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Unsteady Laminar Forced Convection in Confocal Elliptical Pipe

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Abstract- The purpose of this paper is to analyse theoretically the problem of the forced convections in unsteady liquid flow in a confocal elliptical pipe. The results are obtained in terms of Mathieu function for any value of Qin general. As a particular case the results have been obtained when heat source is in the channel and pressure gradient varies as negative exponential power of time.

I. INTRODUCTION

Heat transfer problem of forced convections in channels have contributed an important and useful subject. Laminar forced convection problem under fully developed conditions is one of the most fundamental and important problems in heat transfer as it forms the basis of several other problems of heat transfer.

Tao (1961) and Nigam S.(2021)considered Laminar forced convections in round and flat conduits. Laminar forced convections in unsteady Liquid flow in elliptic pipes has not yet been considered.

In the present work we imagine the following:
a) The flow is Laminar and unsteady and liquid properties are constant.
b) An arbitrary heat source is present in the channel, which has the intensity Q.
c)The flow is fully developed (both hydrodynamically and thermally).
d)The liquid and wall temperatures increase or decrease linearly at the same rate in the direction of flow.
e)Theaxes of the channel and flow direction are in the positive direction of z-axis.

In forced convections, the energy of the motion is appreciably comparable with the amount of heat transferred in the fluid. The presence of the arbitrary source of heat in the channel which has the intensity Q accounts for the heat generation in the fluid.

The purpose of present work is to analyse theoretically the problem of the forced convections in unsteady liquid flow in aconfocal elliptical pipe.

The results are obtained in terms of Mathieu functions for any value of Q in general. As a particular case they have been obtained when heat source is in the channel and pressure gradient varies as negative exponential power of time. Nigam S.(2021)

- Formulation of Problem:

The governing equations after Tao (1) are

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{\mu}\frac{\partial p}{\partial z}+\frac{1}{\nu}\frac{\partial u}{\partial t} \qquad (1)$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=\frac{\rho\Delta}{\kappa}\frac{\partial T}{\partial z}-\frac{Q}{\kappa}+\frac{1}{\kappa}\frac{\partial T}{\partial t} \qquad (2)$$

Where Q is the heat source intensity, κ the thermal conductivity, μ the coefficient of viscosity, ρ the density, u the local velocity in the direction of z-axis and T the modified temperature $T^{'}-T_w, T'$ the local temperature and T_w the wall temperature which remains uniform throughout the wall. Let us further assume

$$C_1(t)=\frac{1}{\mu}\frac{\partial p}{\partial z}$$

$$C_2=\frac{\partial T}{\partial z}\frac{\rho\Delta}{\kappa}$$

$$C_3(t)=\frac{Q}{\kappa}$$

$C_1(t)$ and $C_3(t)$ are functions of time alone. So equations (1) and (2) transform to

- Solutions of the problem:

To solve equation let us choose $q_{2n,m}$ be the roots of the equation

$$C_{e2n}(\xi,q)F_{ey2n}(\xi_{\circ},q)+F_{ey2n}(\xi,q)C_{e2n}(\xi_{\circ},q)=0.$$

Now multiply the above equation by$\beta_{e2}(\xi, q_{2n,m})C_{e2n}(\eta, q_{2n,m})$and integrate ξ to $\xi_\circ$ and $\eta$with in the limits $\circ$ to $2n$where

$\beta_{e2n}(\xi, q_{2n,m}) = \{F_{ey2n}(\xi_\circ, q_{2n,m}) -$ *Fey2nξ1,q2n,mCe2nξ,q2n,m*

$-\{C_{e2n}(\xi_\circ, q_{2n,m}) -$ *Ce2nξ1,q2n,mFey2nξ,q2n,m* (3)

We get

$$-2\bar{u}q_{2n,m} = \frac{C^2}{2}\overline{C_1(t)} + \frac{1}{\nu}\frac{C^2}{2}\frac{d\bar{u}}{dt} \quad (4)$$

and $$-2\bar{T}q_{2n,m} = \frac{C^2}{2}\left[C_2\bar{u} - \overline{C_3(t)} + \frac{1}{\kappa}\frac{d\bar{t}}{dt}\right] \quad (5)$$

$\overline{C_1(t)} = \int_{\xi_\circ}^{\xi_1}\int_\circ^{2n} C_1(t)(\cosh 2\xi -$ cos*2ηβe2nξ,q2n,mCe2nη,q2n,mdξdη*

