, volume of, area of the leaf and area of the cardioid shape, etc. We can compute number of people in the region bounded during the interval with integration method.

Indexed Terms- volume by double integration, area by double integration and average value of function, double integrate a polar form.

I. INTRODUCTION

We consider the integral of a function of two variables $f(x,y)$over a region in the plane and the integral of a function of three variables $f(x,y,z)$over a region in space. To define the double integral of a function over a bounded, nonrectangular region R, this paper is organized with some basic definitions and notations. This paper present finding the volume with double integrals and finding the area with double integrals. And then we stated the double integral in polar form. We express finding the volume and area of the various shapes of the graph in social life. By using the integration methods to solve the problems in our social environment. In this paper, we used double integrals to solve the problems that submit the examples.

II. SOME BASIC DEFINITIONS

A. Definition

If R is a region like the one shown in the xy-plane in Figure.1, bounded 'above' and 'below' by the curve $y = g_2(x)$ and $y = g_1(x)$ and on the sides by the lines $x = a$ and $x = b$.The cross-sectional area,

$$A(x) = \int_{y=g_1(x)}^{y=g_2(x)} f(x,y)dy.$$

B. Definition

If $f(x,y)$ is positive and continuous over R, the volume of the solid region between R and the surface $z = f(x,y)$ to be $\iint_R f(x,y)dA$.

Integrating $A(x)$ from $x = a$ to $x = b$ to get the **volume** as an iterated integral,

$$V = \int_a^b A(x)dx = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)dy\,dx.$$

III. FUBINI'S THEOREM (STRONGER FORM)

Let $f(x,y)$ be continuous on a region R.

1. If R is defined by $a \le x \le b, g_1(x) \le y \le g_2(x)$, with $g_1 and\ g_2$ continuous on [a, b], then(By using vertical cross-sections),

$$\iint_R f(x,y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)dy\,dx.$$

2. If R is defined by $c \le y \le d, h_1(y) \le x \le h_2(y)$, with $h_1 and\ h_2$ continuous on [c, d], then(By using horizontal cross-sections),

$$\iint_R f(x,y)dA = \int_a^b \int_{h_1(y)}^{h_2(y)} f(x,y)dx\,dy.$$

IV. SOME EXAMPLES OF FINDING VOLUME

A. Example

The volume of a region such as tunnel shapes that lies the function $f(x,y) = \frac{x\,e^{2y}}{4-y}$ and x-axis between $0 \le x \le 2$ and $0 \le y \le 4 - x^2$ can be computed.

$$f(x,y) = \frac{x\,e^{2y}}{4-y}$$

$$\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\text{dy dx} = \int_0^4 \int_0^{\sqrt{4-y}} \frac{xe^{2y}}{4-y}\text{dx dy}.$$

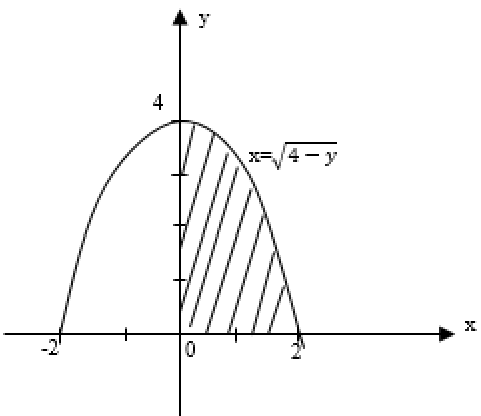

Figure. 1 Tunnel Shape

By using horizontal cross-sections,

$$\int_0^4 \int_0^{\sqrt{4-y}} \frac{xe^{2y}}{4-y}\text{dx dy} = \int_0^4 \left[\frac{x^2 e^{2y}}{2(4-y)}\right]_{x=0}^{x=\sqrt{4-y}} \text{dy}$$

$$= \int_0^4 \frac{e^{2y}}{2}\, dy$$

$$= \left[\frac{e^{2y}}{2}\right]_0^4$$

$$= \frac{e^8 - 1}{4}.$$

B. Example

The volume of the region that triangle shape under the ladder in the xy-plane bounded by the x-axis and the line $y = x$ and $x = 1$ and whose top lies in the plane $f(x,y) = x - y + 2$.

