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Unsteady Flow of a Dusty Viscous Liquid in an Elliptic Tube

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Abstract- The purpose of this paper is to analyses theoretically the problem of the unsteady flow of a dusty viscous liquid in aelliptic tube. The results are obtained in terms of mathieu functions.

I. INTRODUCTION

Saffmann 1962 discussed the stability of laminar flow of a dusty gas in which the dust particles are uniformly distributed. He assumed that the dust particles are uniform in size and shape the bulk concentration of the dust is very small to be neglected. On the other hand the density of the dust material is large compared to gas density so that mass concentration of dust is an appreciable fraction of unity. Mechael (1965) investigated the Kelvi-Helmlutzinstablilty of the dusty gas. Michael and Miller (1966) have discussed the motion induced in the dusty gas in two cases. When the plane more, paralled to itself (1) in simple harmonic motion (ii) impulsively from rest with uniform velocity. Michael &Norcy (1961) have considered the problem of the motion of a dusty gas contained between two co-axial cylinders which start to rotate impulsively form rest. Rao (1969) discussed unsteady laminar flow of a dusty viscous liquid under the influence of exponential pressure gradient through a circular cylinder.

P. D. Verma & A. K. Mathur considered dusty viscous liquid flow in circular tube. In the present paper we have considered dusty viscous liquid flow in elliptic tube. Initially the fluid and the dust particles are at rest. At $T = 0$ a constant pressure gradient is impressed on the system. The change in the velocity profiles with line in determined.

II. EQUATION OF MOTION

The equation of motion for a dusty vi scous liquid are (Seffman 1962)

$$\frac{\partial u}{\partial t}(\vec{u}.\Delta)\vec{u} = -\frac{1}{\ell}\Delta p + v\Delta^2 \mathrm{u} + \frac{KN}{e}(\vec{v} - \vec{u}) \quad (1.1)$$

$$m\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}.\Delta\vec{v})\right] = K(\vec{u} - \vec{v}) \quad (1.2)$$

$$div\vec{u} = 0 \quad (1.3)$$

$$\frac{\partial N}{\partial t} + div(N\vec{v}) = 0 \quad (1.4)$$

In these equations $\vec{u}.\vec{v}$are velocities of fluid and dust particles respectively.p the fluid pressure, $m$the mass of a dust particles, Kthe strokes resistance Coeff, which for spherical particles of radii r is 6πμr,μ being the viscosity of the fluid, N the number density of the dust particles t the time, ϱ the fluid density and $v = \left(\frac{\mu}{\varrho}\right)$. The Kinematic viscosity of the fluid.

III. FORMATION OF THE PROBLEM

In the elliptic cylinder of Semi major axis a we considers the origin on the axis of the cylinder, Z axis along the axis of the cylinder. The motion of the dusty viscous liquid is considered along the axis of the cylinder under the influence of a constant pressure gradient.

In the present case velocity components of fluid are

$$u_x = 0, u_y = 0 \& u_z = u_z(x, y, t) \quad (2.1a)$$

and that of the dust particles are

$$v_x = 0, v_y = 0 \& v_z = v_z(x, y, t) \quad (2.1b)$$

Further for $N = No$a constants, Equ. (1.4) is satisfied throughout motion.

In view of (2.1) Equation (1.2) & (1.3) becomes

$$\frac{\partial u_z}{\partial_t} = -\frac{1}{\varrho}\frac{\partial p}{\partial z} + \gamma\left[\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2}\right] + \frac{KN_\circ}{p}(v_z - u_z) \quad (2.2)$$

$$m\frac{\partial v_z}{\partial t} = K(u_z - v_z) \quad (2.3)$$

We introduce the following non dimensionless quantities,

$$\left.\begin{array}{c} X = \frac{x}{a},\ Y = \frac{y}{a},\ Z = \frac{z}{a}, \bar{p} = \frac{pa^2}{\varrho v^2}, T = \frac{tv}{a^2}, u = \frac{u_z a}{v}, \\ v = \frac{v_z a}{v},\ \beta = \frac{\ell}{r} = \frac{N_\circ K a^2}{\varrho v},\ \varrho = \frac{N_\circ m}{\varrho},\ r = \frac{vm}{ka^2} \end{array}\right\} \quad (2.4)$$

