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Scrutinize Of Growth and Decay Problems by Dinesh Verma Transform (DVT)

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Abstract- The Dinesh Verma Transform (DVT) is a mathematical tool used in solving the differential equations. Dinesh Verma Transform (DVT) makes it easier to solve the problem in engineering application and make differential equations simple to solve. This paper we will scrutinize the applications of Dinesh Verma Transform (DVT) for handling population growth and decay problems.

Indexed Terms- Dinesh Verma Transform (DVT), Growth and decay problems.

I. INTRODUCTION

The Dinesh Verma Transform (DVT) has been applied in different areas of science, engineering and technology [1], [2], [3] [4], [5], [6], [7]. The Dinesh Verma Transform (DVT) is applicable in so many fields and effectively solving linear differential equations, Ordinary linear differential equation with constant coefficient and variable coefficient can be easily solved by the Dinesh Verma Transform (DVT) without finding their general solutions [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18] [19], [20], [21], [22]. The Leguerre polynomial of nth order generally solved by adopting Laplace Transform, Elzaki Transform [23], [24], [25] [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36]. In this paper we will analyze the applications of Dinesh Verma Transform (DVT) for handling population growth and decay problems. These problems have much significance in the field of Physics, Chemistry, Economics, Social Science etc. we have given various numerical applications to express the use of Dinesh Verma Transform (DVT) for handling population growth and decay problems.

The growth of an organ or a cell or a plant is mathematically expressed in term of a first order ordinary linear differential equation [37], [38], [39], [40] is

$$\frac{dQ}{dt} = KQ \ldots\ldots\ldots\ldots\ldots..(I)$$

$(This\ equation\ is\ known\ s\ the\ Malthsian\ of\ population\ growth)$ With initial conditions $(at\ t = 0) = Q_0$, where K is the positive real number, Q is the amount of populations at time t and Q_0 is the initial population at time $t = t_0$.

The Decay problem of the substance is defined mathematically by the following first order linear differential equations [37], [38], [39], [40] is

$$\frac{dQ}{dt} = -KQ \ldots\ldots\ldots\ldots\ldots..(II)$$

With initial conditions $Q(t_0) = Q_0$, where K is the positive real number, Q is the amount of substance at time t and Q_0 is the initial substance at time $t = t_0$.
In equation (II) the negative sign is shows that the mass of substance is decreasing with time.

II. BASIC DEFINITIONS

2.1 DEFINITION OF DINESH VERMA TRANSFORM (DVT)

Dr. Dinesh Verma recently introduced a novel transform and named it as Dinesh Verma Transform (DVT). Let f(t) is a well-defined function of real numbers t ≥ 0. The Dinesh Verma Transform (DVT) of f(t), denoted by $D\{$ {f(t)}, is defined as

$$D\{\,\{f(t)\} = p^5 \int_0^\infty e^{-pt} f(t)dt = \bar{f}(p)$$

Provided that the integral is convergent, where $p$may be a real or complex parameter and D is the Dinesh Verma Transform (DVT) operator.

2.2 DINESH VERMA TRANSFORM OF ELEMENTARY FUNCTIONS

According to the definition of Dinesh Verma transform (DVT),

$$\mathrm{D}\{t^n\} = p^5 \int_0^\infty e^{-pt}\, t^n dt$$
$$= p^5 \int_0^\infty e^{-z} \left(\frac{z}{p}\right)^n \frac{dz}{p} \quad , z = pt$$
$$= \frac{p^5}{p^{n+1}} \int_0^\infty e^{-z} (z)^n dz$$

Applying the definition of gamma function,

$$\mathrm{D}\,\{y^n\} = \frac{p^5}{p^{n+1}} \Gamma(n+1)$$
$$= \frac{1}{p^{n-4}} n!$$
$$= \frac{n!}{p^{n-4}}$$

