Towards A Comprehensive Life Cycle Costing Model for Sustainable Facilities

OLUFOLAHAN ODUYEMI[1], OLAYINKA ABUDU[2], IBRAHIM ADENEKAN[3]
[1, 2, 3] *Southeast Missouri State University, Cape Girardeau, Missouri*

Abstract-
Purpose
The study attempts to develop a methodology for modelling the operating and maintenance costs of an existing sustainable facility using Markov chain and to determine the most appropriate probability distribution that conform with these costs.

Design/methodology/approach
The paper adopts the probabilistic Markov chain model to show the trend of building costs fluctuations. It also obtains the pattern of fluctuations of operating and maintenance costs to determine whether these costs follow a Markov chain model.

Findings
The key findings of the statistical analysis indicated there is an existence of a Markov chain in the observed building costs from January 2011 to January 2020. It also revealed that the building costs series can best be modelled using the Weibull distribution.

Research limitations/implications
This paper does not cover more than one existing case study as access to historical costs is a major concern.

Practical implications
This research will be of interest to industry practitioners and academic researchers with an interest in building modelling. The study can be used to improve the confidence in life cycle costing (LCC) modelling.

Originality/value
This paper contributes with new outlooks aimed at stochastic modelling of sustainable facilities.

Indexed Terms- Costs, Facilities, Life cycle costing, Markov, Modelling, and Sustainable.

I. INTRODUCTION

Life cycle costing is defined as the costs associated with acquiring, using, caring for and disposing of physical assets. One can draw from this definition that LCC quantifies and forecasts choices which can be used to determine the ideal choice of assets (Haugbolle & Raffnsoe, 2019; Zhou et al., 2019).

Yet, there are immense doubts about the accuracy of LCC estimates as they are deemed to be imprecise, inexact, uncertain and vague (Kirkham et al. 2004; Farahani et al., 2019). The above submission unmistakeably shows a variance in prevailing cost estimation techniques and underlines the necessity for re-assessment and potential re-evaluation of LCC methodologies (Oduyemi, 2015; Konstantinidou et al., 2019).

Consequently, the challenge among practitioners is to develop a framework for LCC that is not only universal, but more importantly dynamic as clients now want buildings that demonstrate value for money over the long term and are not interested simply in the design solution which is the least expensive.

This study adopts the probabilistic Markov chain model to show the trend of building maintenance and operating costs fluctuations. This is achieved by obtaining the pattern of fluctuations and determining whether the building maintenance and operating costs follow a Markov chain. It also models these costs by the limit probability of a Markov chain and determines which among the named distributions (Lognormal, Weibull and Gumbel Max) conform with and model the costs with the distribution.

II. MARKOV CHAIN

Markov chains are a fundamental class of stochastic processes. The success of Markov chains is mainly due to their simplicity of use, the large number of available theoretical results and the quality of algorithms developed for the numerical evaluation of various metrics associated with them (Li & Rosenthal, 2019; Betancourt, 2019).

The Markov property means that if the state of the process is known at a given time, predicting its future about this point does not require any information about its past. This property allows for a considerable reduction of parameters necessary to represent the evolution of a system modelled by such a process (Zhang & Li, 2019). It is simple enough for the modelling of systems to be natural and intuitive but also very rich in that it allows general probability distributions in a very precise manner.

Several researchers in the construction industry have addressed potential applications of Markov Chains. Zhang et al (2018) presented a deterioration prediction method for maintenance planning in offshore engineering using the Markov. Sobanjo (2009) presented an investigation of the Markov property underlying the stochastic deterioration models for highway bridges, including transition probabilities between the condition states while Bocchini et al (2013) presented an efficient, accurate, and simple Markov chain model for the life-cycle analysis of individual bridges and bridge groups. Although a substantial amount of research presently exists in the use of Markov chains, none explicitly show the trend of building costs fluctuations or obtain the pattern of fluctuations of operating and maintenance costs of sustainable buildings.