So equations (2.2) and (2.3) transform to

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \left(\frac{\partial^2 u}{\partial X^2} + \frac{\partial^2 u}{\partial Y^2}\right) + \beta(v - u) \quad (2.5)$$

$$r\frac{\partial v}{\partial t} = u - v \quad (2.6)$$

The applied gradient is a constant say C_1, so that

$$-\frac{\partial p}{\partial z} = C_1 \quad (2.7)$$

Eliminating $v$from (2.5) & (2.6) and putting the value of pressure gradient form (2.7), we get

$$r\frac{\partial^2 u}{\partial T^2} + (\ell + 1)\frac{\partial u}{\partial T} - r\frac{\partial}{\partial T}\left(\frac{\partial^2 u}{\partial X^2} + \frac{\partial^2 u}{\partial Y^2}\right) = C_1 + \frac{\partial^2 u}{\partial X^2} + \frac{\partial^2 u}{\partial Y^2} \quad (2.8)$$

Now we introduce elliptic co-ordinate (ξ, η) defined by

$$x + iy = c\cosh(\xi + i\eta) \quad (2.9)$$

We get

$$\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} = \frac{C^2}{2}(\cosh 2\xi - \cos 2\eta) \times \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad (2.10)$$

So equ (2.8) transform to

$$r\frac{\partial^2 u}{\partial T^2} + (\ell + 1)\frac{\partial u}{\partial T} - \frac{2r}{C^2}\frac{\partial}{\partial T}\left(\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2}\right) \times \frac{1}{(\cosh\ 2\xi - \cos\ 2\eta)}$$

$$= C_1 + \frac{2}{C^2(\cosh\ 2\xi - \cos\ 2\eta)}\left(\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2}\right)$$

or $\left[r\frac{\partial^2 u}{\partial T^2}(\ell + 1)\frac{\partial u}{\partial T} - C_1\right]\frac{C^2}{2}(\cosh 2\xi - \cos 2\eta) = r\frac{\partial}{\partial T}\left[\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2}\right] + \left(\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2}\right) \quad (2.11)$

IV. BOUNDRY CONDITIONS

$u = 0$ for $\xi = \xi, 0 \leq \eta \leq 2\Pi$

$u = finite$for $\xi = 0$for$T > 0$ (3.1)

$T = 0$

V. SOLUTION OF PROBLEM

Multiply equation (2.11) by $C_{\ell 2n}(\xi, q_{2n,m})C_{\ell 2n}(\eta, q_{2n,m})d\eta d\xi$ and integrate within the limits o to $\xi_\circ$ and 0 to 2π where $q_{2n,m}$are the roots of the equations

$$C_{\ell 2n}(\xi_\circ, q) = 0 \quad (4.1)$$

We get

$$\frac{C^2}{2}\left[r\frac{\partial^2 \bar{u}}{\partial T^2} + (\ell + 1)\frac{\partial \bar{u}}{\partial T} - \bar{C}_1\right] = -2r\frac{\partial \bar{u}}{\partial T}q_{2n,m} - q_{2n,m}\bar{u}$$

or $r\frac{d^2 \bar{u}}{dT^2} + \left(\ell + 1 + 4r\frac{q_{2n,m}}{C^2}\right)\frac{d\bar{u}}{dT} + 4\frac{q_{2n,m}}{C^2}\bar{u} = \bar{C}_1 \quad (4.2)$

where

$$\bar{u} = \int_\circ^{\xi_\circ}\int_\circ^{2\eta} u(\cosh 2\xi - \cos 2\eta) \times C_{e2n}(\xi, q_{2n,m})C_{e2n}(\eta, q_{2n,m})d\eta d\xi \quad (4.3)$$

and

$$\bar{\zeta} = \int_\circ^{\xi_\circ}\int_\circ^{2\eta} C_1(\cosh 2\xi - \cos 2\eta)C_{e2n}(\xi, q_{2n,m})C_{e2n}(\eta, q_{2n,m})d\xi d\eta \quad (4.4)$$

Now multiply equation (4.4)bye^{-ST} and intregratingwith in the limits 0 to ∞ and putting

$$\bar{u}_L = \int_\circ^\infty e^{-ST}\,\bar{u}dT \quad (4.5)$$

We get

$$\bar{\mathrm{U}}_L\, rS^2 + \left(\ell + 1 + 4r\frac{q_{2n,m}}{C^2}\right)S\,\bar{\mathrm{U}}_L + 4\frac{q_{2n,m}}{C^2}\bar{\mathrm{U}}_L = \frac{\bar{C}_1}{S} \quad (4.6)$$

or
$$\left.\begin{array}{c} \bar{u}_L = \frac{C_1}{S\left[rS^2 + \left(\ell + 1 + \frac{4rq_{2n,m}}{C^2}\right)S + \frac{4q_{2n,m}}{C^2}\right]} \\ \bar{u}_L = \frac{\bar{C}_1}{rS(\alpha - \beta)}\left[\frac{1}{(S-\alpha)} - \frac{1}{(S-\beta)}\right] \end{array}\right\} \quad (4.7)$$

where $\alpha, \beta$are roots of the equation

$$rS^2 + S\left(\ell + 1 + 4r\frac{q_{2n,m}}{C^2}\right) + 4\frac{q_{2n,m}}{C^2} = 0 \quad (4.8)$$