Hence, $D\{t^n\} = \frac{n!}{p^{n-4}}$

Dinesh Verma Transform (DVT) of some elementary Functions

- $D\{t^n\} = \frac{n!}{p^{n-4}}$, $where\ n = 0,1,2,..$
- $D\{e^{at}\} = \frac{p^5}{p-a}$,
- $D\{sinat\} = \frac{ap^5}{p^2+a^2}$,
- $D\{cosat\} = \frac{p^6}{p^2+a^2}$,
- $D\{sinhat\} = \frac{ap^5}{p^2-a^2}$,
- $D\{coshat\} = \frac{p^6}{p^2-a^2}$.
- $D\{\delta(t)\} = p^4$

- The Inverse Dinesh Verma Transform (DVT) of some of the functions are given by
- $D^{-1}\left\{\frac{1}{p^{n-4}}\right\} = \frac{t^n}{n!}$, $where\ n = 0,1,2,..$
- $D^{-1}\left\{\frac{p^5}{p-a}\right\} = e^{at}$,
- $D^{-1}\left\{\frac{p^5}{p^2+a^2}\right\} = \frac{sinat}{a}$,
- $D^{-1}\left\{\frac{p^6}{p^2+a^2}\right\} = cosat$,
- $D^{-1}\left\{\frac{p^5}{p^2-a^2}\right\} = \frac{sinhat}{a}$,
- $D^{-1}\left\{\frac{p^6}{p^2-a^2}\right\} = coshat$,
- $D^{-1}\{p^4\} = \delta(t)$

2.3 DINESH VERMA TRANSFORM (DVT) OF DERIVATIVES

$$D\{f'(t)\} = p\bar{f}(p) - p^5 f(0)$$
$$D\{f''(t)\} = p^2\bar{f}(p) - p^6 f(0) - p^5 f'(0)$$
$$D\{f'''(y)\} = p^3\bar{f}(p) - p^7 f(0) - p^6 f'(0) - p^5 f''(0)$$
And so on.

$D\{tf(t)\} = \frac{5}{p}\bar{f}(p) - \frac{d\bar{f}(p)}{dp}$,

$D\{tf'(\mathrm{t})\} = \frac{5}{p}\left[p\bar{f}(p) - p^5\mathrm{f}(0)\right] - \frac{d}{dp}\left[p\bar{f}(p) - p^5\mathrm{f}(0)\right]$ and

$D\{tf''(\mathrm{t})\} = \frac{5}{p}[p^2\bar{x}(p) - p^6x(0) - p^5x'(0)] - \frac{d}{dp}[p^2\bar{x}(p) - p^6x(0) - p^5x'(0)]$ And so on.

III. METHODOLOGY

(A) For Growth problems By (I),

We have,

$$\frac{dQ}{dt} = KQ$$

This can be written as

$$Q' = KQ$$

with initial conditions $Q(t_0) = Q_0$

Taking Dinesh Verma Transform (DVT) on sides

$$D\{Q'\} = KD\{Q\}$$
$$p\bar{Q}(p) - p^5Q(0) - \mathrm{K}\bar{Q}(p) = 0$$

Applying initial conditions, we get

$$(p-K)\bar{Q}(p) = p^5Q_0$$
$$\bar{Q}(p) = \frac{p^5Q_0}{(p-K)}$$

Applying Inverse Dinesh Verma Transform (DVT), we get,

$$\mathrm{Q}(t) = Q_0 D^{-1}\{\frac{p^5}{(p-K)}\}$$
$$\mathrm{Q}(t) = Q_0 e^{Kt}$$

This is the required amount of the population at time t.

(B) For Decay problems By (II),

We have,

$$\frac{dQ}{dt} = -KQ$$

This can be written as

$$Q' = -KQ$$

with initial conditions $Q(t_0) = Q_0$

Taking Dinesh Verma Transform (DVT) on sides

$$D\{Q'\} = -KD\{Q\}$$
$$p\bar{Q}(p) - p^5Q(0) + \mathrm{K}\bar{Q}(p) = 0$$

Applying initial conditions, we get

$$(p+K)\bar{Q}(p) = p^5 Q_0$$
$$\bar{Q}(p) = \frac{p^5 Q_0}{(p+K)}$$

Applying Inverse Dinesh Verma Transform (DVT), we get,

$$\mathrm{Q}(t) = Q_0 D^{-1}\{\frac{p^5}{(p+K)}\}$$
$$\mathrm{Q}(t) = Q_0 e^{-Kt}$$

This is the required amount of the population at time t.

