Mathematical Model for Behaviour of Blood Flow in Artery through Stenosis

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Abstract- The aim of this paper is to develop a mathematical model for studying the non-Newtonian flow of blood through a stenosed arterial segment. Power law fluid represents the non-Newtonian character of blood. The hemodynamic behavior of the blood flow is influenced by the presence of the arterial stenosis. The problem is solved by using analytical techniques with help of boundary conditions and results are displayed graphically for different flow characteristics like pressure drop, shear stress, velocity profile. For the validation of numerical model, the computation results are compared with the results from published literature.

Indexed Terms- Herschel-Bulkley fluid, power law fluid model, pressure drop, stenosis height, shear stress

I. INTRODUCTION

It is known that stenosis is a constricting of any tubulike structure in the body, including blood vessels, heart valves, vertebral canal and the GI tract. A survey of the existing literature on the experimental and theoretical studies of blood flow various parts of the arterial tree under normal as well as pathological conditions further indicates that in certain situations the Bingham plastic fluid model (a particular case of Herschel- Bulkley model) gives a better description of the rheological properties of blood [2, 16]. Blood vessel narrowing is one of the more common usages of this term. High cholesterol could conduced to the stenosis of an artery when it accumulates on the inner wall of the artery. This accumulation, referred to as atherosclerosis, can build up within the artery to the point where it reduces blood flow to organs of the body. When blood flow is reduced, nutrients and oxygen cannot travel to the tissues that need it. The presence of stenosis can lead to serious circulatory disorders. There is strong evidence that hydrodynamic factors such as resistance to flow, wall shear stress and apparent viscosity may play a vital role in the development and the progression of arterial stenosis. Young [18] described many problems in fluid mechanics of arterial stenosis, while Shukla et al. [12] studied the effect of stenosis on non-Newtonian flow of blood in an artery. Many researchers [15,17] feel that the hydrodynamic factors may be helpful in the diagnosis, treatment and fundamental understanding of many disorders. Verma et. al**.** [14] studied the shape of stenosis to blood flow through an artery with mild stenosis and they evaluated, for a given rate of flow, the wall shear stress increases immediately as the stenosis increases in size. Misra & Shit [3] investigated blood flow through arterial segment assuming blood as Hershal-Bulkley fluid. They obtained that the skin-friction and the resistance to flow is maximum at the throat of the stenosis and minimum at the end. Ali et. al. [8] analyzed the effect of an axially symmetric time dependent growth into the lumen of a tube for constant cross section through which a Newtonian fluid is steadily flowing .They investigated the structure of flow through arterial model with one or two sinusoidal stenosis, assuming tha arterial blood flow is quasi-steady. Shah & Siddiqui [10] studied the influence of peripheral layer viscosity on physiological characteristics of blood flow through stenosed artery using Power–law fluid model. It was observed that the resistance to flow increases as stenosis size and peripheral layer viscosity increases. They found that the peripheral layer viscosity of blood in diabetic patients is higher than in non-diabetic patients, resulting higher resistance to blood flow. Thus diabetic patients with higher peripheral layer viscosity are more prone to high blood pressure. Therefore the resistance to blood flow in case of diabetic patients may be reduced by reducing viscosity of the plasma. A mathematical model of blood flow in porous vessel having double stenosis in the presence of an external magnetic field has been investigated by Sinha et al. [13] while Shit and Roy [11] put forward a mathematical analysis for the unsteady flow of blood through arteries having stenosis, in which blood was treated as a Newtonian viscous incompressible fluid.They also investigated that the wall shear stress decreases as stenosis shape parameter increases but in the case of increasing stenosis size, stenosis length and peripheral layer viscosity wall shear stress is increases. Bali & Awasthi [9] analyzed the effect of external magnetic field on blood flow in stenotic artery. They presented that the resistance to flow increases with stenosis height and red cell, which depends on hematocrit value of blood, on the other side resistance to flow decreases with the increase in the value of Hartmann number. Mathur & Jain [7] studied the pulsatile flow of blood through stenosed arteries, including the effects of body acceleration and a magnetic field. Blood flow problems are more complicated than the fluid flow problems. Some researchers studied the power law fluid model of blood arguing that under certain conditions, blood behaves like a power law fluid. Casson [6] examined the validity of Casson model in studies pertaining to the flow characteristics of blood and reported that at low shear rates the yield stress for blood is non-zero. The properties arise from fact that the vessels walls are formed of different substances such as elastic, collagen and smooth mussels with entirely different properties.

In present paper we have studied the effects of bell shaped stenosis on resistance to flow, wall shear stress and velocity profile in an artery by assuming the blood as power-law fluid.