III. RESEARCH METHODOLOGY

To develop a Markov model, historical cost data was gathered from the International Facilities Management (IFMA) Database The case study is a two-storey office block in Houston, Texas built in August 2010 and has an excellent LEED rating.

The following four steps discussed below are used in this paper to develop the Markov conceptual framework.

i. Identification of project objectives, and project constraints.
ii. Determine the length of the study period.
iii. Cost breakdown structure.
iv. Modelling using Markovian Chain (Variable selection, Training and Validation).

i. Identification of project objectives and project constraints.
The LCC analysis is used in this paper to develop a methodology for modelling the operating and maintenance costs of an existing sustainable facility using Markov chain and to determine the most appropriate probability distribution that conform with these costs.

ii. Determine the length of the study period.
The study period commenced at time zero which was previously defined.

iii. Cost breakdown structure
For each LCC project, cost centre was identified, and information gathered from historic data and building surveys of IFMA and subsequently a cost breakdown structure (CBS) is developed for the building. For this case study, the operating and maintenance costs were summed over the stated years as seen in table one.

Table 1: Historical cost breakdown structure of the operating and maintenance costs of case study

	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Jan	5815	7562	10807	10409	9937	10571	6251	4594	4989	4665
Feb	6915	7473	10585	9073	9974	10837	6131	4769	4617	
Mar	6467	7458	10792	9675	10526	10523	5434	4413	3651	
Apr	7163	7583	10049	10527	10816	10005	4569	4487	4428	
May	6835	7612	10082	10628	10876	9585	4628	4504	4273	
Jun	7408	8172	9985	10341	10543	8608	4696	4929	5276	

Jul	7755	8453	10541	10117	10263	7699	4311	4526	4276	
Aug	7488	9001	10423	10119	10548	6070	3657	5262	4271	
Sep	7712	9269	10707	10510	10210	4711	2978	5359	4312	
Oct	7476	9791	11269	10764	10483	5479	3103	5435	4587	
Nov	7930	10865	11779	10252	10404	5283	3734	5090	4386	
Dec	8418	11624	11367	9885	10487	5754	4075	5216	4184	

iv. Modelling using Markov Chain (Variable selection, Training and Validation)

The historical maintenance and operational cost data were modelled using Markov chain. The variables were tested using the goodness of fit of probability chains and the limit of the transition probability matrix.

IV. ANALYSIS AND DISCUSSION OF RESULTS

4.1 THE BUILDING COSTS SERIES

Table 1 showed the monthly building maintenance and operating costs from January 2011 to January 2020. The costs series feature chaotic characteristics during the 109 months. Figure 1 shows a moving process of the costs with its main distinguishing features including the occurrence of cost prices extraordinary soaring or steeply falling.

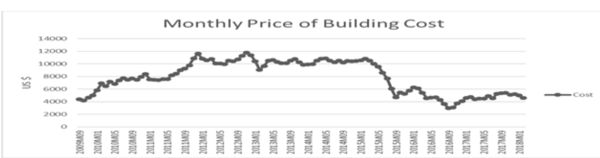

Figure 1. Trend of building costs from January 2011 to January 2020

4.2 THE BUILDING COSTS TRANSITION STATES

From the trend of the costs, the stage transition states can be clearly distinguished. They can be classified as five states: low state, middle - low state, middle state, middle - high state and high state. These states constitute the following full space for stochastic events of building costs:

(0, 4000) [4000, 6000) [6000, 8000) [8000, 10000) [10000, 12000)

The average building cost is US$ 7874. So [6000, 8000) is treated as a middle state of fluctuating costs, the [4000, 6000) interval as a middle - low state, and the [8000, 10000) interval as a middle - high state. The three states can be broadly termed as the middle state. By contrast, the (0, 4000) interval is treated as a low state of costs and the [10000, 12000) interval as a high state. Assuming A represents the (0, 4000) interval of low-state costs, B the [4000, 6000) interval, C the [6000, 8000) interval, D the [8000, 10000) interval, and E the [10000, 12000) interval. There are 109 states and 108 state transitions which constitute a building cost transition process.