Applications (1):

The population of a city grows at a rate proportional to the number of people presently living in the city. If after four years, the population has tripled, and after five years the population is 50,000, estimate the number of people initially in the city.

Mathematically, this can be written as

$$\frac{dQ}{dt} = KQ$$

Where Q denotes the number of people living in the city at ant time t and K is the constant of proportionality. Consider Q_0 is the number of people initially living in the city at t = 0.

This can be written as

$$Q' = KQ$$

With initial conditions $Q(t_0) = Q_0$

Taking Dinesh Verma Transform (DVT) on sides

$$D\{Q'\} = KD\{Q\}$$
$$p\bar{Q}(p) - p^5 Q(0) - \mathrm{K}\bar{Q}(p) = 0$$

Applying initial conditions, we get

$$(p-K)\bar{Q}(p) = p^5 Q_0$$
$$\bar{Q}(p) = \frac{p^5 Q_0}{(p-K)}$$

Applying Inverse Dinesh Verma Transform (DVT), we get,

$$\mathrm{Q}(t) = Q_0 D^{-1}\{\frac{p^5}{(p-K)}\}$$
$$\mathrm{Q}(t) = Q_0 e^{Kt} \ldots\ldots..(III)$$

Now at, $t = 4,\ Q = 3Q_0$, so by (III),

$$3Q_0 = Q_0 e^{4K}$$
$$4K = log3$$
$$K = .275$$

Now, $t = 5,\ Q = 50{,}000, from\ (III)$

$$50{,}000 = Q_0 e^{5K}$$

Putting the value of K,

$$50{,}000 = Q_0 e^{5*.275}$$
$$50{,}000 = Q_0 e^{1.375}$$
$$50{,}000 = Q_0\ 3.955$$
$$Q_0 = 12642\ approx.$$

Applications (2): A radioactive substance is known to decay at a rate proportional to the amount present. If initially there is 50 milligrams of the radioactive substance present and after five hours it is observed that the radioactive substance has lost 20 percent of its original mass, find the half life of the radioactive substance.

Mathematically, this can be written as

$$\frac{dQ}{dt} = -KQ$$

Where Q denotes the amount of radioactive substance at time t and K is the constant of proportionality. Consider $Q_0$is the initial amount of the radioactive substance at time t.

This can be written as

$$Q' = -KQ$$

Taking Dinesh Verma Transform (DVT) on sides

$$D\{Q'\} = -KD\{Q\}$$
$$p\bar{Q}(p) - p^5 Q(0) + \mathrm{K}\bar{Q}(p) = 0$$

Applying initial conditions, we get

$$(p+K)\bar{Q}(p) = 50\ p^5$$
$$\bar{Q}(p) = \frac{50\ p^5}{(p+K)}$$

Applying Inverse Dinesh Verma Transform (DVT), we get,

$$\mathrm{Q}(t) = 50D^{-1}\{\frac{p^5}{(p+K)}\}$$
$$\mathrm{Q}(t) = 50e^{-Kt} \ldots..(I)$$

At $t = 5$, the radioactive substance has lost 20 percent of its original mass so 50-20 = 30,

Now,

$$30 = 50e^{-5K}$$
$$e^{-5K} = \frac{3}{5} = .60$$
$$K = -\frac{1}{5}\log_e 0.60$$
$$K = .102$$

We required t when Q $= \frac{Q_0}{2} = \frac{50}{2} = 25$

From (I),

$$25 = 50e^{-Kt}$$

Because

$$K = .102$$

So,

$$25 = 50e^{-.102t}$$
$$e^{-.102t} = \frac{1}{2}$$

$$\boldsymbol{t} = -\frac{1}{.102}\log_e 0.50$$
$$t = 6.795\ hours$$

This is the required half time of the radioactive substance.

CONCLUSION

This paper has presented the applications of Dinesh Verma Transform (DVT) for handling population growth and decay problems. The given numerical example shows the significance of Dinesh Verma Transform for growth and decay problems. The technique has come out to very effective tool for discussing the population growth and decay problems.

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