On Class (N+K, MBQ) Operators

A.M. NYONGESA[1], WANJALA VICTOR[2], WANJALA WILBERFORCE[3]
[1, 2, 3] *Department of Mathematics and computing, Kibabii University, Bungoma, Kenya.*

Abstract- In this paper, we introduce the class of (n+k, mBQ) operators acting on a complex Hilbert space H. An operator if $T \in B(H)$ is said to belong to class (n+k, mBQ) if $T^{*2m}T^{2(n+k)}$ commutes with $(T^{*m}T^{n+k})^2$ equivalently $[T^{*2m}T^{2(n+k)}, (T^{*m}T^{n+k})^2] = 0$, for a positive integers n and m. We investigate algebraic properties that this class enjoys. have. We analyze the relation of this class to (n+k, m)-powerclass (Q) operators.

Indexed Terms- (n,m)-power Class (Q),Normal ,Binormal operators , n-power class (Q), (BQ) operators , (n+k,mBQ) operators.

I. INTRODUCTION

H denotes Hilbert space over the complex field throughout this paper while B(H) the Banach algebra of all bounded linear algebra on an infinite dimensional separable Hilbert space H. A bounded linear operator T is said to be in class (Q) if $T^{*2}T^2 = (T^*T)^2$ (2), (n,m)-power class (Q) if $T^{*2m}T^{2n} = (T^{*m}T^n)^2$for positive integers n and m (1). The class of (Q) operators was expanded to many classes such as the following classes, almost class (Q) (4), n-power class (Q) (2), (α, β)-class (Q) (3) , K* Quasi-n- Class (Q) Operators (6) and quasi M class (Q) . An operator $T \in B(H)$ is said to belong to class (BQ) if $T^{*2}T^2(T^*T)^2 = (T^*T)^2T^{*2}T^2$ (5), $T \in B(H)$ is said to belong to class (n+k, mBQ) if $T^{*2m}T^{2(n+k)}(T^{*m}T^{n+k})^2 = (T^{*m}T^{n+k})^2T^{*2m}T^{2(n+k)}$. A conjugation on aHilbert space H is an anti-linear operator C from Hilbert space H onto itself that satisfies Cξ, Cζi= hζ, ξifor every ξ, ζ ∈H and $C^2 = I$. An operator T is said to be complex symmetric if $T = CT^*C$.

II. MAIN RESULTS

Theorem 1. Let $T \in B(H)$ be such that $T \in$ (n+k, mBQ), then the following holds for (n+k, mBQ);

i. λT for any real λ
ii. Any $S \in B(H)$ that is unitarily equivalent to T.
iii. The restriction T M to any closed subspace M of H.

Proof. i. The proof is straight forward. ii. Let $S \in B(H)$ be unitarily equivalent to T, then there exists a unitary operator U
∈B(H) with

$S^{n+k} = U^*T^{n+k}U$and $S^{*m} = U^*T^{*m}U$ for non-negative integers n and m. Since $T \in$ (n+k, mBQ), we have;
$T^{*2m}T^{2(n+k)}(T^{*m}T^{n+k})^2 = (T^{*m}T^{n+k})^2T^{*2m}T^{2(n+k)}$, hence

$S^{*2m}S^{2(n+k)}(S^{*m}S^{n+k})^2 = UT^{*2m}U^*UT^{2(n+k)}U^*(UT^{*m}U^*UT^{n+k}U^*)^2$
$= UT^{*2m}U^*U^*T^{2(n+k)}U^*UT^{*m}U^*UT^{*m}U^*UT^{n+k}U^*UT^{n+k}U^*$
$= UT^{*2m}T^{2(n+k)}(T^{*m}T^{n+k})^2U^*$
$= U(T^{*m}T^{n+k})^2T^{*2m}T^{2(n+k)}U^*$
and
$(S^{*m}S^{n+k})^2S^{*2m}S^{2(n+k)} = (UT^{*m}U^*UT^{n+k}U^*)^2UT^{*2m}U^*UT^{2(n+k)}U^*$
$= UT^{*m}U^*UT^{n+k}U^*UT^{*m}U^*UT^{n+k}U^*UT^{*2m}U^*UT^{2(n+k)}U^*$
$= UT^{*m}T^{n+k}T^{*m}T^{n+k}T^{*2m}T^{*2m}U^*$
$=U(T^{*m}T^{n+k})^2T^{*2m}T^{2(n+k)}U^*$
Thus, S is unitarily equivalent to T.