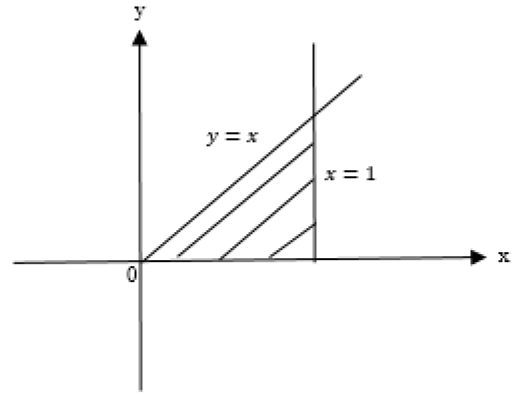

Figure.2 Triangle Shape

$f(x,y) = x - y + 2$

By using the vertical cross-sections,

$$\text{Volume,V} = \int_0^1 \int_0^x (x - y + 2)\,dy\,dx$$

$$= \int_0^1 \left[xy - \frac{y^2}{2} + 2y\right]_{y=0}^{y=x} dx$$

$$= \int_0^1 \left(x^2 - \frac{x^2}{2} + 2x\right) dx$$

$$= \int_0^1 \left(\frac{x^2}{2} + 2x\right) dx$$

$$= \left[\frac{x^3}{6} + x^2\right]_0^1$$

$$= \frac{1}{6} + 1 = \frac{7}{6}.$$

By using the horizontal cross-sections,

$$\text{Volume,V} = \int_0^1 \int_y^1 (x - y + 2)\,dx\,dy$$

$$= \int_0^1 \left[\frac{x^2}{2} - xy + 2x\right]_{x=y}^{x=1} dy$$

$$= \int_0^1 \left(\frac{5}{2} + \frac{y^2}{2} - 3y\right) dy$$

$$= \left[\frac{5y}{2} + \frac{y^3}{6} - \frac{3y^2}{2}\right]_0^1$$

$$= \left[\frac{5}{2} + \frac{1}{6} - \frac{3}{2}\right]$$

$$= \frac{15+1-9}{6} = \frac{7}{6}.$$

C. Example

We will design a can to build at a high place. Therefore, we should volume of the under a can. The volume of a region between the curves $f(x,y) = 3\cos x$ bounded by the x-axis on $(0 \le y \le \sec x$ and $-\frac{\pi}{3} \le x \le \frac{\pi}{3})$ can find.

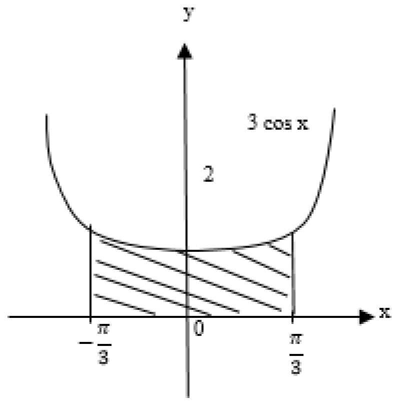

Figure 3.

$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\int_{0}^{\sec x}(3\cos x)dy\,dx = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}[3\cos x\,.\,y]_{y=0}^{y=\sec x}\,dx$

$= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}(3\cos x.\sec x)dx$

$= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}(3\,)dx$

$= 3[x]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$

$= 2\pi.$

V. AREA BY DOUBLE INTEGRATION

A. Definition

The area of a closed, bounded plane region R is

$$Area, A = \iint_{R} dA.$$

B. Definition

For an integrable function of two variables defined on a bounded region in the plane, the average value is the integral over the region divided by the area of the region.

$$Average\ value\ of\ f\ over\ R = \frac{1}{area\ of\ R}\iint_{R} dA.$$

VI. SOME EXAMPLE OF FINDING AREA

A. Example

An engineering design for a building that parabola shape region. He think enclosed area by using the integration method. The area of the region R enclosed by the parabola shape $x = y^2$ and the line $y = x + 2$,can be computed.

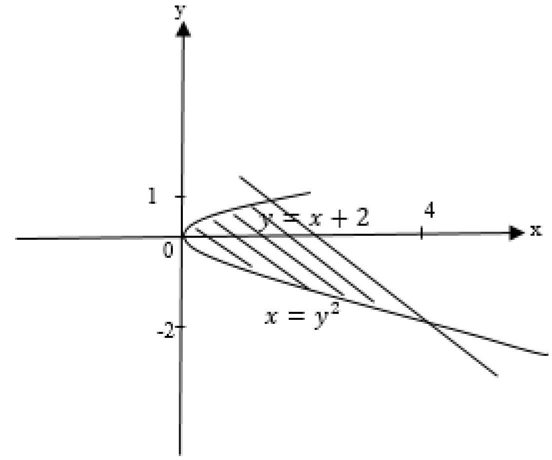

Figure 4.

$x = y^2$ And $y = x + 2$

$y = y^2 + 2$

$y^2 - y + 2 = 0$

$(y-2)(y+1) = 0$

$y = 2\ and\ y = -1.$

Region:$y^2 \leq x \leq y - 2\,, -1 \leq y \leq 2.$

Area, A=$\iint_{R} dA.$

$= \int_{-1}^{2}\int_{y^2}^{y-2} dx\,dy$

$= \int_{-1}^{2}[x]_{y^2}^{y-2}dy$

$= \int_{-1}^{2}[y - 2 - y^2]\,dy$

$= \left[\frac{y^2}{2} - 2y - \frac{y^3}{3}\right]_{-1}^{2}$

$= \left[2 - 4 - \frac{8}{3}\right] - \left[\frac{1}{2} + 2 + \frac{1}{3}\right]$