4.2.1 STATE TRANSITION FREQUENCY OF BUILDING COSTS

Table 2 shows the state transition frequency matrix of building costs five state transition chains.

Table 2: State Transition Frequency of Building Costs from January 2011 to January 2020

	A	B	C	D	E	$n_{i.}$
A	3	1	0	0	0	4
B	1	25	2	0	0	28
C	0	2	15	2	0	19
D	0	0	2	8	4	14
E	0	0	0	4	32	36
$n_{.j}$	4	28	19	14	36	n=101

4.2.2 TESTS FOR THE EXISTENCE OF A MARKOV CHAIN

The five-state transition chain has a χ^2 distribution if it follows a Markov chain. To test this, the formula below is applied.

$$\chi^2 = \sum_{i=1}^{m}\sum_{j=1}^{m}\frac{\left(n_{ij} - n_{i.}n_{.j}/n\right)^2}{\left(n_{i.}n_{.j}/n\right)}$$

Where $n_{i.}$ (i = 1, 2, …, m) and $n_{.j}$ (j = 1, 2, …, m) are the frequency of state i and state j respectively. This has χ^2 distribution with $(m-1)^2$ degrees of freedom, where m refers to the number of states. The test results are reported in Table 3.

Table 3: χ^2 Testing Results of Building Cost Transition Chain

$\frac{(n_{ij} - n_{i.}n_{.j}}{(n_{i.}n_{.j}/n}$	j = 1 (A)	j = 2 (B)	j = 3 (C)	j = 4 (D)	j = 5 (E)
i = 1 (A)	50.9709	0.0107	0.7525	0.5545	1.4257
i = 2 (B)	0.0107	38.279	2.0267	3.8812	9.9802
i = 3 (C)	0.7525	2.0267	36.5244	0.1525	6.7723
i = 4 (D)	0.5545	3.8812	0.1525	18.9202	0.1964
i = 5 (E)	1.4257	9.9802	6.7723	0.1964	28.6342
$\sum_{i=1}^{m}$	53.7123	54.1778	46.2284	23.7048	47.0088
$\sum_{i=1}^{m}\sum_{j=1}^{m}$					224.8321

With m = 5, so the degrees of freedom $(m-1)^2$ = $(5-1)^2$ = 16. Using a 5% significance level (that is, $\alpha = 0.05$) and referring to the χ^2 tables with 16 degrees of freedom, ${\chi^2}_{(16,\ \alpha=0.05)} = 26.296$.

The observed value of the sample statistics χ^2 is 224.8321, much higher than ${\chi^2}_{(16,\ \alpha=0.05)} = 26.296$.

Thus, the null hypothesis is rejected, and the states are independent. As a result, it confirms that a state transition chain of building costs January 2011 to January 2020 follows a Markov Chain.

4.3 LIMIT PROBABILITY OF A MARKOV CHAIN FOR CHANGING TRENDS OF BUILDING COSTS

Let P represent the first – stage transition matrix. Having each element n_{ij} of respective row from Table 3 divided by the sum of its row($n_{i.}$), the first – stage transition matrix P is stated below:

$$P = \begin{bmatrix} 0.7500 & 0.2500 & 0.0000 & 0.0000 & 0.0000 \\ 0.0357 & 0.8929 & 0.0714 & 0.0000 & 0.0000 \\ 0.0000 & 0.1053 & 0.7895 & 0.1053 & 0.0000 \\ 0.0000 & 0.0000 & 0.1429 & 0.5714 & 0.2857 \\ 0.0000 & 0.0000 & 0.0000 & 0.1111 & 0.8889 \end{bmatrix}$$

