Note On k-Prime Labeling of Some Graphs

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Abstract- ***A*** $\boldsymbol{k}$ ***−PL of a graph*** $\boldsymbol{G = (V, E)}$ ***is an injective function*** $\boldsymbol{f: V(G) \to \{k, k+1, \dots, k+|V|-1\}; k \geq 2}$***, a function*** $\boldsymbol{f^+: E(G) \to N}$ ***every edges of*** $\boldsymbol{G}$ ***which induced by*** $\boldsymbol{f^+(pq) = gcd(f(p)f(q)), \forall e = pq \in E(G)}$ ***such that*** $\boldsymbol{gcd(f(p)f(q)) = 1}$***. A graph*** $\boldsymbol{G = (V, E)}$ ***admits a k-PL which is called a k-prime graph. In this paper we investigate*** $\boldsymbol{P_m \odot 3K_1, P_m \odot 4K_1}$***,Ladder graph and a Quadrilateral snake graphs which admit k-PL.***

Indexed Terms- $\boldsymbol{P_m \odot 3K_1}$***,*** $\boldsymbol{P_m \odot 4K_1}$ ***, Ladder graph,*** $\boldsymbol{k}$ ***−PL (Prime Labeling)***

I. INTRODUCTION

Here we discussed about k −PL of some graphs. Note that the graph $G = (V, E)$ consist a $'v'$ vertices and $'e'$ edges, here we take finite, regular and join sum operation of some graphs. In the history of graph theory first idea a thought of PL (prime labeling) was obtainable by Entringer and the problem was conferred by A. Tout, A. N. Dabbooucy and K. Howalla [6]. And then after many years ago S. K. Vaidya and U. M. Prajapati [7] planned the concept of k- PL They showed some interesting results like that P_n graphs, wheel graphs etc…In this paper we will be study about some undirected graphs like $P_m \odot 3K_1, P_m \odot 4K_1$,Ladder graph and Quadrilateral snake graphs which admit k-prime labeling. For all positive integer $k \geq 2$

II. BASIC TERMINOLOGY

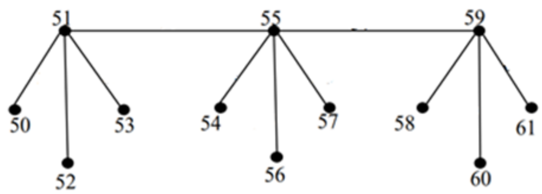
- Definition 2.1[6,7]:A k −prime labeling of a graph $G = (V, E)$ is an injective function $f: V(G) \to \{k, k+1, \dots, k+|V|-1\}; k \geq 2$, a function $f^+: E(G) \to N$every edges of G which induced by $f^+(pq) = \text{gcd}(f(p)f(q)), \forall e = pq \in E(G)$ such that $\text{gcd}(f(p)f(q)) = 1$. A graph $G = (V, E)$ admits a k-prime labeling which is called a k-prime graph.
- Definition 2.2[3]: The Cartesian products of two graphs P_m and K_2 obtained by the ladder graph L_m,Where $P_m$is a path graph with $'m'$ vertices and K_2 is complete graph adjacent with 2 vertices.
- Definition 2.3[4]: A Quadrilateral snake Q_n graph is achieved from a path $\{u_i | i = 1,2, \dots, n\}$ by joining u_i and u_{i+1} with two new vertices v_i and w_i correspondingly and then attaching with w_i and v_i.
- Note: - In this article we take k −PL instead of k −prime labelling

III. NOTE ON k-PRIME LABELING OF CERTAIN GRAPHS

- Theorem 3.1: *The graph* $P_m \odot 3K_1$ *is a* k *−prime graph, for all* $n \geq 2$