$= \frac{-14}{3} - \frac{17}{6} = \frac{-28-17}{6} = \frac{-45}{6}\,.$

B. Example

The area of the regions R can be computed. The following function

$\int_{0}^{1}\int_{x^2-1}^{0} dy\,dx + \int_{0}^{4}\int_{0}^{\sqrt{x}} dy\,dx$

$= \int_{0}^{1}[y]_{x^2-1}^{0}\,dx + \int_{0}^{4}[y]_{0}^{\sqrt{x}}\,dx$

$= \int_{0}^{1}[-x^2 + 1]\,dx + \int_{0}^{4}[\sqrt{x}]\,dx$

$= \left[-\frac{x^3}{3} + x\right]_{0}^{1} + \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_{0}^{4}$

$= -\frac{1}{3} + 1 + \frac{16}{3}$

$= \frac{-1+3+16}{3} = 6.$

C. Example

Finding the average values of $f(x,y) = \cos(x+y)$ over the rectangle $0 \le x \le \pi\,, 0 \le y \le \pi$.

$$\iint_R dA = \int_0^{\pi}\int_0^{\pi} \cos(x+y)\,dy\,dx$$

$= \int_0^{\pi} [\sin(x+y)]_0^{\pi}\, dx$

$= \int_0^{1} [\sin(x+\pi) + \sin x]\, dx$

$= [-\cos(x+\pi) - \cos x]_0^{\pi}$

$= [-\cos(2\pi) - \cos\pi] - [-\cos(\pi) - 1]$

$= [-1+1] - [1-1] = 0.$

Area of rectangle $=\pi \times \pi = \pi^2$.

$$Average\ value\ of\ f\ over\ R = \frac{1}{area\ of\ R}\iint_R dA.$$

Average value of f over R$= \frac{1}{\pi^2} \times 0 = 0.$

VII. DOUBLE INTEGRAL IN POLAR FORM

Area differential in polar coordinates,

$$dA = r\,dr\,d\theta.$$

$$\iint_R f(r,\theta)dA = \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} f(r,\theta) r dr\,d\theta$$

A.Example

The area enclosed by the one leaf of the rose $r = 2\cos 3\theta$ Over the region R:$-\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$, can be stated.

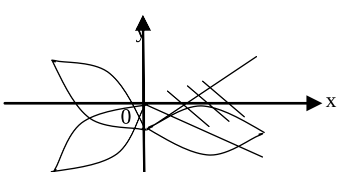

Figure 5.

Area,A$=\iint_R$ dA.

$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\int_0^{2\cos 3\theta} r\,dr\,d\theta$

$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\left[\frac{r^2}{2}\right]_0^{2\cos 3\theta} d\theta$

$= \frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}[4(\cos 3\theta)^2]\,d\theta$

$= 4\int_0^{\frac{\pi}{6}}\left[\frac{1+\cos 6\theta}{2}\right] d\theta$

$= 2\left[\theta + \frac{\sin 6\theta}{6}\right]_0^{\frac{\pi}{6}}$

$= 2\left[\left(\frac{\pi}{6} + \frac{\sin\pi}{6}\right) - 0\right] = \frac{\pi}{3}$.

B.Example

The area of the region R of a piece of moon. The area of the region that lies inside the cardioid $r = 1 + \cos\theta$ over the region R:$0 \le \theta \le \frac{\pi}{2}$ and outside the circle $r = 1$ can be stated.

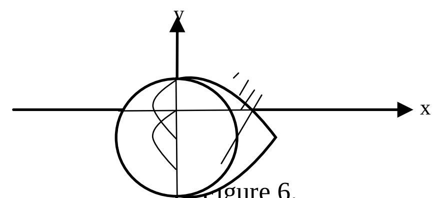

Figure 6.

Area,A$=\iint_R$ dA.

$= 2\int_0^{\frac{\pi}{2}}\int_1^{1+\cos\theta} r\,dr\,d\theta$

$= 2\int_0^{\frac{\pi}{2}}\left[\frac{r^2}{2}\right]_1^{1+\cos\theta} d\theta$

$= \int_0^{\frac{\pi}{2}}[1 + 2\cos\theta + \cos^2\theta - 1]\,d\theta$

$= \int_0^{\frac{\pi}{2}}\left[2\cos\theta + \frac{1+\cos 2\theta}{2}\right] d\theta$

$= \left[2\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{2}}$

$= \left[\left(2 + \frac{\pi}{4} + 0\right) - 0\right]$

$= \frac{8+\pi}{4}$.

CONCLUSION

There are many different rules for mathematics researcher. Among them, integration method is very important method. Especially for engineerings because they are compute in imaginary for their design that is, volume of system and area of design for their practical life. Define and evaluate double integrals over bounded regions in the plane which are more general than rectangles. These double integrals are also evaluated as iterated integrals, with the main practical problem being that of determining the limits of integration. By using the integration method: integrations, double integrations and triple integrations, computed for solving the problems in daily life.

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