Secondly, the derivation of the second – stage transition matrix, …, and the convergence state transition matrix is as follows:

$$P^2 = \begin{bmatrix} 0.5714 & 0.4107 & 0.0179 & 0.0000 & 0.0000 \\ 0.0587 & 0.8136 & 0.1202 & 0.0075 & 0.0000 \\ 0.0038 & 0.1771 & 0.6458 & 0.1433 & 0.0301 \\ 0.0000 & 0.0150 & 0.1944 & 0.3733 & 0.4172 \\ 0.0000 & 0.0000 & 0.0159 & 0.1623 & 0.8219 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 0.2835 & 0.6059 & 0.0978 & 0.0108 & 0.0021 \\ 0.0866 & 0.6709 & 0.1947 & 0.0344 & 0.0135 \\ 0.0206 & 0.2869 & 0.4090 & 0.1477 & 0.1359 \\ 0.0031 & 0.0688 & 0.2004 & 0.2121 & 0.5156 \\ 0.0002 & 0.0105 & 0.0717 & 0.2005 & 0.7171 \end{bmatrix}$$

$$P^{250} = \begin{bmatrix} 0.0396 & 0.2772 & 0.1881 & 0.1386 & 0.3564 \\ 0.0396 & 0.2772 & 0.1881 & 0.1386 & 0.3564 \\ 0.0396 & 0.2772 & 0.1881 & 0.1386 & 0.3564 \\ 0.0396 & 0.2772 & 0.1881 & 0.1386 & 0.3564 \\ 0.0396 & 0.2772 & 0.1881 & 0.1386 & 0.3564 \end{bmatrix}$$

The row – vector, which has been converged to the same value is the Markov chain's limit of building costs series denoted by L

With the initial state probability vector, $X^{(0)} = [0 \quad 1 \quad 0 \quad 0 \quad 0]$

$$X^{(\infty)} = [0 \quad 1 \quad 0 \quad 0 \quad 0] \begin{bmatrix} 0.0396 & 0.2772 & 0.1881 & 0.1386 & 0.3564 \\ 0.0396 & 0.2772 & 0.1881 & 0.1386 & 0.3564 \\ 0.0396 & 0.2772 & 0.1881 & 0.1386 & 0.3564 \\ 0.0396 & 0.2772 & 0.1881 & 0.1386 & 0.3564 \\ 0.0396 & 0.2772 & 0.1881 & 0.1386 & 0.3564 \end{bmatrix}$$

$$\mathrm{X}^{(\infty)} = [0.0396 \quad 0.2772 \quad 0.1881 \quad 0.1386 \quad 0.3564]$$

$X^{(\infty)}$ is the limiting state probability vector

It implies that the building costs series have become stabilized after a continuous transition process. This convergence process is the changing trend of building costs, and the row – vector as the limit probability is the ultimate state of building costs series as seen in table four.

Table 4: Limit Probability Value of Markov Chain

	A	B	C	D	E
Limit Probability Value of Markov Chain	0.0396	0.2772	0.1881	0.1386	0.3564

This limit probability vector indicates the ultimate probability (or proportion) of five states in the building costs series. The probability of low level state A(0,4000) is 0.0396, meaning that the proportion in the series is 3.96%; the probability of middle – low level state B[4000,6000) is 0.2772, the proportion is 27.72%, the probability of middle level state C[6000,8000) is 0.1881, the proportion is 18.81%, the probability of middle – high level state D[8000,10000) is 0.1386, the proportion is 13.86%, the probability of high level state E[10000,12000) is 0.3564, the proportion is 35.64% respectively as seen in figure two.

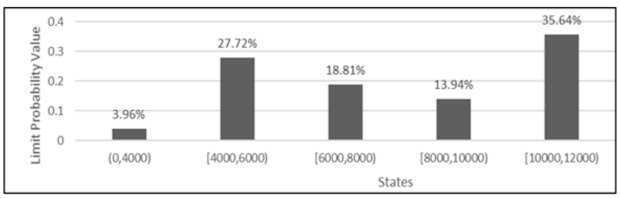

Figure 2: The Limit Probability of Monthly Building Costs

4.4 THE PROBABILITY DISTRIBUTION OF THE CHANGING TRENDS OF BUILDING COSTS

The limit probability of building costs state transition chain as a Markov Chain is the ultimate state of building costs series. It approximates the changing trends of building costs in the medium and long-terms, but not in the short-term. An actual distribution of building costs series reflects the short-term changing trends of building costs.

