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A Novel Three Phase Induction Motor – Analysis of Its Performance Indices

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Abstract- ***The ultimate feature of every electric motor, be it a direct current type or an alternating current type is its torque (T) and speed (ωm) production capabilities. They are the yard stick upon which the machine is measured. The output characteristics (such as output torque, output power, power factor and efficiency) of all induction motors, whether squirrel cage or wound rotor type are much superior to that of three-phase transfer field motor of the same size and rating, and operating in the asynchronous mode. These are the attributes of the fact that; the losses taking place in three phase induction motors are less in percentage compared to their output. Thus, the efficiency of three phase induction motors is quite high of the order of 80 to 90 percent (V. N. Mittle and A. Mittal, 2008). Also the power factor of an induction motor depends upon the magnitude of magnetizing current drawn by the motor. The magnetizing current in case of three-phase induction motor may be of the order of 20 to 25 percent. As such, the power factor of all three phase induction motor is high. More-still, a three phase induction motor does have starting torque, because there is a rotating field at stand-still produced by three phase starter winding. Moreover, it is possible to increase the starting torque in this case by increasing the rotor resistance (as applicable to wound rotor induction motors). Various leakage fluxes such as slot leakage flux, skew leakage flux, zig-zag leakage flux and end leakage flux etc, responsible for leakage reactance are taken care of and are minimized for a minimum loss, for a boost in motor's output characteristics. In the case of squirrel cage motors, it has a smaller rotor overhang leakage, which gives a better power factor, a greater pull out torque and over load capacity. In a bid to achieve the afore-mentioned, the dynamic/steady-state analyses of the machines were made by transformation of the machine's parametric equations in a – b – c reference to arbitrary d-q-o reference frame from which its output characteristics equations necessary for the equivalent circuits/Matlab simulation/plots were achieved.***

Indexed Terms- ***Induction motor, a-b-c/d-q-o reference frame, dynamic/steady state model, equivalent circuit.***

I. INTRODUCTION

The three phase induction motors are widely used for most of the industrial applications, such as centrifugal pump, conveyers, compressors, crushers etc. They run at essentially constant speed from no-load to full load. The speed depends on frequency, hence, not easily adapted for speed control. They are simple, as they have fewer component parts (as in squirrel cage motor), rugged as they can withstand harsh environmental conditions, low priced, due to its simplicity, requires less maintenance as they are not bulky, and can be manufactured with characteristics (such as output torque, output power, power factor, efficiency etc) to suit industrial requirements.

The crude types of three-phase induction motors were initially introduced by Tesla. Subsequently, an improved version of the motors was designed and constructed, using distributed stator winding and cage type of rotor. The starting torque of the motor was increased by developing the slip ring motor. Since then, designs of these two types of motors have undergone lots of improvements, especially in terms of better power factor, higher efficiency and high starting torque.

II. THE MACHINE DESCRIPTION

The poly-phase induction motors are majorly the three phase type. The most popular ac motor for applications exceeding a few horse power is the three-phase induction motor. It is simple, extremely reliable and powerful for its size and has few moving parts. The field of the induction motor is in the stator, and the armature is on the rotor. The field has pairs of pole pieces onto which stator coils are wound. The simplest three-phase induction motor is the two-pole motor, which has pairs of poles, one pair for each phase.

The motor comprises a stator and a rotor mounted on bearings and separated from the stator air gaps. The stator consists of a magnetic core made up of laminations carrying slot-embedded conductors which constitute the stator windings. These windings can be connected in either a delta or three – or four wire star scheme. (See plates 1 and 2). The rotor of induction motor is cylindrical and carries either conducting bars short-circuited at both ends by end rings (See plate 3(a/b) or a poly phase winding connected in a predetermined manner with terminals brought out of slip rings for external connections and short circuited (see plate 4(a/b).

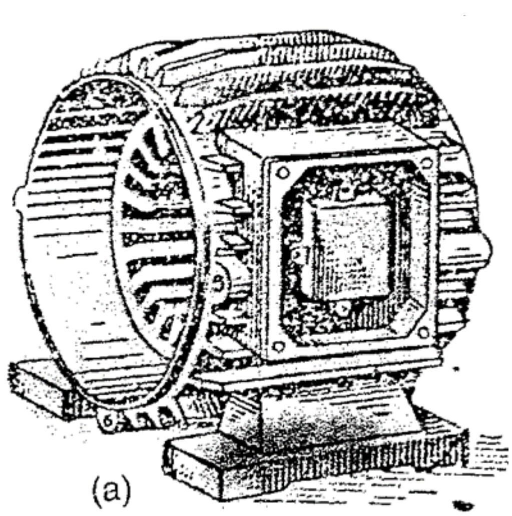


Plate 1: Induction motor frame with unwound stator

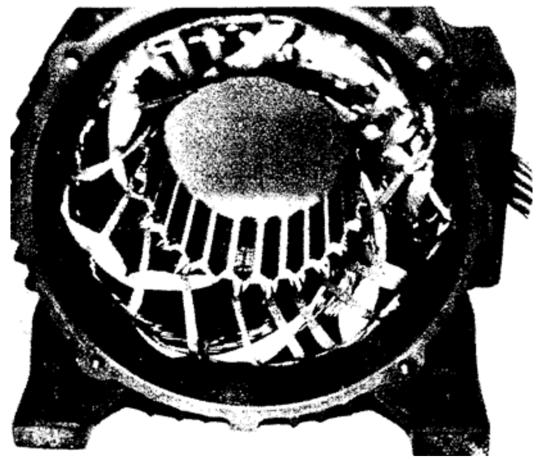
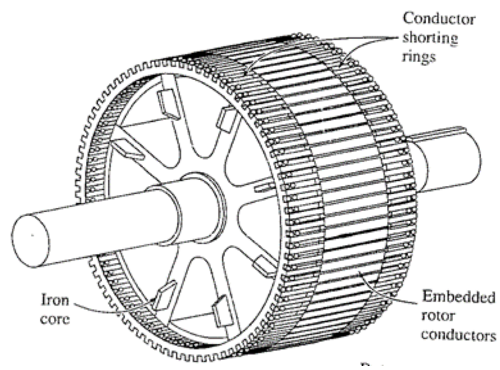
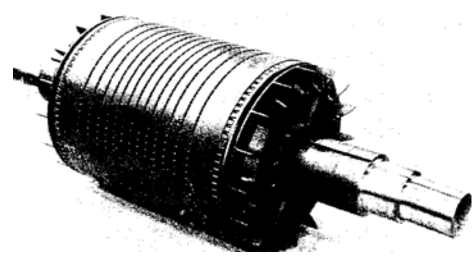


Plate 2: Induction motor frame with wound stator





III. OPERATION OF THREE-PHASE INDUCTION MOTOR

For the principle of operation of three-phase induction motor, let us consider a two-pole, three-phase, star connected, symmetrical induction motor shown in fig. 1

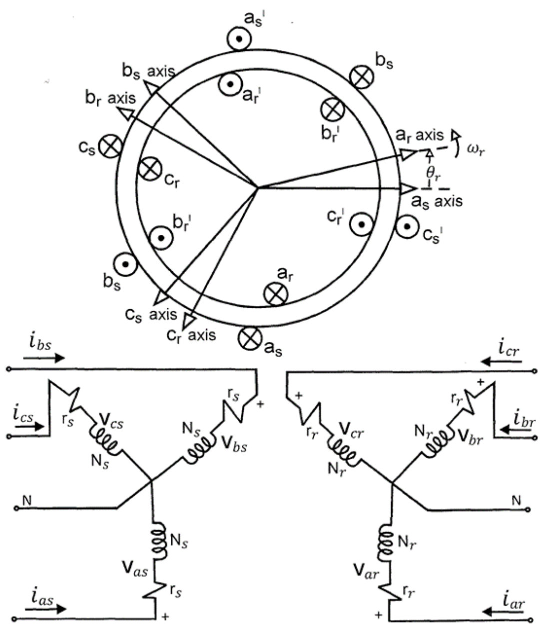


Fig 1: Two pole 3 – phase, star-connected symmetrical induction motor

The fundamental idea behind the operation of an induction machine is simple. In this work, qualitative description of the principles of operation of the machine is adopted. A three phase induction machine of the type provided in figure 1, comprises two major parts namely the stator and the rotor.

The phase displacement between the voltages applied on the stator windings produces a travelling MMF or rotating magnetic field in the uniform air gap.

This field links the short-circuited rotor winding and the relative motion induces short-circuit currents in them, which move about the rotor in exact synchronism with the rotating magnetic field. Obviously, any induced current will react in opposition to the flux linkages producing it, resulting herein a torque on the rotor in the direction of the rotating field. This torque causes the rotor to revolve so as to reduce the rate of change of flux linkages reducing the magnitude of the induced current and the rotor frequency.

If the rotor were to revolve at exactly synchronous speed, there would be no changing flux linkages about the rotor coils and no torque would be produced. However, the practical motor has friction losses requiring some electromagnetic torque, even at no load, and the system will stabilize with the rotor revolving at slightly less than synchronous speed. A mechanical shaft load will cause the rotor to decelerate, but this increases the rotor current, automatically increasing the torque produced, and stabilizing the system at a slightly reduced speed.

The difference in speed between rotor and rotating magnetic field is termed "slip". Slip varies from a fraction of one percent at no-load to a maximum value of three or four percent under full load conditions for most properly designed machines. The speed change between no-load and full-load is so small that the squirrel-cage motor is often termed a constant –speed machine.