II. FORMULATION OF THE PROBLEM AND ITS SOLUTION

Let us consider a bell shaped stenosis geometry given by [3]

$$R(z) = R_0\left[1 - \frac{\delta}{R_0} exp\left(\frac{-m^2\varepsilon^2 z^2}{R_0^2}\right)\right] \quad (1)$$

Where R_0 stands for the radius of the arterial segment outside the stenosis, R(z) is the radius of the arterial segment under consideration of a longitudinal distance z from the left end of the segment, δ is the depth of the stenosis at the throat and m is a parametric constant, ε is characterizes the relative length of the constriction, defined as the ratio of the radius to half length of stenosis,

i.e. $\varepsilon = \frac{R_0}{L_0}$

Considering the stenosis geometry to be of the form

$$\frac{R(z)}{R_0} = \left[1 - ae^{-bz^2}\right] \quad (2)$$

With

$$a = \frac{\delta}{R_0} \text{ and } b = \frac{m^2\varepsilon^2}{R_0^2}$$

Let us consider blood as a power law fluid, the constitutive equation for power law fluid is given as

$$\tau = \mu e^n$$

Or

$$\left(\frac{\tau}{2\mu}\right)^{\frac{1}{n}} = e = \left(\frac{\frac{1}{2}Pr}{\mu}\right)^{\frac{1}{n}} = -\frac{du}{dr}$$

$$u = \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+} r^{\frac{1}{n}+1} + c \quad (3)$$

Using boundary conditions

τ is finite at $r = 0$ (regularity condition) (4)

$u = 0 \; at \; r = R(z)$ (no-slip condition) (5)

$$u = \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+1}\left(R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right) \quad (6)$$

In the absence of the stenosis $(\delta = 0)$

$$u_P = \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+1}\left(R_0^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right) \quad (7)$$

Where the subscript p denotes the poiseuille flow

$$\frac{u}{u_p} = \frac{\left(R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right)}{\left(R_0^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right)} \quad (8)$$

Flow rate can be given as [4]

$Q = \int_0^R u 2\pi r dr$

$$Q = \pi \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{R^{\frac{1}{n}+3}}{\frac{1}{n}+3} \quad (9)$$

$$P = \frac{2\mu}{R^{1+3n}} \left(\frac{Q}{\pi}\left(\frac{1}{n}+3\right)\right)^n \quad (10)$$

Pressure drop across the length of the stenosis

$$\Delta P = \frac{2\mu}{R_0^{1+3n}} \left(\frac{Q}{\pi}\left(\frac{1}{n}+3\right)\right)^n \int_{-z_0}^{z_0} \frac{dz}{\left(\frac{R}{R_0}\right)^{1+3n}} \quad (11)$$

In the absence of the stenosis ($\delta = 0$)

$$(\Delta P)_{\delta=0} = \frac{4\mu}{R_0^{1+3n}} \left(\frac{Q}{\pi}\left(\frac{1}{n}+3\right)\right)^n z_0 \quad (12)$$

$$K = \frac{\Delta P}{(\Delta P)_{\delta=0}}$$

Or

$$K = \frac{1}{2z_0} \int_{-z_0}^{z_0} \frac{dz}{\left(\frac{R}{R_0}\right)^{1+3n}} \quad (13)$$

If 2L is the length of the stenosis artery, pressure drop along the length of the artery can be given as

$$\Delta P = \frac{4\mu}{R_0^{1+3n}} \left(\frac{Q}{\pi}\left(\frac{1}{n}+3\right)\right)^n (2L - 2z_0) + \frac{2\mu}{R_0^{1+3n}} \left(\frac{Q}{\pi}\left(\frac{1}{n}+3\right)\right)^n \int_{-z_0}^{z_0} \frac{dz}{\left(\frac{R}{R_0}\right)^{1+3n}} \quad (14)$$

When there is no stenosis

$$(\Delta P)_{\delta=0} = \frac{4\mu}{R_0^{1+3n}} \left(\frac{Q}{\pi}\left(\frac{1}{n}+3\right)\right)^n L \quad (15)$$

$$K_1 = \frac{P}{(\Delta P)_{\delta=0}} = 1 - \frac{z_0}{L} + \frac{1}{2L} \int_{-z_0}^{z_0} \frac{dz}{\left(\frac{R}{R_0}\right)^{1+3n}} \quad (16)$$

The shear stress on the surface of the stenosis is given by

$$\tau = \frac{1}{2} P(z) R(z)$$

$$\tau = \frac{\mu}{R^{3n}} \left(\frac{Q}{\pi}\left(\frac{1}{n}+3\right)\right)^n \quad (17)$$

Also the ratio of shearing stress on with and without stenosis can be written as;

$$\bar{\tau} = \frac{\tau}{\tau_p} = \left(\frac{R}{R_0}\right)^{3n} \quad (18)$$

