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Designs of Length 2024 From Mathieu Group M_{24}

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Abstract- In this paper, we construct 1-symmetric designs of length 2024 from Mathieu group M_{24} and discuss their properties. We further discuss some codes associated with some of this design
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Indexed Terms- self-dual, doubly even, Mathieu Group M_{24}.

I. INTRODUCTION

Let G be the Mathieu group M_{24}. Group G acts on the triad to generate the point stabilizer $L_3(4)$: S_3. The point stabilizer is a maximal subgroup of degree 2024 in G. The group G acts on this point stabilizer to form orbits. The orbits of this point stabilizers are 1, 63, 210, 630 and 1120. Given a primitive permutation group G acting on a set Ω, it follows from theorem 3.4.1 and 3.4.2 that if we form orbits of the point stabilizer and take their images under the action of the full group, we obtain the blocks of a symmetric 1 - design with the group G acting as an automorphism group.

II. SYMMETRIC 1- DESIGNS

In this section we examine all designs invariant under G. Let Ω be the primitive G - set of degree 2024 and Ω_1, Ω_2, Ω_3, Ω_4, Ω_5, with sub degrees 1, 63, 210,630 and 1120 respectively denote the sub orbits of G on Ω with respect to the point stabilizer $L_3(4)$: S_3. We consider the p - element subsets $\{i_1, i_2, i_3, i_4, i_5\}$ of the set {1,2,3,4,5} to form $\binom{5}{p}$ distinct unions of suborbits Ω_i. Observe that $\left|\bigcup_{i=1}^{p} \Omega\right|$ where $1 \leq p \leq 5$ and $1 < k < 2024$.

In Table 1 the first column represents the 1-design D_k of orbit length k, the second column gives the orbit length, the third column shows the parameters of the 1-designs D_k and the fourth column gives the automorphism group of the design.

Table 1: Designs from primitive group of degree 2024

Design	orbit length	parameters	Automorphism Group
D63	63	1-(2024, 63, 63)	S24
D64	64	1-(2024,64,64)	S24
D210	210	1-(2024,210,210)	M_{24}
D211	211	1-(2024,211,211)	M_{24}
D630	630	1-(2024,630,630)	S 24
D1120	1120	1-(2024,1120,1120)	M_{24}
D631	631	1-(2024,631,631)	S 24
D1121	1121	1-(2024,1121,1121)	M_{24}
D273	273	1-(2024,273,273)	M_{24}
D693	693	1-(2024,693,693)	S 24
D1183	1183	1-(2024,1183,1183)	M_{24}
D840	840	1-(2024,840,840)	M_{24}
D1330	1330	1-(2024,1330,1330)	S 24
D1750	1750	1-(2024,1750,1750)	S 24
D274	274	1-(2024,274,274)	M_{24}
D694	694	1-(2024,694,694)	S 24
D1184	1184	1-(2024,1184,1184)	M_{24}
D841	841	1-(2024,841,841)	M_{24}
D1331	1331	1-(2024,1331,1331)	S 24
D1751	1751	1-(2024,1751,1751)	S 24

D903	903	1-(2024,903,903)	M_{24}
D1393	1393	1-(2024,1393,1393)	S 24
D1960	1960	1-(2024,1960,1960)	S 24
D1813	1813	1-(2024,1813,1813)	M_{24}
D904	904	1-(2024,904,904)	M_{24}
D1394	1394	1-(2024,1394,1394)	S 24
D2023	2023	1-(2024,2023,2023)	S 24
D1961	1961	1-(2024,1961,1961)	S 24
D1814	1814	1-(2024,1814,1814)	M_{24}

- Proposition 2.1. Let G be the Mathieu simple group M_{24}, and Ω the primitive G-set of size 2024 defined by the action on the cosets of $L_3(4)$: S_3. Let M and N be the sets M = [210,211,1120,1121,273,1183, 840,274,1184,841,903,1813,904,1814] and N = [63, 64,630, 631,693,1330,1730,694, 1331,1751, 1393, 1960, 1394, 2023, 1961]. Let $\beta = \{M^g: g \in G\}$ and $D_k = (\Omega, \beta)$. Then the following hold:

i D_k is a primitive symmetric 1 −(2024, |M |, | M |) design.

ii If $k \in M$, then $|Aut(D_k)| \cong M_{24}$

iii If $k \in N$, then $|Aut(D_k)| \cong S_{24}$

- Proof

i. Group G acts as an automorphism group, primitive on points and on blocks of the design and so $G \subseteq Aut(D_k)$.

ii. First, we consider the case when $k \in M$. The only composition factor of Aut (D_k) is

iii. M_{24}. This implies that Aut $(D_k) \cong M_{24}$

iv. We consider the case when $k \in N$. The composition factors of Aut (D_k) are Z2 and A_{24}. This implies that Aut $(D_k) \cong S_{24}$.

III. BINARY CODES

We note that the binary row span of the incidence matrices of each design D_k yield the code denoted C_k. In the following subsections, we examine the properties of some of the codes C_k where computations are possible.

- Proposition 3.1. C_{631} is a projective [2024, 24,253] binary code. The dual code $C^{\perp}_{631}$ of C_{631} is a [2024, 2000, 4].
- Proof

  The minimum weight of $C^{\perp}_{631}$ code is 4. Hence C_{631} is projective.

CONCLUSION

Let *G* be the primitive group of degree 2024 of M_{24} and *C* a linear code and *D* a primitive design admitting *G* as an automorphism group. Then the following holds:

a) There exists a self-orthogonal doubly even projective code.
b) There exist a set of Primitive Symmetric 1-Designs related to M_{24}.

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