IV. THE INDUCTANCE MATRIX AND TRANSFORMATION OF STATOR QUANTITIES TO ARBITRARY Q-D-O REFERENCE FRAME

The winding arrangement of a 2-pole, 3-phase, star connected symmetrical induction machine is shown in figure 1. The stator windings are identical with equivalent turns, (N_s) and resistance (r_s). The rotor windings which may be wound or forged as a squirrel cage winding can also be approximated as identical windings with equivalent turns (N_r) and resistance (r_r)

The air gap of an induction machine is uniform and the stator and rotor windings may be approximated as having a sinusiodally distributed windings

The stator inductance L_s is given as,

$$L_s = \begin{bmatrix} L_{Ls} + L_A - L_B \operatorname{Cos} 2\,\theta_r & -\tfrac{1}{2}\ L_A - L_B \operatorname{Cos} 2\left(\theta_r - \frac{\pi}{3}\right) & -\tfrac{1}{2}\ L_A - L_B \operatorname{Cos} 2\left(\theta_r - \frac{\pi}{3}\right) \\ -\tfrac{1}{2}\ L_A - L_B \operatorname{Cos} 2\left(\theta_r - \frac{\pi}{3}\right) & L_{Ls} + L_A - L_B \operatorname{Cos} 2\left(\theta_r - \frac{2\pi}{3}\right) & -\tfrac{1}{2} L_A - L_B \operatorname{Cos} 2(\theta_r + \pi) \\ -\tfrac{1}{2}\ L_A - L_B \operatorname{Cos} 2\left(\theta_r + \frac{\pi}{3}\right) & -\tfrac{1}{2} L_A - L_B \operatorname{Cos} 2(\theta_r + \pi) & L_{Ls} + L_A - L_B \operatorname{Cos} 2\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \quad (1)$$

Where; L_{Ls} +L_A – L_B cos 2 θr = $La_s a_s$ = Stator self-inductance in winding a (2)

L_{Ls} +L_A – L_B cos 2 ($\theta_r - \frac{2\pi}{3}$) = $L_{bs} b_s$ = Stator self-inductance in winding b (3)

L_{Ls} + L_A– L_B cos 2 ($\theta_r + \frac{2\pi}{3}$) = L_{cs} c_s = Stator self-inductance in winding c (4)

- ½ L_A – L_B cos 2 ($\theta_r - \frac{\pi}{3}$) = $L_{as} b_s$ (5)
- ½ L_A – L_B cos 2 ($\theta_r + \frac{\pi}{3}$) = L_{as} c_s (6)
- ½ L_A – L_B cos 2 (θ_r +π) = L_{bs} c_s (7)

From equation 1, it is evident that all stator self-inductances are equal.

That is; L_{as} a_s = L_{bs} b_s = L_{cs} c_s = L_{cs} c_s and L_{as} a_s = L_{Ls} + L_{ms} (8)

Where;
L_{Ls} = Stator leakage inductance, L_{ms} = Stator magnetizing inductance

The stator magnetizing inductance, (L_{ms},) corresponds to L_A in equation (2) through equation (4), and is mathematically expressed as;

$$L_{ms} = \left(\frac{N_S}{2}\right)^2 \frac{\pi\mu_o r_s l}{g} \quad (9)$$

Where; N_s = stator equivalent turns, μ_o = permeability of free space, r_s = stator resistance, r_s = stator winding length

g = length of uniform gap

Like the stator self-inductances, the stator – to stator mutual inductances are also equal.

This implies that;

$$L_{as}\, b_s = L_{as}\, c_s = L_{bs}\, c_s = -½\, L_{ms} \quad (10)$$

This corresponds to – ½ L_A in equation (5) through equation (7) with L_B = 0. Consequently, equation 1 is now rewritten as;

$$L_s = \begin{bmatrix} L_{LS} + L_{mS} & -½\, L_{ms} & -½\, L_{ms} \\ -½\, L_{ms} & L_{LS} + L_{LS} & -½\, L_{ms} \\ -½\, L_{ms} & -½\, L_{ms} & L_{LS} + L_{ms} \end{bmatrix}$$

(11)

In a related manner, the rotor inductance matrix is obtained as;

$$L_r = \begin{bmatrix} L_{Lr} + L_{mr} & -½\, L_{mr} & -½\, L_{mr} \\ -½\, L_{mr} & L_{Lr} + L_{Lr} & -½\, L_{mr} \\ -½\, L_{mr} & -½\, L_{mr} & L_{Lr} + L_{mr} \end{bmatrix}$$

(12a)

Whereas in stator, the rotor self-inductances are equal that is,

$$L_{ar}\, a_r = L_{br}\, b_r = L_{cr}\, c_r = L_{Lr} + L_{mr} \quad (12b)$$

The rotor magnetizing inductance, L_{mr} is given as;

$$L_{mr} = \left(\frac{N_S}{2}\right)^2 \frac{\pi\mu_o r l}{g} \quad (13)$$

The rotor – to-rotor mutual inductances are equal and expressed as;

$$L_{ar}\, b_r = L_{ar}\, c_r = L_{br}\, c_r = -½\, L_{mr} \quad (14)$$

The mutual inductances between the stator and the rotor windings are obtained as follows;

i. The mutual inductances L_{as} a_r = L_{bs} b_r and L_{cs} c_r are equal, and is given by the expression; L_{as} a_r = L_{bs} b_r = L_{cs} c_r = L_{sr} c_r = L_{sr} cos θr (15)
ii. The mutual inductances L_{as} b_r, L_{bs} c_r and L_{cs} a_r are equal; and is given by the expression L_{as} b_r = L_{bs} c_r = L_{cs} a_r = L_s cos ($\theta_r + \frac{2\pi}{3}$) (16)
iii. The mutual inductances L_{as} c_r, L_{bs} a_r and L_{cs} b_r are equal; and is given by the expression; L_{as} c_r = L_{bs} a_r = L_{cs} b_r = L_{sr} cos ($\theta_r - \frac{2\pi}{3}$) (17)

Equation (15) through equation (17), gives one expression for the mutual inductance between the stator and the rotor windings of an induction machine expressed as;

$$L_{sr} = L_{sr}\begin{bmatrix} \text{Cos } \theta r & \text{Cos } \left(\theta r + \frac{2\pi}{3}\right) & \text{Cos } \left(\theta_r - \frac{2\pi}{3}\right) \\ \text{Cos } \left(\theta r - \frac{2\pi}{3}\right) & \text{Cos } \theta r & \text{Cos } \left(\theta r + \frac{2\pi}{3}\right) \\ Cos\ \left(\theta r + \frac{2\pi}{3}\right) & Cos\ \left(\theta_r - \frac{2\pi}{3}\right) & Cos\ \theta r \end{bmatrix} \quad (18)$$

The L_{sr} on the right hand side of equation (18) represents the amplitude of the mutual inductances between the stator and rotor windings and is given by the expression;

$$L_{sr} = \left(\frac{N_s}{2}\right)\left(\frac{N_r}{2}\right)\left(\frac{\pi \mu_o r l}{g}\right) \quad (19)$$

V. TRANSFORMATION TO ARBITRARY Q-D-O REFERENCE FRAME

The voltage equation in machine variables for the stator and the rotor of a star-connected symmetrical induction machine shown in figure 1 are expressed as follows;

$$\left.\begin{aligned} V_{as} &= i_{as}\, r_s + p\lambda_{as} \\ V_{bs} &= i_{bs}\, r_s + p\lambda_{bs} \\ V_{cs} &= i_{cs}\, r_s + p\lambda_{cs} \end{aligned}\right\} \quad (20)$$

Similarly, for the rotor voltage equations;

$$\left.\begin{aligned} V_{ar} &= i_{ar}\, r_r + p\lambda_{ar} \\ V_{br} &= i_{br}\, r_r + p\lambda_{br} \\ V_{crs} &= i_{cr}\, r_r + p\lambda_{cr} \end{aligned}\right\} \quad (21)$$

In both equations, P = d/dt, the s subscripts denotes variables and parameters associated with the stator circuits and the r subscripts denotes variables and parameters associated with the rotor circuits. Both r_s and r_r are diagonal matrices each with equal non zero elements (Krause et al 1995).

For a magnetically linear system, the flux linkages can be expressed as;

$$\begin{bmatrix} \lambda_s^{abc} \\ \lambda_r^{abc} \end{bmatrix} = \begin{bmatrix} L_{ss}^{abc} & L_{sr}^{abc} \\ L_{rr}^{abc} & L_{rr}^{abc} \end{bmatrix} \begin{bmatrix} i_s^{abc} \\ i_r^{abc} \end{bmatrix} \text{ wb.turn} \quad (22)$$

For an idealized inductance machine, six first order differential equations are used to describe the machine, one differential equation for each machine winding. The stator-to- rotor coupling terms are functions of rotor position and hence when the rotor rotates, the coupling terms vary with time (Chee-mum-ong 1997).

In the analysis of induction machine, it is also desirable to transform the abc variables with the symmetrical rotor windings to the arbitrary q do reference frame (Krause et al 1995).

