Comparative Solution of Heat Transfer Analysis for Squeezing Flow Between Parallel Disks

MUSTAPHA R.A.[1], SALAU A.M.[2]
[1, 2] *Department of Mathematics, Lagos State University, Ojo, Nigeria*

Abstract- The main aim of this present work is to compare two methods employed for solving squeezing flow problems namely Homotopy Perturbation Method (HPM) and the Adomian Decomposition Method (ADM). The Comparison between HPM and ADM is bench-marked against a numerical solution Scheme (FDM). The results show that the ADM is more efficient and reliable than HPM from a computational viewpoint. Although ADM requires tedious computational effort than the HPM, but it yields more accurate and reliable results than the HPM.

Indexed Terms- Squeezing flow, magneto-hydro-dynamics (MHD), Homotopy Perturbation Method (HPM), Adomian Decomposition Method (ADM), Parallel manifolds, Finite Difference Method (FDM).

I. INTRODUCTION

Most problems in real-world engineering and the applied sciences usually rely upon numerical methods to find an approximation of exact solutions. In order to find an approximation of the solution for such problems, we mostly use numerical methods for differential equations, integral equations, nonlinear equations, partial differential equations, boundary value problems etc. Many numerical methods which have been introduced until 1980, represent a discrete approximation of solutions. Since 1980, several numerical methods have been suggested which yield a continuous approximation. These methods approximate the result in the form of a series which converges towards the exact solution. The ADM and the HPM are two examples of such methods which have been applied to many problems in the analysis of functional equations (Adomian, 1994, 1988, 1989; Mirgolbabaei and Ganji, 2009; Ganji et al., 2008; Alnasr and Momani, 2008; Jaradat, 2008; He, 2006, 1999, 2000). They are two powerful methods that consider the approximate solution of nonlinear problems as an infinite series converging to the exact solution (Abbaoui and Cherruault, 1995; Cherruault, 1989; Cherruault and Adomian, 1993; Cherruault et al., 1995). Both methods have been applied to solve a wide range of problems, both deterministic and stochastic, linear and nonlinear, arising from physics, chemistry, biology, engineering, etc. (Adomian, 1976; Fazeli et al., 2008; Ghotbi et al., 2008; Sharma and Methi, 2011; He, 2006; Vahidi and Isfahani, 2011).
The comparison between the ADM and HPM methods, the Homotopy Analysis Method (HAM) and HPM, the HAM and the Variational Iteration Method (VIM), the Taylor series method and ADM, have been given through theoretical anaalysis and numerical analysis, see e.g., (Abbasbandy, 2006; He, 2004; Khatami et al., 2008; Liao, 2004; Ozis and Yildirim, 2008; Sajid and Hayat, 2008; Chowdhury, 2011; Wazwaz, 1998) and other papers where the ADM and HPM methods are applied. For example, Abbasbandy (2006) compared the ADM and HPM methods and by a theorem showed that the ADM is only a special case of the HPM. And Li introduced a comparison between the ADM and the HPM, which showed that these methods are equivalent for solving nonlinear equations (Li, 2009). Recently, the HPM has been successfully compared by the variational iteration method to solve many types of linear and nonlinear problems in science and engineering by many authors (Barari et al., 2008; Choobbasti et al., 2008; Noorzad et al., 2008).

In this study, firstly we explain the Mathematical Formulation of the Problem, and the solution scheme ADM and the HPM to solve nonlinear differential equations of the underlying squeezing flow problem in section 3 and 4, respectively. Then in section 5, we show that the HPM with a specific convex homotopy for solving nonlinear differential equation is equivalent to the ADM. But the ADM converges faster than the HPM, ADM performs well than HPM which can be seen in the table in the discussion of result section

II. MATHEMATICAL FORMULATION OF THE PROBLEM

MHD flow of a viscous incompressible fluid is taken into consideration through a system consisting of two parallel infinite disks distance $h(t) = H(1-at)^{1/2}$ apart. Magnetic field proportional to $B_0(1-at)^{1/2}$ is applied normal to the disks. It is assumed that there is no induced magnetic field. T_W and T_h represent the constant temperatures at $z = 0$ and $z = h(t)$ respectively. Upper disk at $z = h(t)$ is moving with velocity $\frac{aH(1-at)^{-1/2}}{2}$ toward or away from the static lower but permeable disk at $z = 0$ as shown in Fig. 1. We have chosen the cylindrical coordinates system (r, ϕ,z). Rotational symmetry of the flow ($\partial/\partial\phi = 0$) allows us to take azimuthal component v of the velocity $V = (u, v, w)$ equal to zero. As a result, the governing equation for unsteady two-dimensional flow and heat transfer of a viscous fluid can be written as [6]