iv. If T is in class (n, mBQ), then;
$T^{*2m}T^{2(n+k)}(T^{*m}T^{n+k})^2 = (T^{*m}T^{n+k})^2T^{*2m}T^{2(n+k)}$.
Hence;
$(T/M)^{*2m}(T/M)^{2(n+k)}\{(T/M)^{*m}(T/M)^{n+k}\}^2$
$=(T/M)^{*2m}(T/M)^{2(n+k)}\{(T/M)^{*m}(T/M)^{n+k}\}^2$
$= (T^{*2m}/M)(T^{2(n+k)}/M)\{(T^{*m}/M)(T^{n+k}/M)\}\{(T^{*m}/M)(T^{n+k}/M)\}$
$= \{(T^{*m}T^{n+k})^2/M\}\{T^{*2m}T^{2(n+k)}/M\}$
$= \{(T^{*m}/M)(T^{n+k}/M)\}^2(T/M)^{*2m}(T/M)^{2(n+k)}$

Thus $T/M \in$ (n+k, mBQ).

- Theorem 2. If $T \in B(H)$ is in (n+k,m)-power Class (Q), then $T \in$ (n+k,mBQ).

Proof. If $T \in$(n+k)-power (Q), then $T^{*2m}T^{2(n+k)} = (T^{*m}T^{n+k})^2$

post multiplying both sides by $T^{*2m}T^{2(n+k)}$;

$T^{*2m}T^{2(n+k)}\, T^{*2m}T^{2(n+k)} = (T^{*m}T^{n+k})^2 T^{*2m}T^{2(n+k)}$

$T^{*2m}T^{2(n+k)}\, T^{*m}T^{n+k}T^{*m}T^{n+k} = (T^{*m}T^{n+k})^2 T^{*2m}T^{2(n+k)}$

$T^{*2m}T^{2(n+k)} (T^{*m}T^{n+k})^2 = (T^{*m}T^{n+k})^2 T^{*2m}T^{2(n+k)}$.

Theorem 3. Let $S \in$ (n+k, mBQ) and $T \in$ (n+k, mBQ). If both S and T are doubly commuting, then ST is in (n+k, mBQ).

Proof.

$(ST)^{*2m}(ST)^{2(n+k)}\, ((ST)^{*m}(ST)^{n+k})^2$

$=S^{*2m}T^{*2m}S^{2(n+k)}T^{2(n+k)}\, ((ST)^{*m}(ST)^{n+k})((ST)^{*m}(ST)^{n+k}) = S^{*2m}T^{*2m}S^{2(n+k)}T^{2(n+k)}\, ((S^{*m}T^{*m})(S^{n+k}T^{n+k}))\, ((S^{*m}T^{*m})(S^{n+k}T^{n+k}))$

$=S^{*2m}T^{*2m}S^{2(n+k)}T^{2(n+k)}S^{*m}T^{*m}S^{n+k}T^{n+k}S^{*m}T^{*m}S^{n+k}T^{n+k}S^{*m}T^{*m}S^{n+k}T^{n+k}$

$=T^{*2m}T^{2(n+k)}S^{*2m}S^{2(n+k)}S^{*m}S^{n+k}S^{*m}S^{n+k}T^{*m}T^{n+k}T^{*m}T^{n+k}$

$=T^{*2m}T^{2(n+k)}\, S^{*2m}S^{2(n+k)}\, (S^{*m}S^{n+k})^2 T^{*m}T^{n+k}T^{*m}T^{n+k}$

$=T^{*2m}T^{2(n+k)}\, (S^{*m}S^{n+k})^2 S^{*2m}S^{2(n+k)}\, T^{*m}T^{n+k}T^{*m}T^{n+k}$ (Since $S \in$ (n+k, mBQ)).

$= (S^{*m}S^{n+k})^2 T^{*2m}T^{2(n+k)}\, T^{*m}T^{n+k}T^{*m}T^{n+k}S^{*2m}S^{2(n+k)}$

$=(S^{*m}S^{n+k})^2 T^{*2m}T^{2(n+k)}\, (T^{*m}T^{n+k})^2 S^{*2m}S^{2(n+k)}$

$=(S^{*m}S^{n+k})^2 (T^{*m}T^{n+k})^2 T^{*2m}T^{2(n+k)}\, S^{*2m}S^{2(n+k)}$ (Since $T \in$ (n+k, mBQ)).