Proof: Let G be a $V(P_m \odot 3K_1)$ graph. Note that
$|V(G)| = \{v_i, u_i, r_i, x_i | 1 \leq i \leq m\}$ and
$|E(G)| = \{e_i = (v_i, v_{i+1}) | 1 \leq i \leq m\} \cup$
$\{e_i' = (v_i, u_i) | 1 \leq i \leq m\} \cup$
$\{e_i'' = (v_i, r_i) | 1 \leq i \leq m\}| \cup$
$\{e_i''' = (v_i, x_i) | 1 \leq i \leq m\}$
be the vertices and edges of the
given graph $G = (P_m \odot 3K_1)$ respectively.
Define $f: V(G) \to \{k + k + 1, \dots, k + 4m - 1\}$ by
$f(u_i) = k + 4i - 4; i \in \{1,2, \dots, m\}$
$f(v_i) = k + 4i - 3; i \in \{1,2, \dots, m\}$
$f(r_i) = k + 4i - 2;\ i \in \{1,2, \dots, m\}$
$f(x_i) = k + 4i - 1; i \in \{1,2, \dots, m\}$
It is clear that the induced function $f^+: E(G) \to N$ defined by $f^+(t_i, v_i) = \text{gcd}(f(t_i), f(v_i)\} = 1; 1 \leq i \leq m$ Where $t_i \in \{u_i, w_i, x_i | 1 \leq i \leq m\}$
$f^+(v_i, v_{i+1}) = \text{gcd}(f(v_i), f(v_{i+1})) = 1;\ \ 1 \leq i \leq m$
Then it is clear that the $G = P_m \odot 3k_1$
admits a k −PL, hence the graph is k −prime graph.

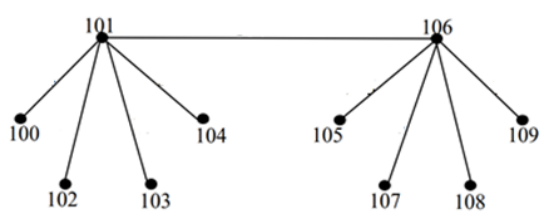
- Example 3.2: k −PL of the path graph $P_3 \odot 3k_1$ for $k = 50$



- Theorem 3.3: *The graph $P_m \odot 4k_1$ is a $k-$prime graph*

Proof: Let G be a $V(P_m \odot 4k_1)$ graph. Note that $|V(G)| = \{v_i, u_i, n_i, x_i, y_i | 1 \le i \le m-1\}$ and
$|E(G)|$ ={e_i=(v_i,v_(i+1))│1≤i≤m-1} ∪
$\{e_i' = (v_i, u_i) | 1 \le i \le m\}$ ∪
$\{e_i'' = (v_i, n_i) | 1 \le i \le m\}$ ∪
$\{e_i''' = (v_i, x_i) | 1 \le i \le m\}$ ∪
$\{e_i''' = (v_i, y_i) | 1 \le i \le m\}$
be the vertices and edges of the given graph $G = P_m \odot 4K_1$ respectively.
Define $f: V(G) \to \{k, k+1, k+2, \dots, k+5m-1\}$
$f(u_i) = k + 5i - 5; i \in \{1,2,\dots,m\}$,
$f(v_i) = k + 5i - 4; i \in \{1,2,\dots,m\}$,
$f(n_i) = k + 5i - 3;\ i \in \{1,2,\dots,m\}$
$f(x_i) = k + 5i - 2; i \in \{1,2,\dots,m\}$,
$f(y_i) = k + 5i - 1; i \in \{1,2,\dots,m\}$
It is note that the induced function $f^+: E(G) \to N$ of edges is well-precise by
$f^+(s_i, v_i) = \gcd(f(s_i), f(v_i)\} = 1; 1 \le i \le m$
Where $s_i \in \{u_i, n_i, x_i, y_i | 1 \le i \le m\}$
$f^+(v_i, v_{i+1}) = \gcd(f(v_i), f(v_{i+1})) = 1; 1 \le i \le m$
Here strongly note that the $G = P_m \odot 4k_1$ admits a $k-$ PL, hence the graph is $k-$prime graph.

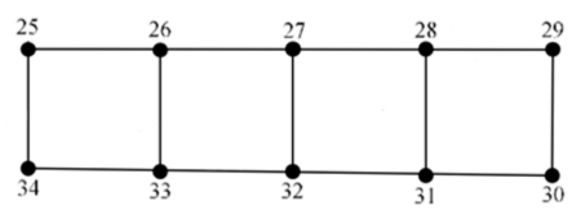
- Example 3.4: $k-$PL of the $P_2 \odot 4K_1$ for $k = 100$