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial \hat{p}}{\partial r} + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) - \frac{\sigma}{\rho}B^2(t)u, \quad (2)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial \hat{p}}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 w}{\partial z^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right), \quad (3)$$

$$C_P\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = \frac{K_0}{\rho}\left(\frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{u}{r^2}\right) + v\left\{2\frac{u^2}{r^2} + \left(\frac{\partial u}{\partial z}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + 2\left(\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{\partial w}{\partial r}\right)^2 + 2\frac{\partial u}{\partial z}\frac{\partial w}{\partial r}\right\}, \quad (4)$$

Auxiliary conditions are [5]

$$u = 0, \quad w = \frac{dh}{dt} \quad at\ z = h(t)$$
$$u = 0, \quad w = -w_0 \quad at\ z = 0. \quad (5)$$
$$T = T_w \quad at\ z = 0$$
$$u = T_h \ \ at\ z = h(t). \quad (6)$$

u and w here are the velocity components in r and z directions respectively, μ is dynamic viscosity, $\hat{p}$ is the pressure and ρ is the density. Further T denotes temperature, K_0 is the thermal conductivity, C_P is the specific heat, v is the kinematic viscosity and w_0 is suction/injection velocity.

Using the following transformations [5]

$$u = \frac{ar}{2(1-at)}f'(\eta), \quad w = -\frac{aH}{\sqrt{1-at}}f'(\eta),$$
$$B(t) = \frac{B_0}{\sqrt{1-at}}, \qquad \eta = \frac{aH}{\sqrt{1-at}},$$
$$\theta = \frac{T - T_h}{T_w - T_h} \quad (7)$$

Into Eqs. (2)-(4) and eliminating pressure terms from the resulting equations, we obtain

$$f^{iv} - S(\eta f''' + 3f'' - 2ff''') - M^2 f'' = 0, \quad (8)$$

$$\theta'' - S\,Pr(2f\theta' - \eta\theta') - \mathrm{Pr}\ \ Ec(f''^2 + 12\delta^2 f'^2) = 0 \quad (9)$$

With the associated conditions

$$f(0) = A, \quad f'(0) = 0, \ \theta(0) = 1, \quad (10)$$
$$f(1) = \tfrac{1}{2}, \quad f'(1) = 0, \ \theta(1) = 0,$$

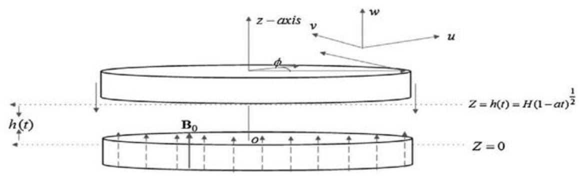
Where S denotes the squeeze number, A is suction/injection parameter, M is Hartman number, Pr Prandtl number, Ec modified Eckert number, and δ denotes the dimensionless length defined as

$$S = \frac{aH^2}{2v}, \quad M^2 = \frac{aB_0^2H^2}{v}, \quad Pr = \frac{\mu C_p}{K_0},$$
$$Ec = \frac{1}{C_p(T_w - T_h)}\left(\frac{ar}{2(1-at)}\right)^2, \quad \delta^2 = \frac{H^2(1-at)}{r^2} \quad (11)$$