$\overline{C_3(t)} = \int_{\xi_\circ}^{\xi_1}\int_\circ^{2\pi} C_3(t)(\cosh 2\xi -$ cos*2ηβe2nξ,q2n,mCe2nη,q2n,mdξdη.*

Now equation (4) and (5) transforms to

$$\frac{d\bar{u}}{dt} + \frac{4q_{2n,m}\nu\bar{u}}{C^2} = -\nu\overline{C_1(t)} \quad (6)$$

and $$\frac{d\bar{T}}{dt} + \frac{4q_{2n,m}\kappa\bar{T}}{C^2} = \kappa C_2\bar{u} - \kappa\overline{C_3(t)} \quad (7)$$

$\bar{T} = \kappa\int_\circ^t\left[\frac{\nu C_1C_2\left(e^{-\alpha t} - e^{-2\lambda_{2n,m}^2 T}\right)}{\nu\lambda_{2n,m} - \alpha} +\right.$ *C3e−βτe−κλ2n,m2t−τdτ*

$\times\int_{\xi_\circ}^{\xi_1}\beta_{e2n}(\xi, q_{2n,m})[2A_\circ^{2n}\cosh 2\xi -$ *A22ndξ,*

$\bar{T} = \kappa\left[\frac{\nu C_1C_2\left(e^{-\alpha t} - e^{-\kappa\lambda_{2n,m}^2 t}\right)}{(\nu\lambda_{2n,m}^2 - \alpha)(\kappa\lambda_{2n,m}^2 - \alpha)} - \frac{\nu C_1C_2\left(e^{-\nu\lambda_{2n,m}^2 t}e^{-\kappa\lambda_{2n,m}^2 t}\right)}{(\nu\lambda_{2n,m}^2 - \alpha)(\kappa - \nu)\lambda_{2n,m}^2}\right.$

$\left. + C_3\frac{e^{-\beta t} - e^{-\kappa\lambda_{2n,m}^2 t}}{(\kappa\lambda_{2n,m}^2 - \beta)}\right]\int_{\xi_\circ}^{\xi_1}\beta_{e2n}(\xi, q_{2n,m})[2A_\circ^{2n}\cosh 2\xi -$ *A22ndξ* (8)

By inversion theorem of Gupta (1)

$$T = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}A'\left[\frac{\nu C_1C_2\left(e^{-\alpha t} - e^{-\kappa\lambda_{2n,m}^2}\right)}{(\nu\lambda_{2n,m}^2 - \alpha)(\kappa\lambda_{2n,m}^2 - \alpha)} - \frac{\nu C_1C_2\left(e^{-\nu\lambda_{2n,m}^2 t} - e^{-\kappa\lambda_{2n,m}^2 t}\right)}{(\nu\lambda_{2n,m}^2 - \alpha)(\kappa - \nu)\lambda_{2n,m}^2} + C_3\frac{\left(e^{-\beta t} - e^{-\kappa\lambda_{2n,m}^2 t}\right)}{(\kappa\lambda_{2n,m}^2 - \beta)}\right]\beta_{e2n}(\xi, q_{2n,m})C_{\ell 2n}(\eta, q_{2n,m}) \quad (9)$$

where $$A' = \frac{\int_{\xi_\circ}^{\xi_1}\beta_{\ell 2n}(\xi, q_{2n,m})\left[2A_\circ^{(2n)}\cosh 2\xi - A_2^{(2n)}\right]d\xi}{\Pi\int_{\xi_\circ}^{\xi_1}\beta_{\ell 2n}^2(\xi, q_{2n,m})[\cosh 2\xi - \Theta_{2n,m}]d\xi} \quad (10)$$

and

$$u = -\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{A'\nu C_1\left(e^{-\alpha t} - e^{-\nu\lambda_{2n,m}^2 t}\right)}{(\nu\lambda_{2n,m}^2 - \alpha)}\beta_{\ell 2n}(\xi, q_{2n,m})C_{\ell 2n}(\eta, q_{2n,m}) \quad (11)$$

Equation (10) & (11) give the temperature and velocity at any point with in the cylinder.

