Newton Raphson Single and Multiple Variable Methods to Obtain the Solutions of Linear and Non Linear Equations

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Abstract -- The paper is about Newton Raphson Method which is all inclusive to solve the non-square and non-linear problems. The study also aims to comparing the rate of performance, rate of convergence of bisection method, root finding of the Newton meted and Secant method. It also represents a new approach of calculation using nonlinear equation and this will be similar to Newton Raphson Method and inverse Jacobian matrix will be used for the iteration process and this will be further used for distributed power load flow calculation and will also be useful in some of the applications. The paper also difference the use of built in derivative function and self-derivative function in solving non-linear equation in scientific calculator. And paper also discuss about single variable and multi variable Newton-Raphson techniques.

I. INTRODUCTION

Finding the solution to the set of nonlinear equation $F(x) = (f1, f2,\ldots\ldots.f^n)=0$ is been a problem for the past years. Here we consider this nonlinear equation and try finding the solution to it and this can be found out by the Newton Raphson method. This method is very familiar for its fast rate convergence and for improving the convergence property,

Root finding is also one of the problems in practical applications [1]. Newton method is very fast and efficient as compared to the others methods. In order to compare the performance, it is therefore very important to observe the cost and speed of the convergence. Newton method requires only one iteration and the derivative evaluation per iteration. Newton-Raphson Method is also used to be find a maximum or minimum of a function $F(x)$. The derivative is zero at a minimum or maximum, so local minima and maxima can be found by applying Newton's method to the derivative. The iteration becomes:-

$$X_{n+1} = X_n - \frac{F\prime(Xn)}{F\prime\prime(Xn)}$$

And there are many more applications of Newton-Raphson Method.[6]

II. NEWTON-RAPHSON METHOD

By solving any linear or non-linear equation using Newton-Raphson method we should have an initial value (X_0). For deriving the formula for Newton-Raphson Method:-[2]

Let the function be $F(X)$ and initial root of the equation be X_0

$F(X)=0$
$X_1=X_0+h$ - (1)

$$F(X) = F(X) + hF'(X) + \frac{h^2 F''(X)}{2!} + \frac{h^3 F'''(X)}{3!} + \cdots, \quad \{\text{using Taylor series}\}$$

Since 'h' is small, neglect higher order terms

$F(X_1)= F(X_0)+hF'(X_0)$
$h= -F(X_0)/F'(X_0)$
Substitute h value in (1)
$X_1=X_0 - F(X_0)/F'(X_0)$
$X_2=X_1 - F(X_1)/F'(X_1)$

There are two important conditions for solving equation by Newton-Raphson Method:
- If $F'(X0) =0$, then change value of X_0
- Use the condition

$F(X_0).F''(X_0)>0$

Example; find the real root of the equation $F(X)=2x^2-3\sin(x)-5=0$ correct upto 3 decimal places, where initial $X_0=1$ using Newton Raphson Method.

Solution:
Let $F(X)= 2x^2-3\sin(x)-5$
$F'(X)= 4x-3\cos(x)$
$F''(X)= 4+\sin(x)$

Now
$F(X_0)= 2(1)^2-3\sin(1)-5$
$\quad = -5.524412$
$F''(X)= 4+\sin(1)$
$\quad =6.524412$

It satisfies the condition
$F(X0). F''(X0)>0$

Therefore we can use Newton Raphson Method by taking initial value as $X_0=1$

1st Iteration:
$F'(X)= 4(1)-3\cos(1)$
$\quad =2.379093$
$X_1=X_0 - F(X_0)/ F'(X_0)$
$X_1=1- \left\{\dfrac{-(5.524412)}{2.2379093}\right\}$
$X_1=1+ \left\{\dfrac{(5.524412)}{2.2379093}\right\}$
$X_1= 3.322066$