The transformation equation from the abc quantities to the q do reference frame is given by (Chee-mum-ong 1997).

$$\begin{bmatrix} f_q \\ f_d \\ f_o \end{bmatrix} = \left[T_{qdo}\ (\theta)\right] \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \quad (23)$$

Where the variable f can be the phase voltage, current or flux linkages of the machine.

$$\left[T_{qdo}\ (\theta)\right] = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos\ \left(\theta - \frac{2\pi}{3}\right) & \cos\ \left(\theta + \frac{2\pi}{3}\right) \\ sin\theta & sin\left(\theta - \frac{2\pi}{3}\right) & sin\ \left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (24)$$

and the inverse of equation (24) is

$$\left[T_{qdo}\ (\theta)\right]^{-1} = \begin{bmatrix} cos\theta & sin\theta & 1 \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\ \left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\ \left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

(25)

VI. VOLTAGE EQUATIONS IN Q-D-O REFERENCE FRAME

From equation (20), the stator winding abc voltage equations can be expressed as;

$$V_s^{abc} = r_s^{abc} \quad i_s^{abc} \ + p\lambda_s^{abc} \qquad (26)$$

Applying the transformation, $[T_{qdo}(\theta)]$, to equation (26), yields;

$$V_s^{qdo} = [T_{qdo}(\theta)] \quad r_s{}^{abc}[T_{qdo}(\theta)]^{-1} \quad [i_s^{qdo}] + [T_{qdo}(\theta)]\, p[T_{qdo}(\theta)]^{-1}\,[\lambda_s^{qdo}] \qquad (27)$$

The above equation (27), simplifies to ;

$$V_s^{qdo} = r_s^{qdo} \ i_s^{qd0} + \rho\lambda_s^{qd0} + \omega\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\lambda_s^{qdo} \quad (28)$$

Where; $r_s^{qdo} = r_s\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; p = d/dt; $\omega = \frac{d\theta}{dt}$

Similarly, the rotor quantities must be transformed into the same q-d-o frame. Now the transformation angle for the rotor phase quantities is (θ-θr). When the transformation Tqdo (θ-θr) is applied to the rotor voltage equation in the same manner as the stator, we have;

$$V_r^{qdo} = r_r^{qdo} \ i_r^{qdo} + p\lambda_r^{qdo} + (\omega\text{-}\omega r)\begin{bmatrix} 0 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\lambda_r^{qdo} \quad (29)$$

VII. FLUX LINKAGE IN Q-D-O REFERENCE FRAME

From equation (22), the stator and rotor flux linkages are given as;

$$\lambda_s^{abc} = L_s^{abc} \ i_s^{abc} + L_{sr}^{abc} \ i_r^{abc} \qquad (30)$$

$$\lambda_r^{abc} = L_{rs}^{abc} \ i_s^{abc} + L_{rr}^{abc} \ i_r^{abc} \qquad (31)$$

The stator flux linkages in q-d-o reference frame are obtained by applying T_{qdo} (θ) to equation (30) to give;

$$\lambda_s^{qdo} = [T_{qdo}(\theta)]\,[L_{ss}^{abc} \ i_s^{abc} + L_{sr}^{abc} \ i_r^{abc}]$$

$$= T_{qdo}(\theta) \ L_{ss}^{abc} \ T_{qdo}{}^{-1}(\theta) \ i_s^{qdo} + T_{qdo}(\theta) \ L_{rs}^{abc} \ T_{qdo}{}^{-1}(\theta)\, i_r^{qdo} \qquad (32)$$

Equation (32) simplifies to (Chee – Mum ong 1997);

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix} = \begin{bmatrix} L_{Ls}+\frac{3}{2}L_{ss} & 0 & 0 \\ 0 & L_{Ls}+\frac{3}{2}L_{ss} & 0 \\ 0 & 0 & L_{Ls} \end{bmatrix}\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{os} \end{bmatrix} + \begin{bmatrix} \frac{3}{2}L_{sr} & 0 & 0 \\ 0 & \frac{3}{2}L_{sr} & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} i_{qr} \\ i_{dr} \\ i_{or} \end{bmatrix} \quad (33)$$

In a similar manner, if the transformation, T_{qdo} (θ – θ_r) is applied to equation (31) the rotor q-d-o flux linkage becomes;

$$\lambda_r^{qdo} = [T_{qdo}(\theta-\theta r)]L_{rs}^{abc}[T_{qdo}(\theta-\theta r)]^{-1} \ i_s^{qdo} + [T_{qdo}(\theta-\theta r)]L_{rr}^{abc}[T_{qdo}(\theta-\theta r)]^{-1} \ i_r^{qdo} \qquad (34)$$

Equation 34 can be simplified as;

$$\begin{bmatrix} \lambda_{qr} \\ \lambda_{dr} \\ \lambda_{0r} \end{bmatrix} = \begin{bmatrix} \frac{3}{2}L_{sr} & 0 & 0 \\ 0 & \frac{3}{2}L_{sr} & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{os} \end{bmatrix} + \begin{bmatrix} L_{Lr}+\frac{3}{2}L_{rr} & 0 & 0 \\ 0 & L_{Lr}+\frac{3}{2}L_{rr} & 0 \\ 0 & 0 & L_{Lr} \end{bmatrix}\begin{bmatrix} i_{qr} \\ i_{dr} \\ i_{or} \end{bmatrix} \qquad (35)$$

Merging equation (33) and (35) give the stator and rotor flux linkage equations in q-d-o reference frame as depicted in equation (36)

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \\ \acute{\lambda}_{qr} \\ \acute{\lambda}_{dr} \\ \acute{\lambda}_{0r} \end{bmatrix} \begin{bmatrix} L_{Ls}+L_m & 0 & 0 & L_m & 0 & 0 \\ 0 & L_{Ls}+L_m & 0 & 0 & L_m & 0 \\ 0 & 0 & L_{Ls} & 0 & 0 & 0 \\ L_m & 0 & 0 & \acute{L}_{Lr}+L_m & 0 & 0 \\ 0 & L_m & 0 & 0 & \acute{L}_{Lr}+L_m & 0 \\ 0 & 0 & 0 & 0 & 0 & \acute{L}_{Lr} \end{bmatrix}\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{as} \\ \acute{i}_{qr} \\ \acute{i}_{dr} \\ \acute{i}_{ar} \end{bmatrix} \qquad (36)$$

In equation (36), the primed quantities are rotor values referred to the stator side and are related thus;

$$\left.\begin{aligned} \acute{\lambda}_{qr} &= \frac{N_s}{N_r}\lambda_{qr} \\ \acute{i}_{dr} &= \frac{N_s}{N_r}\lambda_{dr} \end{aligned}\right\} \qquad (37)$$

$$\left.\begin{aligned} \acute{i}_{qr} &= \frac{N_r}{N_s}\, i_{qr} \\ \acute{i}_{dr} &= \frac{N_r}{N_s}\, i_{dr} \end{aligned}\right\} \quad (38)$$

Also from equation (36), L_m is the magnetizing inductance on the stator side and can be expressed as;

$$L_m = \frac{3}{2}\, L_{ss} = \frac{3}{2}\frac{N_r}{N_s} L_{sr} = \frac{3}{2}\frac{N_s}{N_r} L_{rr} \quad (39)$$

VIII. STEADY – STATE ANALYSIS OF 3-PHASE INDUCTION MOTOR

In steady state operation of the three phase induction machine (IM), the derivative terms in the voltage equations become zero.

Referring to equation (32) earlier expressed as;

$$V_s^{qdo} = r_s^{qdo} \quad i_s^{qdo} + p\lambda_s^{qdo} + \omega \begin{bmatrix} 0\ 1\ 0 \\ -1\ 0\ 0 \\ 0\ 0\ 0 \end{bmatrix} \lambda_s^{qdo}$$ and

substituting,

$$r_s^{qdo} = r_s \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$, gives;

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{os} \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} + p \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix} \begin{bmatrix} 0 & \omega & O \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix} \quad (40)$$

Equation (40) suggests that;

$$\left.\begin{aligned} V_{qs} &= r_s\, i_{qs} + p\lambda_{qs} + \omega\lambda_{ds} \\ V_{ds} &= r_s\, i_{ds} + p\lambda_{ds} + \omega\lambda_{qs} \\ V_{0s} &= r_s\, i_{os} + p\lambda_{os} \end{aligned}\right\} \quad (41)$$

Also,

$$\left.\begin{aligned} \acute{V}_{qr} &= \acute{r}_r\, \acute{i}_{qr} + p\acute{\lambda}_{qr} + (\omega-\omega_r)\, \acute{\lambda}_{dr} \\ \acute{V}_{dr} &= \acute{r}_r\, \acute{i}_{dr} + + p\acute{\lambda}_{dr} - (\omega-\omega_r)\, \acute{\lambda}_{qr} \\ \acute{V}_{ar} &= \acute{r}_r\, \acute{i}_{ar} + p\acute{\lambda}_{as} \end{aligned}\right\} \quad (42)$$

When the derivation terms (P=d/dt) are set equal to zero and is applied to the stator and rotor voltage equation (41) and (42) for the q-axis, we have;

$$V_{qs} = r_s\, i_{qs} + \omega\lambda_{ds} \quad (43)$$

$$\acute{V}_{qr} = \acute{r}_r\, \acute{i}_{qr} + (\omega-\omega_r)\, \acute{\lambda}_{dr} \quad (44)$$

Since q and d are in space quadrature, if we apply the transformation $F_{ab} = jF_{qs}$ to equation (43) and (44), we have;

$$V_{qr} = r_s\, i_{qs} + j\omega\lambda_{qs} \quad (45)$$

$$\acute{V}_{ds} = \acute{r}_r\, \acute{i}_{qr} + (\omega-\omega_r)\, \acute{\lambda}_{qr} \quad (46)$$

From equation (36), the stator flux linkage along the q-axis is;

$$\lambda_{qs} = (L_{Ls} + L_m)\, i_{qs} + L_m\, \acute{i}_{qr}$$

$$= L_{Ls}\, i_{qs} + L_m\, i_{qs} + L_m\, \acute{i}_{qr}$$

$$\Rightarrow \lambda_{qs} = L_{Ls}\, i_{qs} + L_m\, (i_{qs} + \acute{i}_{qr}) \quad (47)$$

Also, the rotor flux linkage along the q-axis, is,

$$\acute{\lambda}_{qr} = L_m\, i_{qs}\, (\acute{L}_{Lr} + L_m)\, \acute{i}_{qr}$$

$$= L_m\, i_{qs} + \acute{L}_{Lr} + \acute{i}_{qs} + L_m\, \acute{i}_{qr}$$

$$\Rightarrow \acute{\lambda}_{qr} = \acute{L}_{Lr}\ \ i_{qr} + L_m\, (i_{qs} + \acute{i}_{qr}) \quad (48)$$