- Theorem 3.5: *Any ladder L_m is a $k-$prime graph, for all $m \ge 2$*

Proof: Let G be any Ladder graph L_m. Note that $|V(G)| = \{u_i, v_i | 1 \le i \le m\}$ and
$|E(G)| = \{u_i v_i, v_i w_i, w_i m_i, m_i n_i | i = 1,2\}$ ∪
$\{u_i n_j, v_i m_j, w_i w_j, m_i v_j, n_i u_j | i = 1, j = 2\}$
be the cardinality of vertices and edges of $G = L_m$ respectively.
Define $f: V(G) \to \{k, k+d, k+2d, \dots, k+5m-3\}$ by
For $\forall i \in \{1,2,\dots,(n-3)\}$
$f(u_i) = k + 5i - 5$
$f(v_i) = k + 5i - 4$;
$f(w_i) = k + 5i - 3$;
$f(m_i) = k + 5i - 2$;
$f(n_i) = k + 5i - 1$;
By the definition of $k-$PL the induced function $f^+: E(G) \to N$ of edges is archived by
$f^+(r, s) = \gcd(f(r), f(s)\} = 1$
Where $r \in \{u_i, v_i, w_i, m_i | i = 1,2\}$ and
$s \in \{v_i, w_i, m_i, n_i | \ i = 1,2\}$
Now for $\forall i = 1, j = 2$, we have
$f^+(u_i, n_j)$ =g.c.d.$\left(f(u_i), f(n_j)\right) = 1$,
$f^+(v_i, m_j)$ = g.c.d. .$\left(f(v_i), f(m_j)\right) = 1$,
$f^+(w_i, w_j)$ =g.c.d.$\left(f(w_i), f(w_j)\right) = 1$,
$f^+(m_i, v_j)$ =g.c.d.$\left(f(m_i), f(v_j)\right) = 1$,
$f^+(n_i, u_j)$ =g.c.d. .$\left(f(n_i), f(u_j)\right) = 1$,
Then the following graph $G = L_m$ admits a $k-$PL, hence the graph is $k-$prime graph.

- Example 3.6: $k-$PL of Ladder graph L_m for $k = 25$.

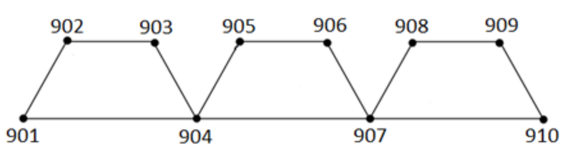


Theorem 3.7: *Any Quadrilateral snake Q_m is a $k-$prime graph*

Proof: Let $V(Q_m) = \{u_i, v_i, w_i | 1 \le i \le m\}$ and
$E(L_m) = \{e_i = (u_i, u_{i+1}) | 1 \le i \le m-1\}$ ∪
$\{e_i' = (u_i, v_i) | 1 \le i \le m-1\}$ ∪
$\{e_i'' = (u_{i+1}, w_i) | 1 \le i \le m-1\}$ ∪
$\{e_i''' = (v_i, w_i) | 1 \le i \le m-1\}$
be the vertices and edges of Q_m.
Define $f: V(Q_m) \to \{k, k+1, k+2, \dots, k+7m-7\}$ by
For $\forall i \in \{1,2,\dots,(n-3)\}$
$f(u_i) = k + 3i - 3;\ i \in \{1,2,\dots,m\}$
$f(v_i) = k + 3i - 2;\ i \in \{1,2,\dots,m-1\}$

$f(w_i) = k + 3i - 1; i \in \{1,2,\ldots,m-1\}$
According to the induced function $f^+: E(G) \to N$ of edges is well-defined by
$f^+(u_i, v_i) = \gcd(f(u_i), f(v_i)\} = 1,$
$i \in \{1,2,\ldots,m-1\}$
$f^+(v_i, w_i) = \gcd(f(v_i), f(w_i)\} = 1,$
$i \in \{1,2,\ldots,m-1\}$
$f^+(w_i, u_i) = \gcd(f(w_i), f(u_i)\} = 1,$
$i \in \{1,2,\ldots,m-1\}$
$f^+(u_i, u_{i+1}) = \gcd(f(u_i), f(u_{i+1})\} = 1,$
$i \in \{1,2,\ldots,m-1\}$
From above discussion clear that the Quadrilateral snake graph Q_m admits a k −PL, hence the graph is k −prime graph.

- Example 3.6: k −PL of Ladder graph Q_m for $k = 901$



CONCLUSION

From the above article we examined some lot of acyclic graphs such that the Ladder graph, Quadrilateral snake graph, the graph $P_m \odot 4k_1$, the graph $P_m \odot 3K_1$ those all are k −prime graph.

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