III. RESULTS & CONCLUSION

In this paper, we have dealt with the effects of stenosis in an artery by considering the blood as power-law fluid. It has been concluded that the pressure drop and shear stress increases as the size of the stenosis increases for a given non-Newtonian model of the blood. The expression for velocity in Eqs. (8), for pressure drops across the length of the stenosis in Eqs. (13), for pressure drops along the length of the artery in Eqs. (16) and for shear stress in Eqs.(18) and displayed graphically. Figure (1) & (2) shows variation of pressure drop across the length of the stenosis w. r. t. stenosis height and the relative length of the constriction and highlights that the pressure drop increases with $\frac{\delta}{R_0}$ increases and decreases with $\boldsymbol{\varepsilon}$ increases. Eqs. (16) Depicts that the pressure drop along the length of the artery increases with $\frac{\delta}{R_0}$ and increases with $\frac{z_0}{L}$ increases for fixed n , this is in conformity with the results obtained by Kapur [4]. Variation of wall shear stress with axial distance z, stenosis height and with $\boldsymbol{\varepsilon}$ are presented in fig. (2), fig. (3) & fig. (5) respectively. Form the figure 3, it can be clearly observed that shear stress increases from its approached magnitude (i.e. at z = -2.5) in the upstream of the throat with the axial distance and achieves its maximal at the throat of the stenosis and then decreases in the downstream and attains a lower magnitude at the end of the constriction profile (i.e. at z =2.5). Magnitude of wall shear stress in uniform tube is lower than its magnitude in stenosed artery; this is in conformity with the results obtained by Biswas and Chakraborty [1]. It can be seen from fig. (4) that shear stress increases with the height of stenosis and it is always greater than unity and increases as value of n increases, for fixed $\frac{\delta}{R_0}$. Fig. (5) Shows shear stress decreases with the relative length of the constriction.

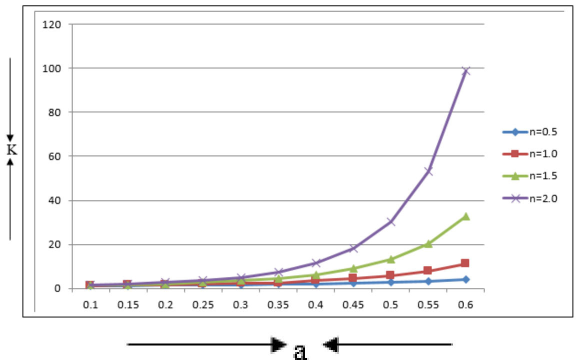

Figure 1. Variation of pressure drop across the length of the stenosis w.r.t. stenosis hight at ε=0.64 and z=2.5

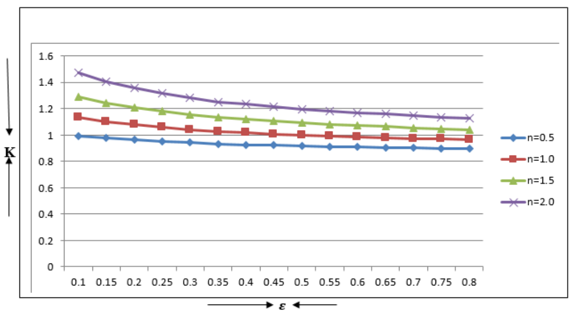

Figure 2. Variation of pressuredrop across the length of the stenosis with the relative length of the constriction

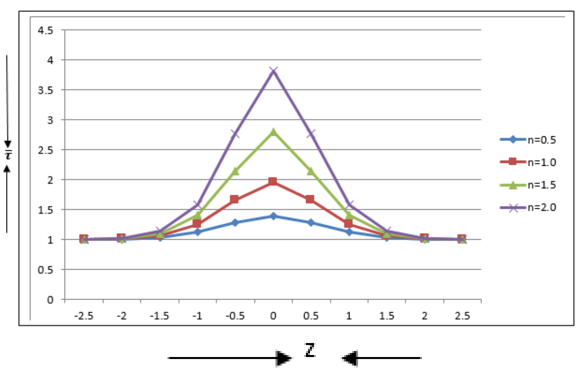

fig. (3) Variation of shear stress along the length of the arterial segment

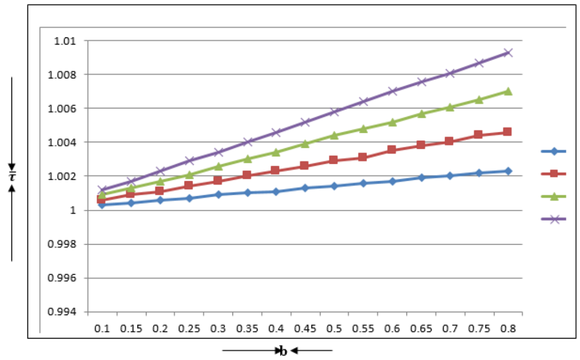

fig.(4) Variation of shear stress with stenosis height

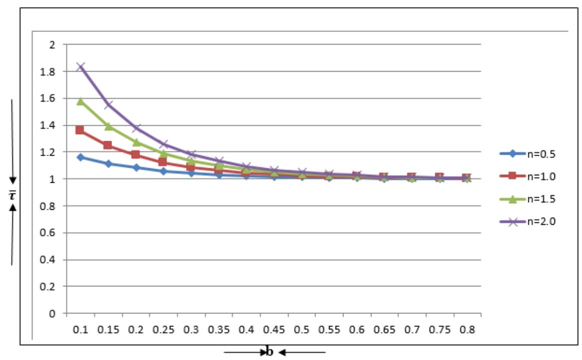

fig.(5) Variation of shear stress with b

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