Substituting equation (47) back into equation (25), we have;

$$V_{qs} = r_s\, i_{qs} + j\omega\, \left[L_{Ls}\acute{i}_{qr} + L_m\, (i_{qs} + \acute{i}_{qr}\,)\right]$$

$$= r_s\, i_{qs} + j\omega\, L_{Ls}\, i_{qs} + j\omega\, L_m\, (i_{qs} + \acute{i}_{qr})$$

$$\Rightarrow V_{qs} = r_s\, i_{qs} + jX_{LLs}\, i_{qs} + jX_{Lm}\, (i_{qs} + \acute{i}_{qr}) \quad (49)$$

But from equation (46), we have that;

$$\acute{V}_{qr} = \acute{r}_r\, \acute{i}_{qr} + j\, (\omega-\omega_r)\, \acute{\lambda}_{qr}$$

$$\Rightarrow \acute{V}_{qr} = \acute{r}_r\, \acute{i}_{qr} + js\omega\, \acute{\lambda}_{qr}$$

$$= \acute{r}_r\, \acute{i}_{qr} + js\omega\, \left[\acute{L}_{Lr} + L_m\, (i_{qs} + \acute{i}_{qr}\,\right]$$

$$= \acute{r}_r\, \acute{i}_{qr} + js\omega\, \acute{L}_{Lr}\, \acute{i}_{qr} + js\omega\, L_m\, (i_{qs} + \acute{i}_{qr})$$

$$\Rightarrow \acute{V}_{qr} = \acute{r}_r\, \acute{i}_{qr} + js\, \acute{X}_{LLr}\, \acute{i}_{qr} + jsx_{Lm}\, (i_{qs} + \acute{i}_{qr}) \quad (50)$$

Where s = slip of the induction machine.

If we let us suppose that the **q**-axis is aligned with phase **a** of the induction motor such that;

$V_{qs} = V_{as}$; $\acute{V}_{qr} = \acute{V}_{ar}$

$i_{qs} = i_{as}$; $\acute{i}_{qr} = \acute{i}_{ar}$, then equation (49) boils down to;

$$V_{as} = r_s\ i_{as} + jX_{Ls}\ + jX_{Lm}\ (i_{as} + \acute{i}_{as}\,) \quad (51)$$

Similarly, equation (50) yields;

$$\acute{V}_{ar} = \acute{r}_r\ \acute{i}_{ar} + js\acute{X}_{LLr}\ \acute{i}_{ar} + jsX_{Lm}\, (i_{as} + \acute{i}_{ar}) \quad (52)$$

Dividing equation (52) through by the slip(s), we have;

$$\frac{\acute{V}_{ar}}{s} = \frac{\acute{r}_r}{s}\, i_{\acute{a}r} + j\acute{X}_{LLr}\, \acute{i}_{ar} + jX_{Lm}\, (i_{as} + \acute{i}_{ar}) \quad (53)$$

Equations (51) and (53) are used to draw the steady state equivalent circuit of an induction motor as depicted in figure 2

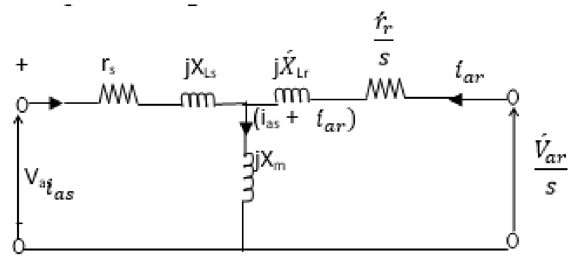


Fig 2: Per phase equivalent circuit of an induction motor

Normally, for an induction motor, (squirrel cage type) the rotor conductors (windings) are short-circuited. Hence, rotor voltage $\acute{V}_{ar}$ is equal to zero. The per phase equivalent circuit of figure 2 with the rotor short circuited, yields;

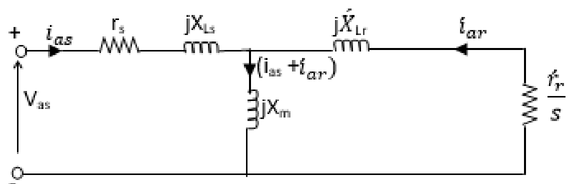


Fig 3: Per phase equivalent circuit of an induction motor at run condition with (s<1) and the rotor short circuited

To account for the copper loss $(\acute{\iota}_{ar})^2{}_r$ in the rotor circuit, figure (3) is redrawn as shown in fig (4)

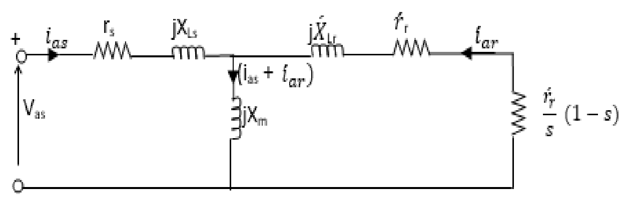


Fig 4: Per phase equivalent circuit of an induction motor which account for copper loss $(\acute{\iota}_{ar})^2_r$ in the rotor circuit.

With reference to figure 4, under run condition of the machine (motor), the rotor speed (n_r)>0, and s<1. Hence the load resistance $\acute{r}_r$ is affected by slip (s).

IX. POWER ACROSS AIR GAP, OUTPUT POWER AND ELECTROMECHANICAL TORQUE OF THREE-PHASE INDUCTION MOTOR

With reference to the equivalent circuit of fig 3, the power crossing the terminals of the shunt mutual inductance (jX_m) is the electrical power input per phase minus the stator losses (stator copper and iron loss). It is the power that is transferred from the stator to the rotor through the air-gap magnetic field. This is known as the power across the air gap (P_g). Its 3-phase value (Pg) is given by (Obute et al 2017);

$$P_g = 3(\acute{\iota}_{ar})^2 \frac{\acute{r}_r}{s} \text{ Watts} \quad (54)$$

Similarly, rotor Copper loss (P_{cr}) as in figure 4 = $3(\acute{\iota}_{ar})^2 \acute{r}_r$ Watts (55)

From equations (54) and (55);

Power across the air gap $P_g = \frac{P_{cr}}{s}$

$$\Rightarrow P_{cr} = sP_g \text{ Watts} \quad (56)$$

The mechanical (gross) Output power (P_m) of the motor is obtained by subtracting equation 56 from 54 as below;

$$P_m = P_g - P_{cr} = 3(\acute{\iota}_{ar})^2 \frac{r_r}{s} - 3(\acute{\iota}_{ar})^2 r_r$$

$$= 3(\acute{\iota}_{ar})^2 \frac{r_r}{s}(1\text{-}s) \text{ Watts} \quad (57)$$

Similarly, the electromagnetic torque (T_e) developed by the motor is given by;

$$T_e = \frac{P_g}{\omega_r} = \frac{(1-s)P_g}{\omega_s(1-s)} = \frac{P_g}{\omega_s}$$

$$\Rightarrow T_e = \frac{3(\acute{\iota}_{ar})2\frac{\acute{r}_r}{s}}{\omega_s} \text{ N-m} \quad (58)$$

Where $\omega_r = 2\pi n_r$ = rotor speed in mechanical radian per second, $\omega_s = 2\pi n_s$ = synchronous speed in mechanical radian per second

More-still, the mechanical net power or shaft power (P_{sh}) = P_m – mechanical losses (friction and windage losses)

⇒ Output or shaft torque (T_{sh}) of the machine is given by;

$$T_{sh} = \frac{P_{sh}}{(1-s)\omega_s} \text{ N-m} \quad (59)$$

Equation (58) is an interesting and significant result according to which torque is obtained from the power across the air gap by dividing it with synchronous speed (ω_s) in rad/s as if power was transferred at synchronous speed.

X. OUTPUT CHARACTERISTICS OF 3-PHASE INDUCTION MACHINE (MOTOR).

The performance indices of an induction motor can be studied for better if the per-phase equivalent circuit of figure 3 is slightly modified as in fig 5 below;

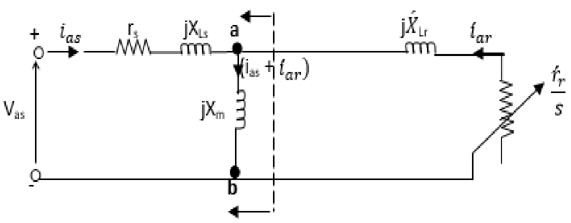


Fig 5: Modified per-phase equivalent circuit of an induction motor at run condition.

10.1 Torque/Slip characteristics of 3-phase induction motor

The expressions for characteristics easily obtained by finding the Thevenin equivalent of the circuit of figure 5, to the left of **ab**, as shown below;

$Z_{TH} = R_{TH} + jX_{TH}$

$= (r_s + jX_{Ls}) \,//\, jX_m$

$= \frac{(r_s + jX_{Ls})jX_m}{(r_s + jX_{Ls} + jX_m)}$

$$= \left[\frac{(r_s + jX_{Ls})jX_m}{(r_s + j(X_{Ls} + X_m))}\right] \Omega \qquad (60)$$

Assuming $X_{Ls} + X_m >>> r_s$ then;

$$Z_{TH} = \left(\frac{r_s\, X_m}{X_{Ls+X_m}} + \frac{jX_{Ls}\, X_m}{X_{Ls+X_m}}\right) \Omega \qquad (61)$$

$$\left.\begin{array}{l} \Rightarrow R_{TH} = \frac{r_s\, X_m}{X_{Ls} + X_m} \text{ (real component of } Z_{TH}\text{)} \quad (62) \\ \\ X_{TH} = \frac{jX_{ls}\, X_m}{X_{Ls+}\, X_m} \text{ (imaginary component of } Z_{TH}\text{)} \end{array}\right\}$$

$$\text{Similarly, } V_{TH} = \left[\frac{jX_m}{(r_s + jX_{Ls} + X_m)}\right] V_{as} \text{ volts} \qquad (63)$$

For negligible value of r_s compared to $j(X_{Ls} + X_m)$;

$$V_{TH} = \left[\frac{jX_m}{(jX_{Ls} + X_m)}\right] V_{as} \text{ volts} \qquad (64)$$

Hence, the circuit of figure (5), reduces to that of figure (6), in which it is convenience to take V_{TH} as the reference voltage.