By Laplace inverse transformation, we get

$$\bar{u} = \frac{\bar{C}_1}{r(\alpha - \beta)}\left\{\frac{(1 - e^{\alpha T})}{\alpha} - \frac{(1 - e^{\beta T})}{\beta}\right\} \quad (4.9)$$

Now

$$\bar{C}_1 = \int_\alpha^{\xi_\circ}\int_\circ^{2\eta} C_1 C_{e2n}(\xi, q_{2n,m})C_{e2n}(\eta, q_{2n,m})(\cosh 2\xi - \cos 2\eta)d\xi d\eta$$

$$= C \int_{\circ}^{\xi_\circ} C_{e2n}\left(\xi, q_{2n,m}\right)[2A_{\circ}^{2n} \cosh 2\xi - A_2^{2n}]d\xi$$

By inversion theorem of Mathieu functions by R. K. Gupta (1964)

$$u = \sum_{n=0}^{\infty} \frac{\sum_{n=1}^{\infty} \varsigma \int_{e2n}^{\xi_\circ}(\xi, q_{2n,m})[2A_{\circ}^{2n} \cosh 2\xi - A_2^{2n}]d\xi\, C_{e2n}(\xi, q_{2n,m}) C_{e2n}(\eta, q_{2n,m})}{\Pi \int_{\circ}^{\xi_\circ} C_{e2n}^2(\xi, q_{2n,m})[\cosh 2\xi - \Theta_{2n,m}]d\xi}$$

$$\times \frac{1}{r(\alpha-\beta)}\left[\frac{(1-e^{\alpha T})}{\alpha} - \frac{(1-e^{\beta T})}{\beta}\right] \qquad (4.10)$$

To solve Equ (2.6) multiply equation by e^{-ST} and intregratewith in the limits 0 to ∞ and putting

$$v_L = \int_{\circ}^{\infty} e^{-ST}\, v dT \qquad (4.11)$$

We get

$$rSv_L = u_L - v_L \qquad (4.12a)$$

or $$v_L = \frac{u_L}{1+rS} \qquad (4.12b)$$

Now multiply Equ (4.12a) and (4.12b) by $C_{e2n}(\xi, q_{2n,m}) C_{e2n}(\eta, q_{2n,m})$and integrate$\xi_\circ$and $\eta$with in the limits 0 to 2Π, we get

$$\bar{v}_L = \frac{\bar{u}_L}{1+rS} \qquad (4.13)$$

Substituting the value of $\bar{u}_2$ from (4.7) and (4.13), we get

$$\bar{v}_L = \frac{\bar{C}_1}{rS(\alpha-\beta)(1+rS)}\left[\frac{1}{S-\alpha} - \frac{1}{S-\beta}\right] \qquad (4.14)$$

By Laplace inverse transform we get

$$\bar{v} = \frac{\bar{C}_1}{r}\left[\left\{-\frac{1}{\alpha} + \frac{e^{\alpha T}}{\alpha(1+\alpha r)} - \frac{re^{-T/r}}{(1+\alpha r)}\right\} + \frac{1}{\beta} - \frac{e^{-\beta T}}{\beta(1+\beta r)} + \frac{re^{-T/r}}{(1+\beta r)}\right] \qquad (4.15)$$

By inverse transform of mathieu functions, we get

$$\bar{v} = \sum_{n=0}^{\infty} \frac{\sum_{i=0}^{n} \frac{C_1}{r} C_{e2n}(\xi, q_{2n,m}) C_{e2n}(\eta, q_{2n,m}) \int_{\circ}^{\xi_\circ} C_{e2n}(\xi, q_{2n,m})[2A_{\circ}^{2n} \cosh 2\xi - A_2^{2n}]d\xi}{\Pi \int_{\circ}^{\xi_\circ} C_{e2n}^2(\xi, q_{2n,m})[\cosh 2\xi - \Theta q_{2n,m}]d\xi}$$

$$\times \left[-\frac{1}{\alpha} + \frac{e^{\alpha T}}{\alpha(1+\alpha r)} - \frac{re^{-T/r}}{(1+\alpha r)} + \frac{1}{\beta} - \frac{e^{\beta T}}{\beta(1+\beta r)} + \frac{re^{-T/r}}{1+\beta r}\right]. \qquad (4.16)$$

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