Skin friction coefficient and the Nusselt number are defined in terms of variables (7) as

$$\frac{H^2}{r^2}Re_r C_{fr} = f''(1), \ \ (1-at)^{1/2}Nu = -\theta'(1), \quad (13)$$

$$Re_r = \frac{raH(1-at)^{1/2}}{2v}. \quad (14)$$



III. SOLUTION OF THE PROBLEM BY ADM

The decomposition method was introduced by Adomian [3]. Consider the general equation:

$$\varphi[u(y)] = g(y) \tag{15}$$

Where φ represents a general non-linear ordinary (or partial) differential operator involving both linear and non-linear terms. The linear terms is decomposed into the form$L + R$, where L is usually taken as the highest order derivative which is assumed to be easily invertible and R is the linear differential operator of order less than L. Therefore, equation (15) can be expressed as

$$Lu + Ru + Nu = g(y) \tag{16}$$

Where Nu represents the non-linear terms of $\varphi[u]$. Applying the inverse operator, L^{-1} to both sides of equation (16) gives

$$u = L^{-1}g - L^{-1}(Ru + Nu). \tag{17}$$

If L is a fourth order operator, then L^{-1} is a 4-fold integral. Now, solving equation (17), we have

$$u = \sum_{j=0}^{3} \alpha_j \frac{y}{j!} + L^{-1}g - L^{-1}(Ru + Nu), \tag{18}$$

Where $\alpha_j (j = 1..3)$ are constants of integration and can be determined from the given boundary conditions.

The standard Adomian decomposition method defines the solution u by the infinite series

$$u = \sum_{n=0}^{\infty} u_n \tag{19}$$

And the non-linear term by the infinite series

$$Nu = \sum_{n=0}^{\infty} A_n \tag{20}$$

Where A_n are the Adomian polynomials determined formally from the relation;

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N\left(\sum_{i=0}^{\infty} \lambda^i u_i\right)\right]_{\lambda=0}, \qquad i = 0,1,2,\ldots \tag{21}$$

The u_n are determined from the recursive algorithm

$$u_0 = \sum_{j=0}^{3} \alpha_j \frac{y}{j!} + L^{-1}g$$

$$u_{n+1} = -L^{-1}(Ru + Nu) \quad n \geq 0, \tag{22}$$

Where u_0 is the zeroth component. For numerical computation, the truncated series solution is obtained as

$$S_n(y) = \sum_{k=0}^{n-1} u_k \tag{23}$$

Where S_n denotes the n-term approximation of $u(y)$. In this section, the solution to the system of non-linear differential equations (8 – 9) subject to the boundary condition (10) is obtained via ADM. The method is imperative because of the non-linearity involved. Equations (8 – 9) can be written in operator form as:

$$L_1 f = S(\eta f''' + 3f'' - 2ff''') + M^2 f'' \tag{24}$$

$$L_2\theta = S\,Pr(2f\theta' - \eta\theta') + \Pr\ Ec(f''^2 + 12\delta^2 f'^2) \tag{25}$$

Where $L_1 = \frac{d^4}{d\eta^4}$ and $L_2 = \frac{d^2}{d\eta^2}$ is a fourth order and second order differential operator respectively, with inverse operators $L_1^{-1} = \int_0^\eta \int_0^\eta \int_0^\eta \int_0^\eta (\cdot) d\tau d\tau d\tau d\tau$ and $L_2^{-1} = \int_0^\eta \int_0^\eta (\cdot) d\tau d\tau$ respectively. Applying L_1^{-1} to both sides of equation (24) and L_2^{-1} to both sides of equation (25) and imposing the boundary conditions at $\eta = 0$ yields

$$f = A + \frac{\eta^2}{2}\alpha_1 + \frac{\eta^3}{6}\alpha_2 + L_1^{-1}[S(\eta f''' + 3f'' - 2N_1(f)) + M^2 f''] \tag{26}$$

$$\theta = 1 + \eta\alpha_3 + L_2^{-1}\left[SPr\left(2N_2(f,\theta) - \eta\theta'\right) + \Pr\ Ec(N_3(f) + 12\delta^2 N_4(f))\right] \tag{27}$$

Where $\alpha_1 = f''(0)$, $\alpha_2 = f'''(0)$, $\alpha_3 = \theta'(0)$ are to be determined later using the boundary conditions at $\eta = 1$ and $N_1(f)$, $N_2(f,\theta)$, $N_3(f)$, $N_4(f)$ are the non-linear terms.