2nd iteration
$F(X_1)= 2(3.322066)^2-3\sin(3.322066)-5$
$F(X_1)= 17.610730$
$F'(X_1)= 2(3.322066)-3\cos(3.322066)$
$F(X_1)= 16.239540$
$X_2=X_1- F(X_1)/ F'(X_1)$
$X_2=3.322066 -\dfrac{17.610730}{16.239540}$
$X_2=2.237630$

3rd iteration:
$F(X_2)= 2(2.237630)^2-3\sin(2.237630)-5$
$F(X_2)=2.656624$
$F'(X_2)= 2(2.237630)-3\cos(2.237630)$
$F'(X_1)= 10.806023$
$X_3=X_2- F(X_2)/ F'(X_2)$
$X_3=2.237630 -\dfrac{2.656624}{10.806023}$

$X_3=1.991783$

4th iteration:
$F(X_3) = 2(1.991783)^2-3\sin(1.991783)-5$
$F(X_3) =0.196340$
$F'(X_3) = 2(1.991783)-3\cos(1.991783)$
$F'(X_3) =9.193115$

$X_4=2.237630 -\dfrac{0.196340}{9.193115}$

$X_4=1.970425$

5th iteration:
$F(X_4)= 2(1.970425)^2-3\sin(1.970425)-5$
$F(X_4)=1.843943$
$F'(X_4)= 2(1.970425)-3\cos(1.970425)$
$F'(X_4)=9.048928$
$X_5=X_4- F(X_4)/ F'(X_4)$
$X_5=1.970425 -\dfrac{1.843943}{9.048928}$

$X_5=1.970255$

So the real root of the equation $F(X)=2x^2-3\sin(x)-5$ is 1.970255.

So this was the procedure of Newton Raphson Method.

Now there are two types of Newton Raphson method there are as follow:
2.1 Newton-Raphson method for Single variable
2.2 Newton-Raphson method for Multi variables[5]

2.1 Newton-Raphson Method For Single Variable

In this method we can find more than one root of the given linear equation. Formula for this method is same as Newton Raphson method except there is one more term in this method
$\qquad X_{n+1}=X_n–[P]\{F(X_n)/ F'(X_n)\}$
Or
$\qquad X_1=X_0 – [P-1]\{F'(X_0)/ F''(X_0) \}$

Here 'P' is the number of roots which we want to find. And this method is only applicable when the given equation is a Linear equation.[3]

2.2: Newton-Raphson method for Multi variables:
This method is used to find roots of multi variable i.e.. There are two different variables. Consider two non-linear equation having two variable 'x' and 'y'.

F(x,y)=0
G(x,y)=0

The solution is given by

$$X_{n+1}=X_n-\frac{\begin{vmatrix} F(x,y) & G(x,y) \\ \frac{dF(x,y)}{dy} & \frac{G(x,y)}{dy} \end{vmatrix}}{J(F,G)}$$

$$Y_{n+1}=Y_n-\frac{\begin{vmatrix} F(x,y) & G(x,y) \\ \frac{dF(x,y)}{dx} & \frac{G(x,y)}{dx} \end{vmatrix}}{J(F,G)}$$

And where $J(F,G)$ is

$$J(F,G)=\begin{vmatrix} \frac{dF(x,y)}{dx} & \frac{dF(x,y)}{dy} \\ \frac{G(x,y)}{dx} & \frac{G(x,y)}{dy} \end{vmatrix}$$

This method is used only when the equation is non-linear.[4]

III. CONCLUSION

In this review paper we have discussed about Newton Raphson method and its types. We have also solved a example using Newton Raphson Method and we have also derived the formula for Newton Raphson method .we have learnt how to solve Newton Raphson single variable and multi variable method.

The effectiveness of using scientific calculator in solving problem in linear and non-linear equation by Newton-Raphson method is also less time consuming. By using scientific calculator we can solve equation faster and effectively. The chances of error are less and we can solve iteration faster by calculator.

Newton-Raphson Method is a easy method to solve linear and non-linear equation and it is less time consuming method too.

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