This study considered three named distribution, Lognormal, Gumbel Max and Weibull distribution by carrying out some statistical tests among the competing models to determine which distribution best fits the building costs data.

4.4.1 PARAMETER ESTIMATES OF THE DISTRIBUTIONS

To determine the probability distribution which the data follows, maximum likelihood estimates of each of the parameters of the distributions are obtained as it is summarized in table five below.

Table 5: Parameter estimates for the Distribution

DISTRIBUTIONS	ESTIMATED PARAMETERS
LOGNORMAL $\mathrm{LN}(\mu, \sigma^2)$	$\hat{\mu} = 8.9097505$ $\hat{\sigma} = 0.3649262$
GUMBEL MAX $\mathrm{GB}(\mu, \sigma)$	$\hat{\mu} = 6571.838$ $\hat{\sigma} = 2393.921$
WEIBULL $\mathrm{WE}(\gamma, k)$	$\hat{\gamma} = 8782.601809$ $\hat{k} = 3.588844$

4.4.2 LOG – LIKELIHOOD RATIO VALUES OF THE DISTRIBUTIONS

The log-likelihood theory provides rigorous and omnibus inference methods if the model is given, that is, after the parameters of a distribution have been obtained. The log-likelihoods form the basis of the selection of the distribution that fits the data. The distribution with the maximum log – likelihood value is considered the most suitable. The log – likelihood and log – likelihood ratio values are provided in tables 6 and 7.

Table 6: Log – likelihood values of the Distributions

DISTRIBUTIONS	LOGNORMAL $LN(\mu, \sigma^2)$	GUMBEL MAX $GB(\mu, \sigma)$	WEIBULL $WE(\gamma, k)$
LOG-LIKELIHOOD	-950.7042	-951.1071	-942.9634

The log – likelihood ratio value is obtained by obtaining the difference between likelihood values of competing distributions

Table 7: Log – likelihood ratio values of the Distributions

Competing Distribution	Decision	Inference
$LN(\mu,\ \sigma^2)$ and $GB(\mu,\sigma)$	0.4029 > 0	LOGNORMAL
$WE(\gamma,k)$ and $GB(\mu,\sigma)$	8.1437 > 0	WEIBULL
$WE(\gamma,k)$ and $LN(\mu,\ \sigma^2)$	7.7408 > 0	WEIBULL

The Weibull distribution emerges the most suitable distribution for the data since it has the highest log – likelihood value.

4.4.3 GOODNESS OF FITS TEST

The idea behind the goodness of fit tests is to measure the "distance" between the data and the distribution being tested and compare the distance to some threshold value. Since the goodness of fit test statistics indicate the distance between the data and the fitted distributions, it is obvious that the distribution with the lowest statistic value is the best fitting model.

Table 8: Goodness of fit test

	LOGNORMAL	GUMBEL MAX	WEIBULL
Kolmogorov-Smirnov	0.1819568	0.1793120	0.1753222
Anderson-Darling	4.4835393	4.2710097	4.2692543
Cramer-von Mises	0.7405879	0.6804194	0.6753404

The Weibull distribution is shown to have the best fit of the three distributions. The Kolmogorov – Smirnov, Anderson - Darling statistics and Cramer-von Mises have the least figure under the Weibull distribution. These results show that in comparison to Lognormal and Gumbel Max distribution, the Weibull distribution is a more acceptable fit for the building costs.

4.4.4 COMPARISON OF MODEL

This section compares the appropriateness of the distribution models. Specification measures such as Akaike Information Criteria (AIC) and Bayes Information Criteria (BIC) were employed. It was observed that the AIC and BIC for the Weibull distribution is smaller than those of the gamma, Lognormal and Gumbel Max distribution. Thus, the Weibull distribution is a better fit for the building costs.

Table 9: Comparison of the results of the models

	LOGNORMAL	GUMBEL MAX	WEIBULL
AIC	1905.408	1906.214	1889.927
BIC	1910.658	1911.464	1895.177

From Table 6 to 9, it can be deduced that the building costs series conform to a Weibull distribution.

