Prime Labeling Of Special Graphs

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***Abstract-* In this paper we investigate the prime labeling of newly constructed graphs. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph$G$. A graph $G = (V(G), E(G))$ with $|V(G)|$ vertices is said to have prime labeling if there exist a bijection map$f$: $V(G) \rightarrow \{1, 2, 3, \ldots, |V(G)|\}$ such that for each edge $e = uv$ in$E(G)$, f (u) and f (v) are relatively prime. Two integers are said to be relatively prime, if their greatest common divisor(gcd) is 1. We proved that the graphs obtained by replacing every edge of a star graph $K_{1,n}$ by C_m is a prime graph, where $n \geq 1$ and $m \geq 4$.**

***Indexed Terms-* Prime labeling, prime graphs, Greatest common divisor, Star Graphs, Cyclic Graphs**

I. INTRODUCTION

Graph theory is an important area in mathematics with many real world application. Graph labeling is a one of the active research area in Graph theory and lot of researchers are followed literature of graph labeling. In a graph labeling, various types of labeling methods can be seen.[3] Prime labeling is one of the most active labeling method from those methods. The notion of a prime labeling was introduced by Roger Entringer around 1980's with a conjecture which is *every tree is prime*.[10] Current work on prime labeling admits for known graphs.

In our previous researches have been done for Tripartite graphs, Roach graphs, Combining Star graphs and Complete Bipartite graphs and Scorpion graphs.[6],[7] In this paper, we consider simple finite undirected graph $G = (V(G), E(G))$ with the vertex set $V(G)$ and edge set $E(G)$ obtained by replacing every edge of star graph $K_{1,n}$by cyclic graph C_m , where $n \geq 1$ and $m \geq 4$.

Some main explanations for this research are given below.

Definition (Prime Graphs)
A graph $G = (V(G), E(G))$ with $|V(G)|$ vertices is said to have *prime labeling* if its vertices can be labeled with distinct positive integers not exceeding $|V(G)|$ such that the label of each pair of adjacent vertices are relatively prime. Two integers are said to be relatively prime, if their greatest common divisor (gcd) is 1. A graph G which admits prime labeling is called a *prime graph*. [4]

Definition (Star Graph)
A *complete bipartite graph* is a simple bipartite graph in which each vertex in one partite set is adjacent to all the vertices in the other partite set. A $K_{p,q}$ graph is a complete bipartite graph which has p vertices in one partite set and q vertices in the other partite set where $p, q \geq 1$. If $p = 1$, then $K_{1,q}$ graph is called a *star graph*. [9]

Definition (Cyclic Graph)
A connected graph that is regular of degree 2 is a *Cyclic Graph*. The cycle graph on n vertices is denoted by C_n. [8]

II. METHODOLOGY

Theorem
The graph H obtained by replacing every edge of star graph $K_{1,n}$ by C_m is a prime graph, where $n \geq 1$ and $m \geq 4$.

Proof:
Let H be a graph obtained by replacing every edge of star graph $K_{1,n}$ by C_m , where $n \geq 1$ and $m \geq 4$.

Let the vertices of $K_{1,n}$ be $u_0, u_1, u_2, \ldots, u_n$ with u_0 as the center vertex and every edge u_0u_i of $K_{1,n}$ replace by C_m by joining $u_{(i-1)(j-1)}u_{ij}$, $u_{nm}u_0$ for $1 \leq i \leq n$ and $1 \leq j \leq m-1$ where $n \geq 1$ and $m \geq 4$.

Then the new vertex set is $V(H) = \{u_0, u_{ij}\}$ for $1 \leq i \leq n$ and $1 \leq j \leq m-1$ where $n \geq 1$ and $m \geq 4$. Also new edge set is $E(H) = \{u_{(i-1)(j-1)}u_{ij},\ u_{nm}u_0\}$ for $1 \leq i \leq n$ and $1 \leq j \leq m-1$ where $n \geq 1$ and $m \geq 4$. So, $|V(H)| = n(m-1)+1$ where, $n \geq 1$ and $m \geq 4$.