Mean temperature is given by

$$T_m = \frac{I}{A}\int_D T\,dA$$

$$T_m = \frac{\int_{\xi_\circ}^{\xi_1}\int_\circ^{2\Pi}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{A'\nu C_1\left(e^{-\alpha t} - e^{-\nu\lambda_{2n,m}^2 t}\right)}{(\nu\lambda_{2n,m}^2 - \alpha)}\beta_{e2n}(\xi, q_{2n,m})C_{\ell 2n}(\eta, q_{2n,m}) \times \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}A^1\left[\frac{\nu C_1C_2\left(e^{-\alpha t} - e^{-\kappa\lambda_{2n,m}^2 t}\right)}{(\nu\lambda_{2n,m}^2 - \alpha)(\kappa\lambda_{2n,m}^2 - \alpha)} - \frac{\nu C_1C_2\left(e^{-\nu\lambda_{2n,m}^2} - e^{-\kappa\lambda_{2n,m}^2}\right)}{(\nu\lambda_{2n,m}^2 - \alpha)(\kappa - \nu)\lambda_{2n,m}^2} + \frac{C_3\left(e^{-\beta t} - e^{-\kappa\lambda_{2n,m}^2}\right)}{(\kappa\lambda_{2n,m}^2 - \beta)}\right]\beta_{e2n}(\xi, q_{2n,m})C_{\ell 2n}(\eta, q_{2n,m})\frac{C^2}{2}(\cosh 2\xi - \cos 2\eta)d\xi d\eta}{\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{A'\nu C_1\left(e^{-\alpha t} - e^{-\nu\lambda_{2n,m}^2 t}\right)}{\Pi(\nu\lambda_{2n,m}^2 - \alpha)(\sinh 2\xi - \sinh 2\xi_\circ)}\int_{\xi_\circ}^{\xi_1}\beta_{e2n}(\xi, q_{2n,m})[2A_\circ^{2n}\cosh 2\xi - A_2^{2n}]d\xi \times \frac{\Pi C^2}{2}(\sinh 2\xi_1 - \sinh 2\xi_\circ)}$$

.

Using orthogonal property and on solving (Mchachalem 2), Nigam S.(2021,VI)we get

$$T_m = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left[\frac{\nu C_1 C_2\left(e^{-\alpha t} - e^{-\nu\lambda_{2n,m}^2 t}\right)}{(\nu\lambda_{2n,m}^2-\alpha)(\kappa\lambda_{2n,m}^2-\alpha)} - \frac{\nu C_1 C_2\left(e^{-\nu\lambda_{2n,m}^2 t} - e^{-\kappa\lambda_{2n,m}^2 t}\right)}{(\nu\lambda_{2n,m}^2-\alpha)(\kappa-\nu)\lambda_{2n,m}^2} + \frac{C_3\left(e^{-\beta t} - e^{-\kappa\lambda_{2n,m}^2 t}\right)}{(\kappa\lambda_{2n,m}^2-\beta)}\right] \quad (12)$$

Overall heat transfer rate at the boundary $\bar{q}$ is given by

$$\bar{q} = [C_2 u_m - C_3(t)]\kappa A$$

$$\bar{q} = -\kappa\left[\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{C_1 C_2 A'\nu\left(e^{-\alpha t} - e^{-\nu\lambda_{2n,m}^2 t}\right)}{\Pi(\nu\lambda_{2n,m}^2-\alpha)(\sinh 2\xi_1 - \sinh 2\xi_\circ)}\int_{\xi_\circ}^{\xi_1}\beta_{e2n}\left(\xi, q_{2n,m}\right)[2A_\circ^{2n}\cosh 2\xi - A_2^{2n}]d\xi + C_3 e^{-\beta t}\frac{\Pi C^2}{2}\right](\sinh 2\xi_1 - \sinh 2\xi_\circ) \quad (13)$$

or $\bar{q}$ can also be written as

$$\bar{q} = -\left[\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{A'\nu\left(e^{-\alpha t} - e^{-\nu\lambda_{2n,m}^2 t}\right)}{\Pi(\nu\lambda_{2n,m}^2-\alpha)(\sinh 2\xi_1 - \sinh 2\xi_\circ)}\int_{\xi_\circ}^{\xi_1}\beta_{e2n}\left(\xi, q_{2n,m}\right)\times + [2A_\circ^{2n}\cosh 2\xi - A_2^{2n}]d\xi C_5 e^{-\beta t}\right]\kappa A C_1 C_2$$

where $C_1 C_2 = C_4$, $C_5 = \frac{C_3}{C_1 C_2}$.

Nusselts number N_u based on the mixed mean temperature are given by

$$N_u = \frac{hD_e}{\kappa}$$

where D_e is equivalent hydraulic diameter i.e.,$\frac{4A}{S}$ and h is heat transfer coefficient based on mixed mean temperature.