In terms of Adomian decomposition methods $f(\eta)$ and $\theta(\eta)$ are assumed to be a series solution of the form

$$f(\eta) = \sum_{n=0}^{\infty} f_n(\eta) \text{ and } \theta(\eta) = \sum_{n=0}^{\infty} \theta_n(\eta) \tag{28}$$

And the non-linear terms are decomposed as series
$N_1(f) = \sum_{n=0}^{\infty} A_n$, $N_2(f,\theta) = \sum_{n=0}^{\infty} B_n$, $N_3(f) = \sum_{n=0}^{\infty} C_n$ and $N_4(f) = \sum_{n=0}^{\infty} D_n$ (29)

Where A_n, B_n and C_n are the Adomian's polynomials which are generated by equation (21). Here

$A_0 = f_0 f_0^{'''}, A_1 = f_0 f_1''' + f_1 f_0''' ,\ldots,$

$B_0 = f_0 \theta_0^{'} , B_1 = f_0 \theta_1^{'} + f_1 \theta_0^{'} ,\ldots,$

$C_0 = \left(f_0^{''}\right)^2, C_1 = 2f_0^{''} f_2^{''} + \left(f_1^{''}\right)^2,\ldots,$

$D_0 = \left(f_0^{'}\right)^2 , D_1 = 2f_0^{'} f_1^{'} ,\ldots,$

Substituting equations (28-29) in equations (26-27) we obtain:

$\sum_{n=0}^{\infty} f_n(\eta) = A + \frac{\eta^2}{2}\alpha_1 + \frac{\eta^3}{6}\alpha_2 + L_1^{-1}[S(\eta \sum_{n=0}^{\infty} f_n''' + 3\sum_{n=0}^{\infty} f_n'' - 2\sum_{n=0}^{\infty} A_n) + M^2 \sum_{n=0}^{\infty} f_n''],$ (30)

$\sum_{n=0}^{\infty} \theta_n(\eta) = 1 + \eta\alpha_3 + L_2^{-1}[\, SPr(2\sum_{n=0}^{\infty} B_n - \eta\theta') + \Pr \ Ec(\sum_{n=0}^{\infty} C_n + 12\delta^2 \sum_{n=0}^{\infty} D_n)]$ (31)

From integral equations (30-31), the recursive relations for the approximate analytical solution of system (8-10) are given as:

$f_0 = A + \frac{\eta^2}{2}\alpha_1 + \frac{\eta^3}{6}\alpha_2$ (32)

$f_{n+1} = L_1^{-1}[S(\eta f''' + 3f'' - 2A_n) + M^2 f''],$
$n \geq 0,$ (33)

$\theta_0 = 1 + \eta\alpha_3$ (34)

$\theta_{n+1} = L_2^{-1}[\, SPr(2B_n - \eta\theta') + \Pr \ Ec(C_n + 12\delta^2 D_n)] \ \ n \geq 0$ (35)

The following partial sum
$f(\eta) = \sum_{n=0}^{\infty} f_k(\eta)$ and $\theta(\eta) = \sum_{n=0}^{\infty} \theta_k(\eta)$ (36)

Are the approximate solutions. Equations (32-35) are coded using algebraic symbolic package called Maple.

IV. SOLUTION OF THE PROBLEM USING HPM

In this section, the solution to the system of non-linear differential equations (8 – 9) subject to the boundary condition (10) is obtained via HPM.

According to HPM, we can construct an homotopy for (8,9) as follows

$H(f,p) = (1-p)(f^{iv}) + p(f^{iv} - S(\eta f''' + 3f'' - 2ff''') - M^2 f''$ (37)

$H(\theta,p) = (1-p)(\theta'') + p(\theta'' - S\,Pr(2f\theta' - \eta\theta') - \Pr Ec(f''^2 + 12\delta^2 f'^2))$ (38)

Where primes denote differentiation with respect to η to third degree and f^{iv} is representing f''''.