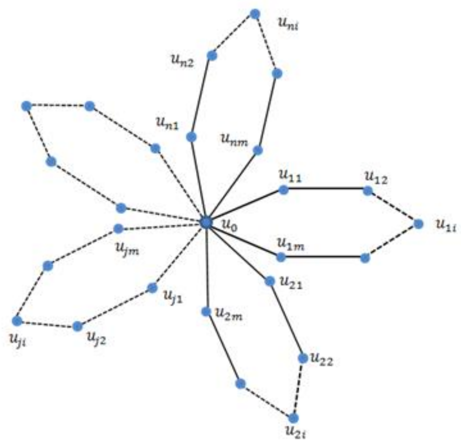

Figure 1: Prime labeling of the graph obtained by replacing every edge of the star graph $K_{1,n}$ by C_m

Define a function $f: V(H) \rightarrow \{1,2,3,\ldots,n(m-1)+1\}$ as follows, $f(u_0) = 1$ and $f(u_{ij}) = [(i-1)(m-1)+1]+j$ for $1 \leq i \leq n$ and $1 \leq j \leq m-1$ where $n \geq 1$ and $m \geq 4$.

Note that,
$\gcd\big(f(u_0), f(u_{i1})\big) = \gcd\big(1, f(u_{i1})\big) = 1$ for $1 \leq i \leq n$ where $n \geq 1$.
$\gcd\big(f(u_0), f(u_{im})\big) = \gcd\big(1, f(u_{im})\big) = 1$ for $1 \leq i \leq n$ where $n \geq 1$.
For all $1 \leq i \leq n$ and $2 \leq j \leq m-1$ where $n \geq 1$ and $m \geq 4$,

$$\gcd\Big(f\big(u_{i(j-1)}\big), f\big(u_{ij}\big)\Big) = \gcd([(i-1)(m-1)+1] + (j-1), [(i-1)(m-1)+1] + j) =$$
$$\gcd\big((i-1)(m-1)+j,\ (i-1)(m-1)+j+1\big)$$
$$= \gcd(j, j+1)$$
$= 1$ (consecutive positive integers)

Therefore,
$\gcd\Big(f\big(u_{i(j-1)}\big), f\big(u_{ij}\big)\Big) = 1$ for $1 \leq i \leq n$ and $1 \leq j \leq m-1$, where $n \geq 1$ and $m \geq 4$.

Clearly, vertex labels are distinct. Thus, labeling defined above gives a prime labeling for the graph H. Therefore, H is a prime graph.

III. RESULTS AND DISCUSSION

In general, all graphs are not prime, it is very interesting to investigate graphs which admit prime labeling. In this research, we prove that the graph obtained by replacing every edge of star graph $K_{1,n}$ by C_m is a prime graph, where $n \geq 1$ and $m \geq 4$.

In this section, Prime graph obtained for $n = 4$ and $m = 6$ values and it is illustrated below.
Note that,
$\gcd\big(f(u_0), f(u_{i1})\big) = \gcd\big(1, f(u_{i1})\big) = 1$ for $1 \leq i \leq 4$,
$\gcd\big(f(u_0), f(u_{im})\big) = \gcd\big(1, f(u_{im})\big) = 1$ for $1 \leq i \leq 4$ and $m = 6$,
For all $1 \leq i \leq 4$ and $2 \leq j \leq 5$,

$$\gcd\Big(f\big(u_{i(j-1)}\big), f\big(u_{ij}\big)\Big) =$$
$$\gcd([(i-1)5+1] + (j-1), [(i-1)5+1] + j)$$
$$= \gcd\big((i-1)5+j,\ (i-1)5+j+1\big)$$
$$= \gcd(j, j+1)$$
$= 1$ (consecutive positive integers)

Hence, graph H for $n = 4$ and $m = 6$ has a prime labeling. The above theorem can be used to give a prime labeling for the Crab graph.

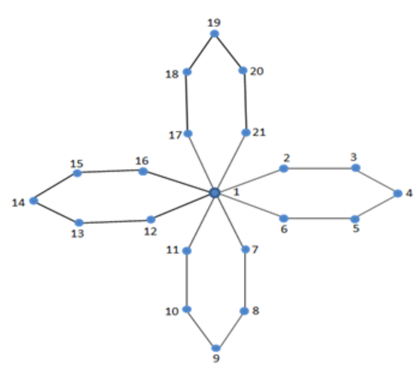

Figure 2: Prime labeling of the graph obtained by replacing every edge of star graph $K_{1,4}$ by C_6

CONCLUSION

Study of relatively prime numbers is very interesting in number theory and it is challenging to investigate prime labeling of some graphs. In this work, we investigate one result about prime labeling of a special type of graphs.

As a future work we are planning to do prime labeling of center less double wheel graphs.

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