So
$$D_e = \frac{2\Pi C^2(\sinh 2\xi_1 - \sinh 2\xi_\circ)}{4C(\cosh\xi_1 + \cosh\xi_\circ)\int_\circ^{\Pi/2}\sqrt{1-e^1\cos^2\theta}\,d\theta} \quad (14)$$

Now heat transfer coefficient based on mixed temperature is given by

$$h = -\frac{\bar{q}}{ST_m}$$

$$h = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{\left[\frac{C_1 C_2 A'\nu\left(e^{-\alpha t} - e^{-\nu\lambda_{2n,m}^2 t}\right)}{\Pi(\nu\lambda_{2n,m}^2-\alpha)(\sinh 2\xi_1 - \sinh 2\xi_\circ)}\int_{\xi_\circ}^{\xi_1}\beta_{e2n}\left(\xi, q_{2n,m}\right)[2A_\circ^{2n}\cosh 2\xi - A_2^{2n}]C_3 e^{-\beta t}\right]\times\Pi C^2[\sinh 2\xi_1 - \sinh 2\xi_\circ]}{\left[\frac{\nu C_1 C_2\left(e^{-\alpha t} - e^{-\kappa\lambda_{2n,m}^2 t}\right)}{(\nu\lambda_{2n,m}^2-\alpha)(\kappa\lambda_{2n,m}^2-\alpha)} - \frac{\nu C_1 C_2\left(e^{-\nu\lambda_{2n,m}^2 t} - e^{-\kappa\lambda_{2n,m}^2 t}\right)}{(\nu\lambda_{2n,m}^2-\alpha)(\kappa-\nu)\lambda_{2n,m}^2} + C_3\frac{\left(e^{-\beta t} - e^{-\kappa\lambda_{2n,m}^2 t}\right)}{(\kappa\lambda_{2n,m}^2-\beta)}\right]\times C_1(\cosh\xi_1 - \cosh\xi_\circ)\int_\circ^{\Pi/2}\sqrt{1-e^1\cos^2\theta}\,d\theta} \quad (15)$$

So Nusselts number is given by

$$N_u = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{\Pi^2 C^2(\sinh 2\xi_1 - \sinh 2\xi_\circ)\left\{\frac{A'\left(e^{-\alpha t} - e^{-\nu\lambda_{2n,m}^2 t}\right)}{\Pi(\nu\lambda_{2n,m}^2-\alpha)(\sinh 2\xi_1 - \sinh 2\xi_\circ)}\times\int_{\xi_\circ}^{\xi_1}\beta_{\ell 2n}\left(\xi, q_{2n,m}\right)\times[2A_\circ^{2n}\cosh 2\xi - A_2^{2n}]\,d\xi + C_5 e^{-\beta t}\right\}}{4\left[\frac{\nu\left(e^{-\alpha t} - e^{-\kappa\lambda_{2n,m}^2 t}\right)}{(\nu\lambda_{2n,m}^2-\alpha)(\kappa\lambda_{2n,m}^2-\alpha)} - \frac{\nu\left(e^{-\nu\lambda_{2n,m}^2 t} - e^{-\kappa\lambda_{2n,m}^2 t}\right)}{(\nu\lambda_{2n,m}^2-\alpha)(\kappa-\nu)\lambda_{2n,m}^2} + C_5\frac{\left(e^{-\beta t} - e^{-\kappa\lambda_{2n,m}^2 t}\right)}{(\kappa\lambda_{2n,m}^2-\beta)}\right]\times\left[\cosh\xi_1 - \cosh\xi_\circ\int_\circ^{\Pi/2}\sqrt{1-e^1\cos^2\theta}\,d\theta\right]} \quad (16)$$

So dimensionless heat transfer rate $\frac{\bar{q}}{C_4 \kappa A^2}$ is given by

$$q' = \left\{ \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A'\nu\left(e^{-\alpha t} - e^{-\nu \lambda_{2n,m}^2 t}\right)}{\Pi\left(\nu \lambda_{2n,m}^2 - \alpha\right)(\sinh 2\xi_1 - \sinh 2\xi_\circ)} \int_{\xi_\circ}^{\xi_1} \beta_{e2n}\left(\xi, q_{2n,m}\right)[2A_\circ^{2n} \cosh 2\xi - A_2^{2n}] \right.$$

$$\left. + \quad C_5 e^{-\beta t} \right\} \frac{2}{\Pi C^2 (\sinh 2\xi_1 - \sinh 2\xi_\circ)}.$$

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