We are going to consider a three term-solution for f and θ in the infinite series solution which can be seen below as follows

$f = f_0 + pf_1 + p^2 f_2$ (39a)

$\theta = \theta_0 + p\theta_1 + p^2\theta_2$ (39b)

We substitute 39a – 39b into (37) and (38) and we do some algebraic manipulation to obtain the below set of equations:

$p^0: f_0''' = 0$,

$p^1: -m^2 f_0'' + 2Sf_0 f_0''' - S\eta f_0''' - 3Sf_0'' + f_1^{iv} = 0$
(40)

$p^2: -m^2 f_1'' + 2Sf_1 f_0''' + 2Sf_0 f_1''' - S\eta f_1''' - 3Sf_1'' + f_2^{iv} = 0$,

Which is associated with the below initial conditions

$f_0(0) = A\, , f_0'(0) = 0\, , f_0(1) = \frac{1}{2}\, , f_0'(1) = 0$

$f_1(0) = A\, , f_1'(0) = 0\, , f_1(1) = \frac{1}{2}\, , f_1'(1) = 0$ (41)

$f_2(0) = A\, , f_2'(0) = 0\, , f_2(1) = \frac{1}{2}\, , f_2'(1) = 0$

$p^0: \theta_0'' = 0$,

$p^1: -12PrEc\delta^2(f_0')^2 - PrEc(f_0'')^2 + 2PrSf_0\theta_0' - PrS\eta\theta_0' + \theta_1'' = 0$ (42)

$p^2: -24PrEc\delta^2 f_0' f_1' - 2PrEcf_0'' f_1'' + 2PrSf_1\theta_0' + 2PrSf_0\theta_1' - PrS\eta\theta_1' + \theta_2'' = 0$

Which is associated with the below initial conditions

$f_0(0) = A\,, f_0'(0) = 0, \theta_0(0) = 1\ , f_0(1) = \frac{1}{2}\,, f_0'(1) = 0, \theta_0(1) = 0$

$f_1(0) = A\,, f_1'(0) = 0, \theta_1(0) = 1\ , f_1(1) = \frac{1}{2}\,, f_1'(1) = 0, \theta_1(1) = 0$ (43)

$f_2(0) = A\,, f_2'(0) = 0, \theta_2(0) = 1\ , f_2(1) = \frac{1}{2}\,, f_2'(1) = 0, \theta_2(1) = 0$

We obtained an analytical solution using the symbolic algebra package Maple16 to solve (40) and (42) with the associated boundary conditions (41) and (43)

$$f_0(\eta) = \frac{1}{6}(-6 + 12A)\eta^3 + \frac{1}{2}(3 - 6A)\eta^2 + A$$

And

$$\theta_0(\eta) = -\eta + 1$$

The above are the first term of the series solutions

V. COMPARISON OF THE ADOMIAN DECOMPOSITION AND HOMOTOPY PERTURBATION METHOD

In other to compare the two methods used in this present work, first we need to set up a bench mark numerical solution as a guide. We shall use the table of values in [] and [] for ADM and HPM by Mustapha, R.A. and Salau, A.M. we note here that the ADM is closer to the numerical solution than the HPM.

It's worth putting in consideration that as we increase the power series solution of HPM the HPM solution gets progressively worse, whereas the ADM solution maintains its accuracy.

From the below table, we see that in this particular instance the ADM is clearly a cutting-edge choice to choose. Although the HPM is a good method in solving non-linear problems. It's noteworthy to say that both methods provide infinite series solutions.

Table 1.0 Comparison of Numerical and ADM solutions for diverging channel for $\delta = 0.1$, $A = 0.1$, $S = 0.1$, $M = 0.2$, $Pr = 0.3$, $Ec = 0.2$

	$f'(\eta)$					$\theta\ (\eta)$				
η	FDM	HPM	ADM	Error (HPM)	Error (ADM)	FDM	HPM	ADM	Error (HPM)	Error (ADM)
0	0	0	0	0	0	1	1	1	1	0
0.2	0.384801	0.38400	0.384801	8×10^{-4}	0	0.793392	0.8000	0.806144	0.8000	1.3×10^{-2}
0.4	0.575554	0.57600	0.575554	4.5×10^{-4}	0	0.592274	0.6000	0.607022	0.6000	1.5×10^{-2}
0.6	0.575175	0.57600	0.57517	8.25×10^{-4}	0	0.392257	0.4000	0.407004	0.4000	1.5×10^{-2}
0.8	0.384040	0.38400	0.384040	-4×10^{-5}	0	0.193382	0.2000	0.206121	0.2000	1.3×10^{-2}
1	0	0	0	0	0	0	0	0	0	0

CONCLUSION

In this present work, the HPM and ADM have been successfully applied to solve the non-linear equation (8-9) along with the boundary conditions. And we see that both methods generated a series solution where the ADM performs better than HPM.

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