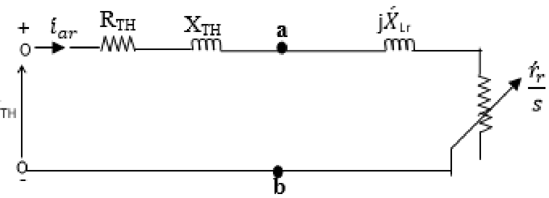


Fig 6: Thevenin equivalent of 3 – phase induction machine (motor) circuit model

From figure 6;

$$\acute{i}_{ar} = \frac{V_{TH}}{\left[\left(R_{TH} + \frac{r_r}{s}\right) + j\,(X_{TH} + X_{Lr})\right]} \quad \text{Amperes} \qquad (65)$$

Putting equation (65) into (58) we obtain;

$$T_e = \frac{3\acute{r}_r}{S\omega_s}\left[\frac{(V_{TH})^2}{\left(R_{TH} + \frac{r_r}{s}\right)^2 + j\,(X_{TH} + X_{Lr})^2}\right] \text{N-m} \qquad (66)$$

Equation (66) is the expression for torque developed as a function of voltage and slip. If the machine parameters of table (1) are properly used in equation (66), a simulation plot for the average value of the torque developed at various slip values is obtained as in fig (7). This is termed the motoring /generating regions of the machine, operation.

In motoring mode of operation, the rotor rotates in the same direction of operating magnetic field produced by the stator current, the speed is between zero and synchronous speed, and the corresponding slip is between 1.0 and 0. The speed of the rotor (N_r) is usually less than the synchronous speed (N_s)

10.2 Efficiency/slip characteristics of a conventional 3-phase induction machine

With reference to squirrel cage induction motor type, the resistance is fixed, and less compared to its reactance. There is no provision for addition of external resistance efficiency (ε) of the machine is given by;

$$\varepsilon = \frac{Power\ Output}{Power\ input} \qquad (67)$$

For the per-phase equivalent circuit of an induction machine of fig 3

The input impedance looking at the input terminals is given by;

$$Z = r_s + j_{Xls} + \left[\frac{(j_{Xm})\left(j_{Xlr} + \frac{r_{\acute{r}}}{s}\right)}{\frac{r_{\acute{r}}}{s} + j\,(X_m + _{Xlr})}\right] \qquad (68)$$

The current in the main (stator) winding; $i_{as} = \frac{V_{as}}{Z}$ (69)

The current in the rotor winding is given by; $\acute{i}_{ar}$ = $\left[\frac{jx_m}{\frac{r_{\acute{r}}}{s}+j\,(X_m+X_{lr})}\right] i_{as}$ (70)

Copper losses in the stator and rotor windings = 3 $r_{\acute{r}}$ $(i_{as} + \acute{i}_{ar})^2$ (71)

For the machine;

Input power = Output power + copper losses in stator and rotor windings, excluding windage and frictional losses. (72)

Adding equation 57 to equation 71, then equation 72 becomes;

$$\text{Input power} = 3\, r_{\acute{r}} \left(\frac{1-s}{s}\right) (i_{\acute{a}r})^2 + 3\, r_{\acute{r}}\, (i_{as} + i_{\acute{a}r})^2$$
$$= 3\, r_{\acute{r}} \left[\left(\frac{1-s}{s}\right) (i_{\acute{a}r})^2 + (i_{as} + i_{\acute{a}r})^2\right] \quad (73)$$

∴ From equation 67;

$$\text{Efficiency } \varepsilon = \frac{3\, r_{\acute{r}} \left(\frac{1-s}{s}\right) (i_{\acute{a}r})^2}{3\, r_{\acute{r}} [\left(\frac{1-s}{s}\right) (i_{\acute{a}r})^2 + (i_{as}+i_{\acute{a}r})^2]}$$
$$= \frac{\left(\frac{1-s}{s}\right) (i_{\acute{a}r})^2}{\left(\frac{1-s}{s}\right) (i_{\acute{a}r})^2 + (i_{as}+i_{\acute{a}r})^2} \quad (74)$$

A plot of machine efficiency against slip is shown in figure 8

10.3 Power factor/Slip characteristics of 3-phase induction motor

From the Thevenin equivalent of a 3-phase induction machine circuit model of fig 6, the machine's power factor (cosθ) is given as;

$$\text{Power factor } (\cos\theta) = \frac{Real\,(Z)}{\sqrt{Real\,(Z)^2 + Imag(Z)^2}}$$
$$= \frac{R_{TH} + \frac{r_{\acute{r}}}{s}}{\sqrt{(R_{TH}+\frac{r_{\acute{r}}}{s})^2 + (X_{TH}+X_{Ir})^2}} \quad (75)$$

A plot of the power factor (cos θ) against slip (s) is shown in figure 9

10.4 Rotor current ($\acute{i}_{ar}$)/Slip(s) Characteristics of 3-Phase induction Motor

Using equation 65, a plot of rotor current ($\acute{i}_{ar}$) against slip(s) is obtained as in figure 10.

XI. THE DYNAMIC MODEL OF 3-PHASE INDUCTION MOTOR

The dynamic analysis of a 3-phase induction machine can be done using, three particular cases of the generalized mode in arbitrary reference frames. They are of general interest and they include;

i) Stator reference frame model
ii) Rotor reference frame model
iii) Synchronously rotating reference model

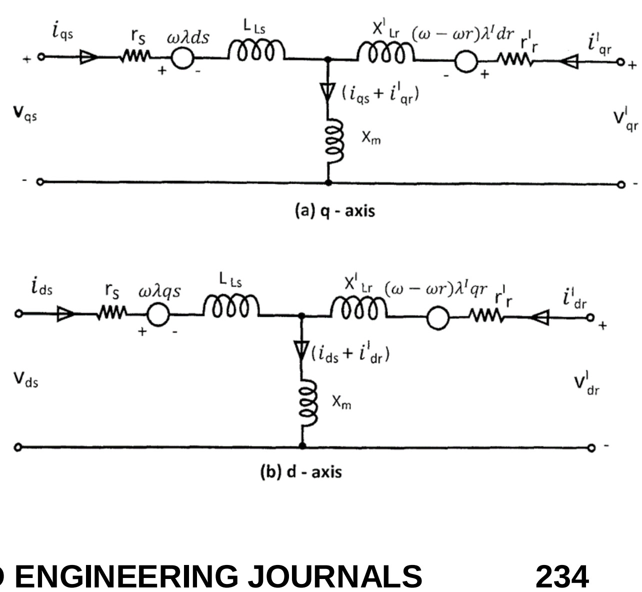
The dynamic analysis of the induction machine is carried out in the d-q-rotor reference frame. As earlier derived in the steady state analysis of equations (41) and (42), the stator and rotor voltage equation in the arbitrary reference frames are expressed as;

$$\left.\begin{aligned}
V_{qs} &= r_s\, i_{qs} + P\lambda_{qs} + \omega\lambda_{ds}\\
V_{ds} &= r_s\, i_{ds} + P\lambda_{ds} - \omega\lambda_{qs}\\
V_{os} &= r_s\, i_{ds} + P\lambda_{0s}\\
\acute{V}_{qr} &= \acute{r}_r \acute{i}_{qr} + P\acute{\lambda}_{qr} + (\omega\text{-}\omega_r)\, \acute{\lambda}_{dr}\\
\acute{V}_{dr} &= \acute{r}_r \acute{i}_{dr} + P\acute{\lambda}_{dr} - (\omega\text{-}\omega_r)\, \acute{\lambda}_{qr}\\
\acute{V}_{or} &= r_r\, i_{or} + P\lambda_{or}
\end{aligned}\right\} \quad (76)$$

While the stator and rotor flux linkages in the arbitrary reference frame are given as;

$$\left.\begin{aligned}
\lambda_{qs} &= (L_{Ls} + L_m) i_{qs} + L_m\, \acute{i}_{qr}\\
\lambda_{ds} &= (L_{Ls} + L_m) i_{ds} + L_m\, \acute{i}_{dr}\\
\acute{\lambda}_{qr} &= L_{Ls}\, i_{os}\\
\acute{\lambda}_{qr} &= (\acute{L}_{Lr} + L_m)\, \acute{i}_{qr} + L_m\, i_{qs}\\
\acute{\lambda}_{dr} &= (\acute{L}_{Lr} + L_m)\, \acute{i}_{dr} + L_m\, i_{ds}\\
\acute{\lambda}_{qr} &= L_{Lr}\, i_{or}
\end{aligned}\right\} \quad (77)$$

With equations (76) and (77), the dynamic equivalent circuits of an induction motor in the arbitrary reference frame is as shown in figure 11.



