© MAR 2021 | IRE Journals | Volume 4 Issue 9 | ISSN: 2456-8880

A Self Dual and Doubly Even Code Related to Mathieu Group M_{24}

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Abstract- In this paper, we determine a self-dual and doubly even [24,12,8] code. We determine and discuss the properties of designs related to this code 2010 Mathematics Subject Classification: 94B 05C

Indexed Terms- self-dual, doubly even, Mathieu Group M_{24}.

I. INTRODUCTION

For a permutation group G acting on a finite set Ω, of degree 24 we construct a 24dimensional permutation module invariant under *G*. We take the permutation module to be our working module and recursively find all submodules. The recursion stops as soon as we obtain all submodules. We find that permutation module breaks into submodules of dimension 1, 12 and 23. These submodules are the building blocks for the construction of a submodule lattice as shown in Figure 1

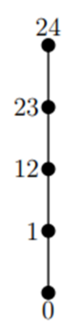

Figure 1: Submodule lattice of the 24-dimensional permutation module

The lattice diagram shows that there is only one irreducible submodule of dimension 1. The authors in [1] used slightly different approach to the one described above. They split the permutation module into maximal submodules. They took the permutation submodule to be the working module and recursively found all maximal submodules of each module. The recursion terminated as soon as it reached an irreducible maximal submodule. In so doing they determined all codes associated with the permutation module invariant under G. This approach involves every time testing equivalence and filtering out isomorphic copies. Our approach has the advantage that the permutation module shows up as non-isomorphic submodules. In this case we produce the submodules more directly. We obtain only one non trivial submodule of dimension 12.

II. BINARY LINEAR CODE

The binary linear code with minimum distance from this representation is [24,12,8]. We shall denote the code $C_{24,1}$ and its dual $C_{24,1}^{\perp}$ Table 1 shows the weight distribution of these codes.

Table 1: Weight distribution of codes of length 24

name	dim	0	8	12	16	24
$C_{24,1}$	12	1	759	1256	759	1
$C_{24,1}^{\perp}$	12	1	759	1256	759	1

We make some observations about the properties of these codes in Proposition 2.1.

- Proposition 2.1. Let G be a primitive group of degree 24 of the Mathieu group M_{24} and $C_{24,1}$ a binary code of dimension 12. Then $C_{24,1}$ is self-dual, doubly even and projective $[24,12,8]_2$ code of weight 8 with 759 words. Furthermore Aut($C_{24,1}$) is isomorphic to S_{24}.
- Proof

From weight distribution, we deduce that codewords have weights divisible by 4. Since the weights of this codewords are divisible by 4, $C_{24,1}$ is doubly even.

Since the dimension of $C_{24,1}$ is half its length $C_{24,1}$ is self-dual. The code $C^{\perp}_{24,1}$ is of weight 8. Hence $C_{24,1}$ is projective. For the structure of the automorphism group, let G be isomorphic to $C_{24,1}$. G has only one composition factor M_{24} . We conclude that G is isomorphic to M_{24}

III. DEIGNS OF CODEWORDS OF MINIMUM WEIGHT IN $C_{24,1}$

We determine designs held by the support of codewords of minimum weight w_m in $C_{24,1}$. In Table 2 columns one, two, three and four respectively represents the code $C_{24,1}$ of weight m, the parameters of the 5-designs D_{wm}, the number of blocks of D_{wm}, and tests whether or not a design D_{wm} is primitive under the action of Aut(C).

Table 2: Deigns of codewords of minimum weight in $C_{24,1}$

weight m	Dw_m	No of Blocks	primitivity
8	5-(24, 8, 1)	759	yes

Remark 3.1. *From the results in table 2 we observe that D_{wm} is primitive.*

IV. SYMMETRIC 1- DESIGNS

Using the orbits of the point stabilizers, we construct symmetric 1 - designs from the simple Mathieu group M_{24}. We examine symmetric 1-design invariant under G constructed from orbits of the rank - 2 permutation representation of degree 24. The first column in Table 3 represents the 1-design D_k of orbit length k, the orbit length in the second column, the parameters of the symmetric 1-design D_k in the third column and the automorphism group of the design the last column.

Table 3: symmetric 1-Design

Design	orbit length	parameters	Automorphism Group
D23	23	1-(24,23,23)	S24

- *Let G be the primitive group of degree 24 of the mathieu group M_{24}. Let $\beta = \{M^g : g \in G\}$ and $D_k = (\Omega, \beta)$. Then the Aut (D_k) is isomorphic to* S_{24}
- Proof

The composition factors of D_k are $\mathbf{Z}_2$ and A_{24}. This implies that AutD_{23} is isomorphic to S_{24}

CONCLUSION

Let G be the primitive group of degree 24 of M_{24} and C a linear code admitting G as an automorphism group. Then the following holds:

a) There exists a self-dual doubly even projective code.
b) There exist a Primitive Design related to M_{24}.
c) AutD_{23} is isomorphic to S_{24} for primitive symmetric 1-design

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