The function of the Weibull distribution is as follows:

$f(x,\gamma,k) = \frac{k}{\gamma}\left(\frac{x}{\gamma}\right)^{k-1} e^{-\left(\frac{x}{\gamma}\right)^k}$, $x > 0$, where $k > 0$ is the shape parameter and $\gamma > 0$ is the scale parameter.

The estimated parameters are: $k = 3.588$ and $\gamma = 8782.602$

Substituting $k = 3.588$ and $\gamma = 8782.602$ in the Weibull distribution,

$$f(x) = \frac{3.588}{8782.602}\left(\frac{x}{8782.602}\right)^{3.588-1} e^{-\left(\frac{x}{8782.602}\right)^{3.588}}$$

$$= 0.00041\left(\frac{x}{8782.602}\right)^{2.588} e^{-\left(\frac{x}{8782.602}\right)^{3.588}}$$

Hence, the fitted probability density function (pdf) is given as:

$$f(x) = 0.00041\left(\frac{x}{8782.602}\right)^{2.588} e^{-\left(\frac{x}{8782.602}\right)^{3.588}}$$

V. CONCLUSION

The value of LCC is its ability to provide more comprehensive and accurate cost predictions as there is an increasing realisation of the importance of considering operation and maintenance costs as opposed to capital costs throughout the life of an asset.

The results obtained from this exercise provide the researcher with a great deal of information about modelling inputs into life cycle costing exercises. The conceptual framework is generic and can thus be applied to any sustainable building, at any level from sub-elemental to the whole cost scenario.

This study is different from previous ones in terms of the input parameters used and a different case study.

This paper further demonstrates that the development of a stochastic model using Markov chain methodology is feasible. There is a however the need to evaluate Markov chains with more projects for future developments.

REFERENCES

[1] Betancourt, M. (2019). The convergence of Markov chain Monte Carlo methods: from the Metropolis method to Hamiltonian Monte Carlo. Annalen der Physik, 531(3), 1700214.

[2] Bocchini, P., Saydam, D., & Frangopol, D. M. (2013). Efficient, accurate, and simple Markov chain model for the life-cycle analysis of bridge groups. *Structural safety*, *40*, 51-64.

[3] Farahani, A., Wallbaum, H., & Dalenbäck, J. O. (2019). Optimized maintenance and renovation scheduling in multifamily buildings–a systematic approach based on condition state and life cycle cost of building components. Construction management and economics, 37(3), 139-155.

[4] Konstantinidou, C. A., Lang, W., Papadopoulos, A. M., & Santamouris, M. (2019). Life cycle and life cycle cost implications of integrated phase change materials in office buildings. International Journal of Energy Research, 43(1), 150-166.

[5] Li, B., & Rosenthal, J. S. (2019). Exploration of Markov Chain Monte Carlo Algorithms.

[6] Oduyemi, O. I. (2015). *Life cycle costing methodology for sustainable commercial office buildings* (Doctoral dissertation).

[7] Haugbølle, K., & Raffnsøe, L. M. (2019). Rethinking life cycle cost drivers for sustainable office buildings in Denmark. Facilities.

[8] Sobanjo, J. O. (2011). State transition probabilities in bridge deterioration based on Weibull sojourn times. *Structure and Infrastructure Engineering*, *7*(10), 747-764.

[9] Zhang, Y., Kim, C. W., & Tee, K. F. (2017). Maintenance management of offshore structures using Markov process model with random transition probabilities. *Structure and Infrastructure Engineering*, *13*(8), 1068-1080.

[10] Zhang, G., & Li, L. (2019). Analysis of Markov Chain Approximation for Option Pricing and Hedging: Grid Design and Convergence Behavior. Operations Research, 67(2), 407-427.

[11] Zhou, Z., Chi, Y., Dong, J., Tang, Y., & Ni, M. (2019). Model development of sustainability assessment from a life cycle perspective: A case study on waste management systems in China. Journal of cleaner production, 210, 1005-1014.