(a) q - axis

(b) d - axis

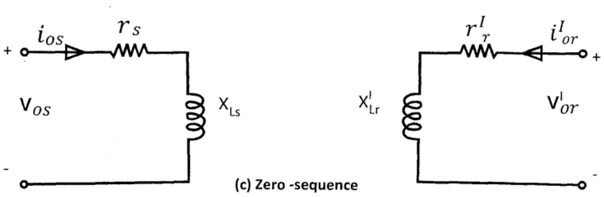


Fig 11: Dynamic equivalent circuit for 3-phase symmetrical induction machine in an arbitrary reference frame

Substituting the flux linkage values into the voltage equations gives;

$$\left.\begin{aligned}
V_{qs} &= (r_s + L_s P)\, i_{qs} + \omega L_s\, i_{ds} + L_m P \acute{i}_{qr} + \omega L_m \acute{i}_{dr} \\
V_{ds} &= \omega L_s\, i_{qs}\, (r_s + L_s P)\, i_{ds} + \omega L_m\, \acute{i}_{qr} + L_m P \acute{i}_{dr} \\
V_{os} &= (r_s + L_{Ls} P)\, i_{as} \\
\acute{V}_{qr} &= L_m P\, i_{qs} + (\omega - \omega_r)\, L_m\, i_{ds} + (\acute{r}_r + \acute{L}_r P)\acute{i}_{qr} + (\omega - \omega_r)\, \acute{L}_r\, \acute{L}_{dr} \\
\acute{V}_{dr} &= (\omega - \omega_r)\, L_m\, i_{qs} + L_m P\, i_{ds} - (\omega - \omega_r)\, \acute{L}_r\, \acute{i}_{qr} + (\acute{r}_r + \acute{L}_r P)\acute{i}_{dr} \\
\acute{V}_{or} &= (\acute{r}_r + L_{Lr} P)\, \acute{i}_{0r} \\
\text{Where, } L_s &= L_{Ls} + L_m,\ \acute{L}_r = \acute{L}_{Lr} + L_m
\end{aligned}\right\}$$

(78)

From equation (77) through equation (78), it can be observed that only leakage inductance and phase resistances influence the zero-sequence voltages and currents unlike in the d-q-component variables which are influenced by the self and mutual inductances and phase resistances. Again in a balances 3-phase machine, the sum of the three-phase current is zero which leads to a zero sequence current of zero value (R.krishnan, 2001).

This therefore implies that the analysis can be carried out with the voltage and flux linkage equations ignoring the Zero-sequence components. And when this is done, the flux linkage equation of (77) can be rewritten as;

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \acute{\lambda}_{qr} \\ \acute{\lambda}_{dr} \end{bmatrix} = \begin{bmatrix} (L_{Ls}+L_m) & 0 & L_m & 0 \\ 0 & (L_{Ls}+L_m) & 0 & L_m \\ L_m & 0 & (\acute{L}_{Lr}+L_m) & 0 \\ 0 & L_m & 0 & (\acute{L}_{Lr}+L_m) \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ \acute{i}_{qr} \\ \acute{i}_{0r} \end{bmatrix} \quad (79)$$

The voltage equation for the induction motor in the arbitrary reference frame becomes;

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ \acute{V}_{qr} \\ \acute{V}_{dr} \end{bmatrix} = \begin{bmatrix} (r_s+L_sP) & \omega_{Ls} & L_m P & \omega_{Lm} \\ -\omega L_s & (r_s+L_sP) & -\omega L_m & L_m P \\ L_m P & (\omega-\omega_r)L_m & (\acute{r}_s+\acute{L}_r P) & (\omega-\omega_r)\acute{L}_r \\ -(\omega-\omega_r)L_m & L_m P & -(\omega-\omega_r)\acute{L}_r & (\acute{r}_r+\acute{L}_r P) \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ \acute{i}_{qr} \\ \acute{i}_{dr} \end{bmatrix} \quad (80)$$

In q-d-rotor reference frame $\omega = \omega_r$.

Substituting this into equation (80), we have;

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ \acute{V}_{qr} \\ \acute{V}_{dr} \end{bmatrix} = \begin{bmatrix} (r_s+L_sP) & \omega_r L_s & L_m P & \omega_r L_m \\ -\omega_r L_s & (r_s+L_sP) & -\omega_r L_m & L_m P \\ L_m P & 0 & (\acute{r}_r+\acute{L}_r P) & 0 \\ 0 & L_m P & 0 & (\acute{r}_r+\acute{L}_r P) \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ \acute{i}_{qr} \\ \acute{i}_{dr} \end{bmatrix} \quad (81)$$

The transformation from **a-b-c** to **q-d-o** variables is still the same and given as;

$$[T_{abc}] = {}^2/_3 \begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \sin\theta_r & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (82)$$

The inverse matrix is;

$$[T_{abc}]^{-1} = \begin{bmatrix} \cos\theta_r & \sin\theta_r & 1 \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \quad (83)$$

The electromagnetic torque for the d-q-rotor reference is expressed as;

$$T_e = \frac{3}{4} P_n L_m \left(i_{qs}\, \acute{i}_{dr} - \acute{i}_{ds}\, \acute{i}_{qr}\right) \quad (84)$$

Also, the electromechanical (rotor) dynamic equation is given by;

$$T_e = J\, \frac{dw_m}{dt} + T_L + B\omega_m \quad (85)$$

Where;

J = moment of inertia of motor, T_L = Load torque, B = friction coefficient of the load and motor, ω_m = mechanical rotor speed.

XII. CIRCUIT PARAMETER VALUES AND CONSTANTS FOR THE DYNAMIC/STEADY-STATE SIMULATION OF THE INDUCTION MOTOR

The dynamic simulation of the 3-phase induction machine is carried out with parameters in table 1.

Table 1

S/No	Parameter	Value
1	L_s	5.79mH
2	L_r	5.79mH
3	$r_s = r_{\acute{r}}$	3.0Ω
4	J	1.98×10^{-3}kgm^3
5	V	220V
6	F	50H$_Z$
7	P	4

With values in table 1 and using equation (79); and equation (81) through equation (85) the dynamic simulation plots for the 3-phase induction machine is a shown in figure 12 through figure 15.

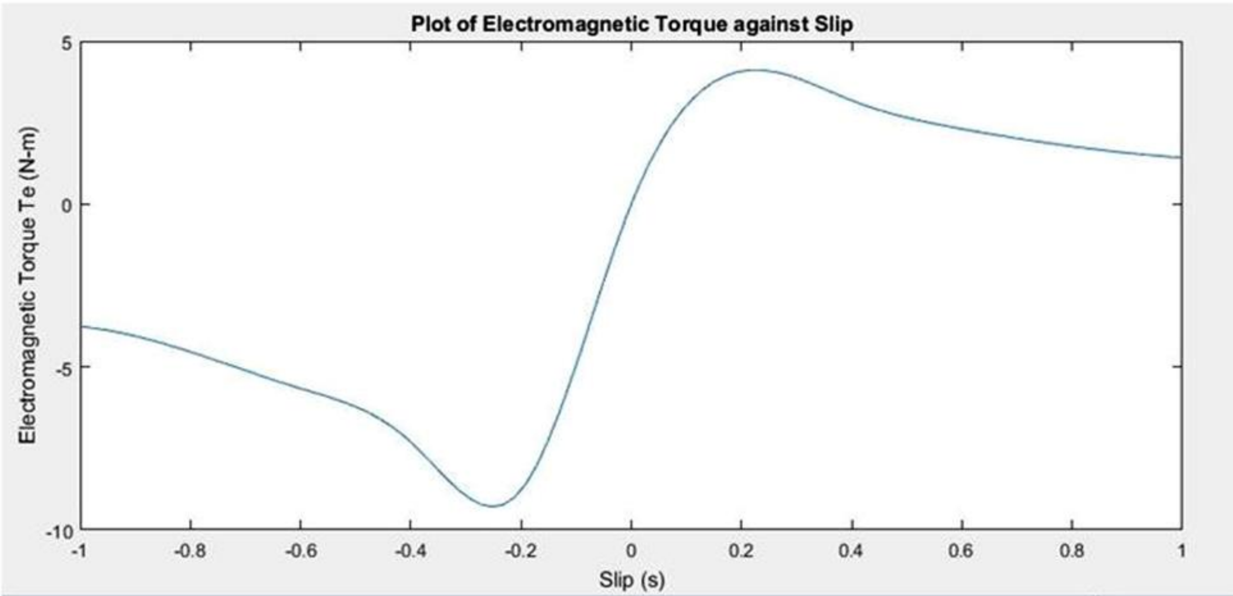


Fig 7: Torque/slip curve of a typical three –phase induction machine, operating at motoring/generating modes.

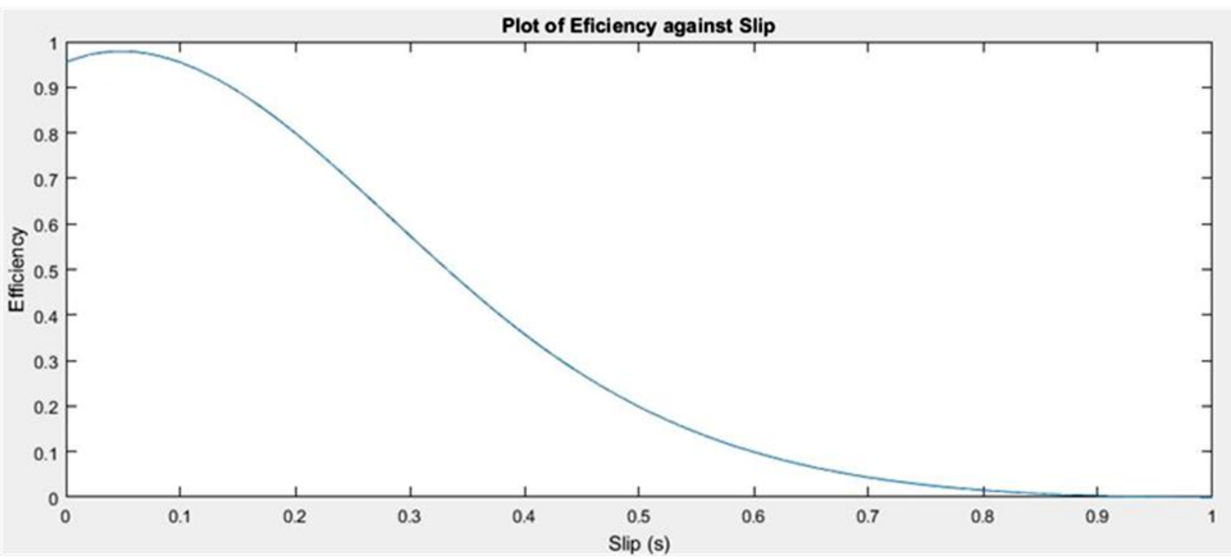


Fig 8 – Efficiency/Slip characteristics of 3-phase induction motor

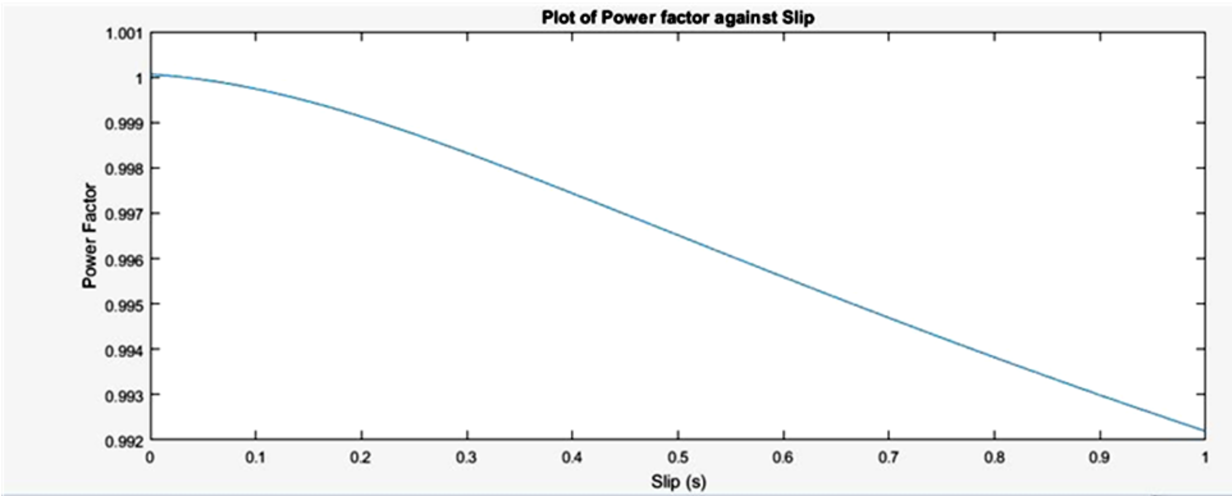


Fig 9 - Power factor/Slip characteristics of 3-phase induction motor

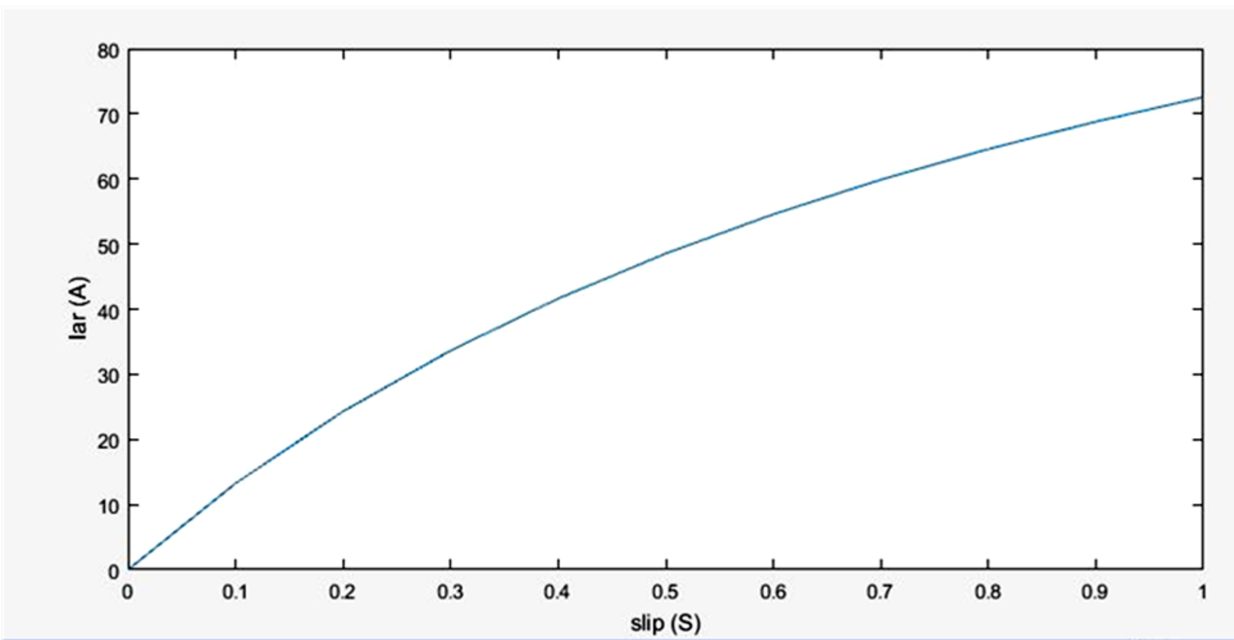


Fig 10: Rotor current against slip of 3-phase induction motor

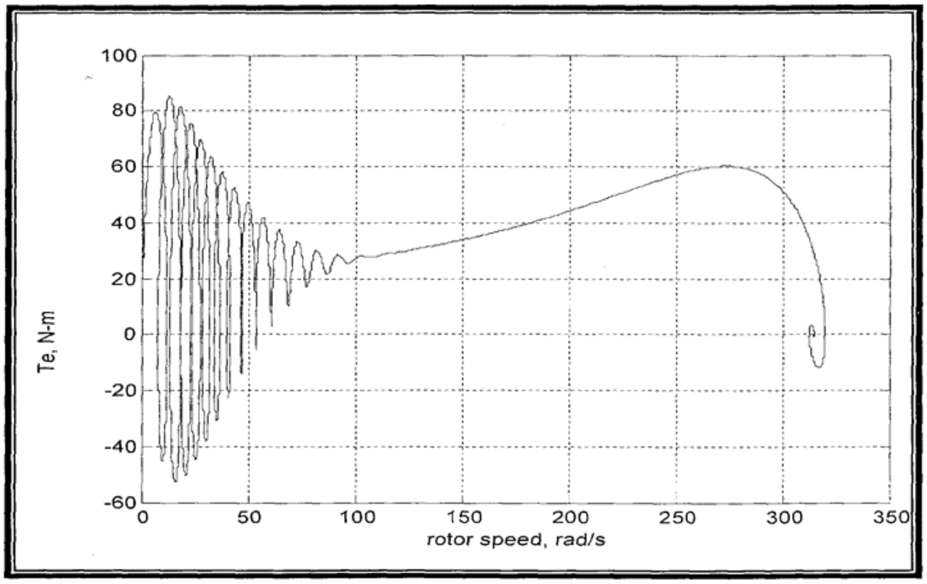


Fig 12 – The Electromagnetic Torque, T_e against rotor speed for the 3-phase induction machine (IM)

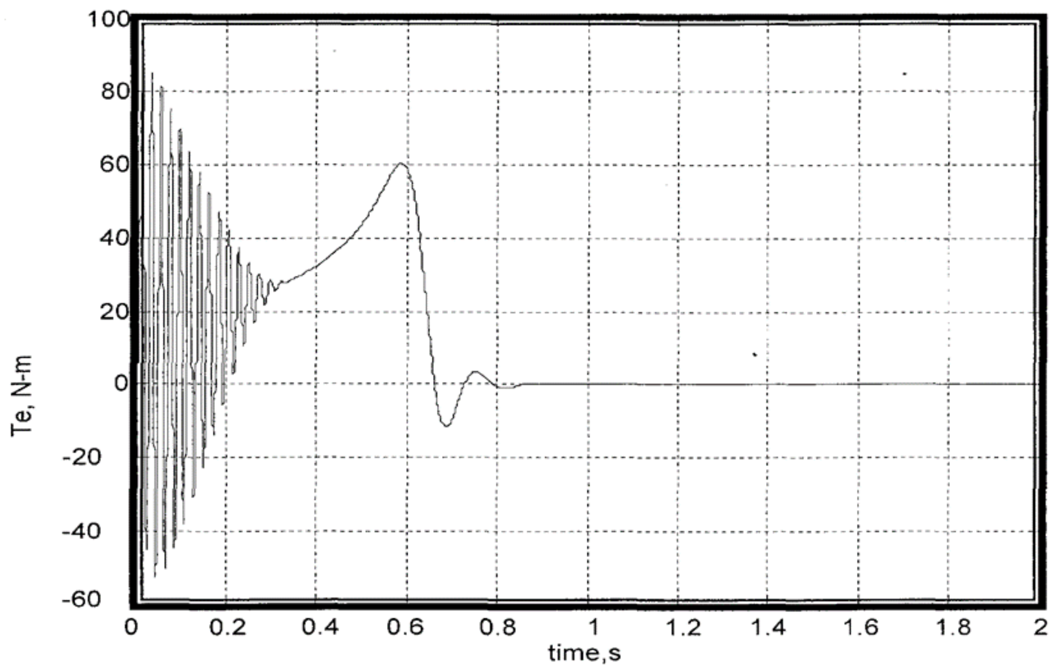
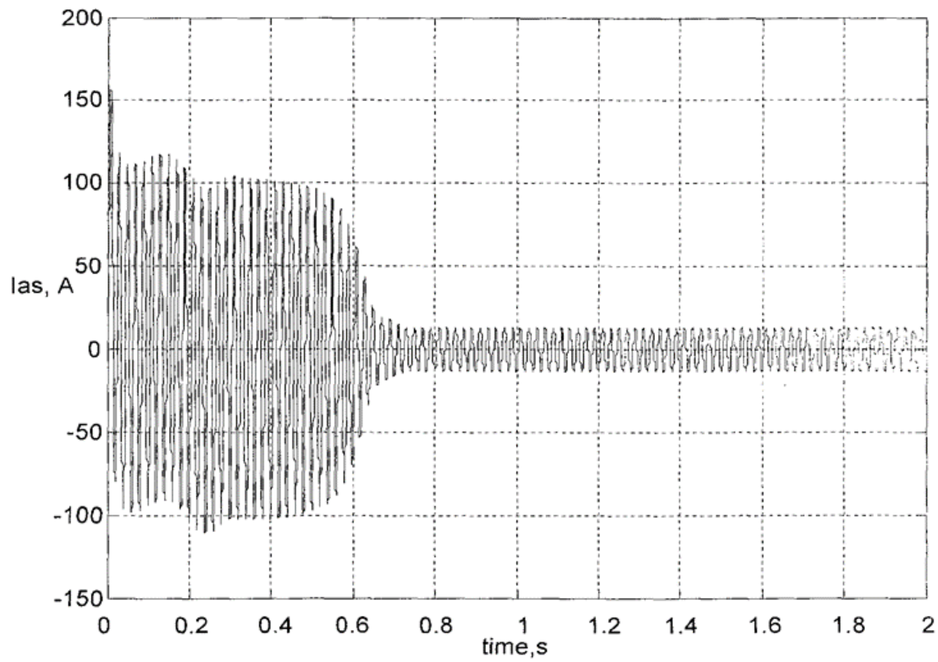
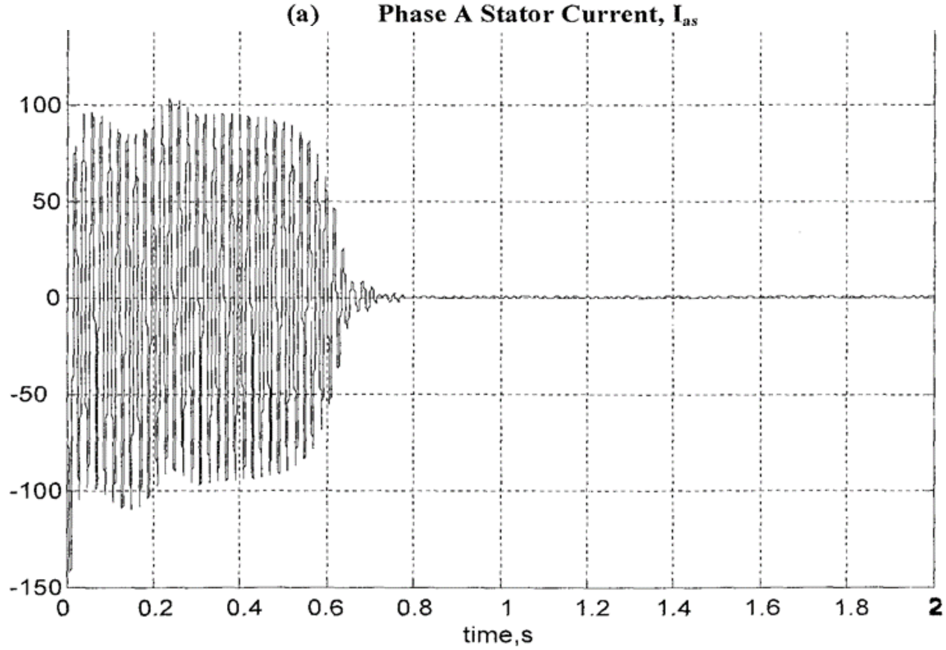


Fig 13 – The electromagnetic Torque, T_e against time for the 3-phase induction machine (IM)

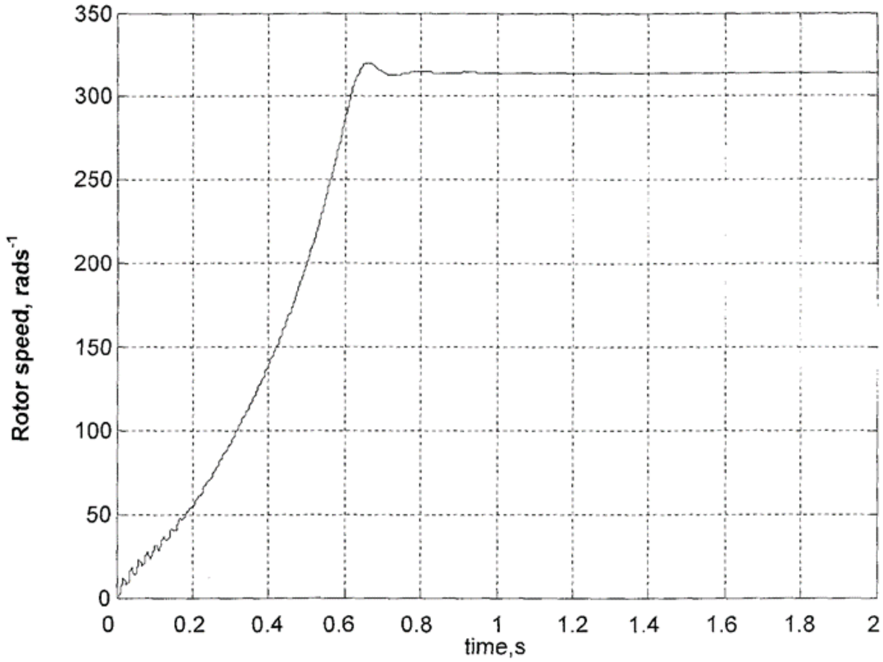


(a) Phase A Stator Current, I_{as}

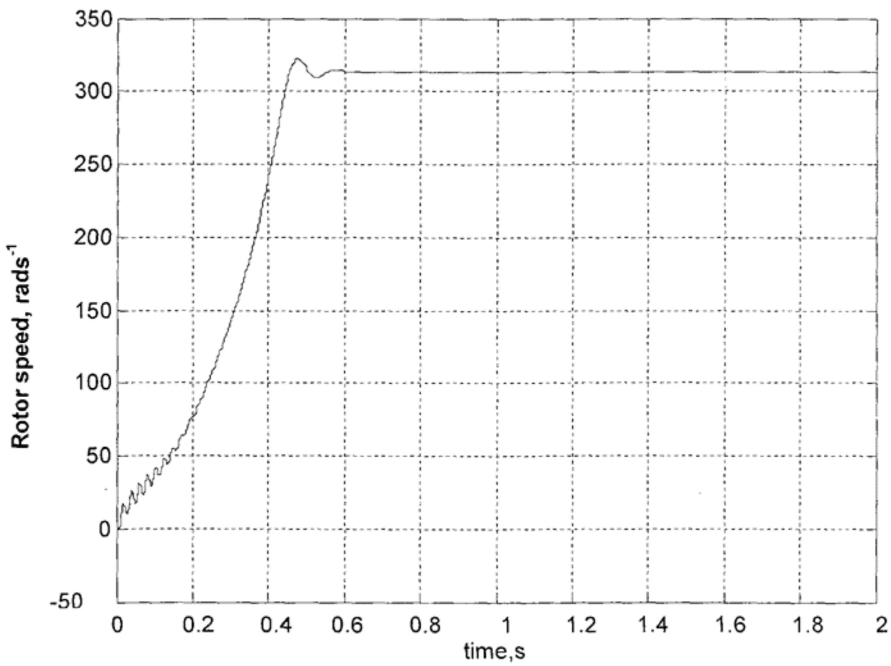


(b) Phase A rotor current, I_{ar}

Fig 14: Plotting of phase A current for stator and rotor windings of 3-phase induction machine (IM).



(a) At no-load



(b) At a torque load, T_L, of 3.5 N-m

Fig 15: Rotor Speed ω_r, against time for the 3-phase induction machine.

ANALYSIS OF RESULT

The analysis of the output characteristics of the motor as seen in the graphs are as follows;

The steady-state condition, the starting torque of the steady-state electromagnetic torque of the motor was observed as in figure 7 to be approximately 2.1N-m, while the maximum torque is 4.2N-m.

More-still, at stand still condition of the motor (i.e. $N_r = 0$ or $s = 1$), the power factor is 0.9921, as in figure 9. Also the starting current is as high as 72A (see figure 10). This is a problem in most induction motors, especially the squirrel cage type with end rings short circuited cage windings.

In the dynamic mode, the torque versus speed characteristics of the machine oscillated at a rotor speed of 314 rads^{-1}, that is at synchronous speed. The plot is shown as in figure 12.

Also, the machine was simulated to start on no load as in figure 15a. It accelerated freely, and after some damped oscillation settles at a synchronous speed $\omega_r = \omega_s$ (314rads^{-1}) at a time of 0.6seconds.

When it was simulated with load torque T_L of 3.5N-m, the rotor speed did not accelerate freely as it did under no load, rather it has few seconds of delayed acceleration, oscillated very briefly and settles at less than the synchronous speed (ω_r = 312 rads^{-1}) at oscillation time of 1.1seconds. This is shown in figure 15b.

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