$=((S^{*m}S^{n+k})\,(T^{*m}T^{n+k}))^2 T^{*2m}S^{*2m}T^{2(n+k)}\, S^{2m}$

$=((S^{*m}T^{*m})\,(S^{n+k}T^{n+k}))^2 S^{*2m}T^{*2m}S^{2(n+k)}\, T^{2(n+k)}$

$= ((ST)^{*m}(ST)^{n+k})^2 (ST)^{*2m}(ST)^{2(n+k)}$

Thus $ST \in$ (n+k, mBQ).

- Theorem 4. Let $T \in B(H)$ be a class (n+k, mBQ) operator such that $T = CT^{*}C$ for positive integers n and m with C being aconjugation on H. If C is such that it commutes with $T^{*2m}T^{2(n+k)}$ and $(T^{*m}T^{n+k})^2$, then T is an (n+k,m)-power class (Q) operator.

Proof. Let $T \in$ (n+k, mBQ) and complex symmetric, then we have; $T^{*2m}T^{2(n+k)} (T^{*m}T^{n+k})^2 = (T^{*m}T^{n+k})^2 T^{*2m}T^{2(n+k)}$ and $T = CT^{*}C$.

hence;

$T^{*2m}T^{2(n+k)} (T^{*m}T^{n+k})^2 = (T^{*m}T^{n+k})^2 T^{*2m}T^{2(n+k)}$

$T^{*2m}T^{2(n+k)}CT^{n+k}CCT^{*m}CCT^{n+k}CCT^{*m}C$

$=(T^{*m}T^{n+k})^2 CT^{n+k}CCT^{*m}CCT^{n+k}CCT^{*m}C$.

$T^{*2m}T^{2(n+k)}CT^{n+k}T^{*m}T^{n+k}T^{*m}C$

$=(T^{*m}T^{n+k})^2 CT^{n+k}T^{*m}T^{n+k}T^{*m}C$

$T^{*2m}T^{2(n+k)}CT^{2(n+k)}T^{*2m}C = (T^{*m}T^{n+k})^2 CT^{*m}T^{n+k}T^{*m}T^{n+k}C$

$T^{*2m}T^{2(n+k)}\, CT^{*2m}T^{2(n+k)}\, C = (T^{*m}T^{n+k})^2 C\, (T^{*m}T^{n+k})^2 C$.

C commutes with $T^{*2m}T^{2(n+k)}$ and $(T^{*m}T^{n+k})^2$ hence we obtain;

$T^{*2m}T^{2(n+k)}\, T^{*2m}T^{2(n+k)} = (T^{*m}T^{n+k})^2 (T^{*m}T^{n+k})^2$.

which implies;

$T^{*2m}T^{2(n+k)} = (T^{*m}T^{n+k})^2$ and thus $T \in$ (n+k,m)-power class(Q).

Theorem 5. Let $T \in B(H)$ be (n+k-1, m)-class (Q) operator, if T is a complex symmetric operator such that C commutes with $(T^{*m}T)^2$ for a positive integerm, then T is an (n+k, m)-power class (Q) operator.

Proof. With T being complex symmetric and (n-1, m)-class (Q), we have;

$T = CT^{*}C$ and $T^{*2m}T^{2(n+k-1)} = (T^{*m}T^{n+k-1})^2$.

We obtain;

$T^{*2m}T^{2(n+k-1)}\, T^{2} = (T^{*m}T^{n+k-1})^2 T^{2}$.

hence;

$T^{*2m}T^{2(n+k)} = (T^{*m}T^{n+k-1})^2 T^{2}$.

$T^{*2m}T^{2(n+k)} = T^{*2m}T^{2(n+k-1)}\, T^{2} = T^{2(n+k-1)}\, T^{*2m}T^{2}$

$T^{*2m}T^{2(n+k)} = T^{2(n+k-1)}\, T^{*m}T^{*m}TT=$

$T^{2(n+k-1)}\, CTCCTCCT^{*m}CCT^{*m}C = T^{2(n+k-1)}\, CTTT^{*m}T^{*m}C$.

$=T^{*2m}T^{2(n+k)} = T^{2(n+k-1)}\, CT^{2}T^{*2m}C = T^{2(n+k-1)}\, C\,(T^{*m}T)^2 C$

Since C commutes with $(T^{*m}T)^2$ we obtain; $T^{*2m}T^{2(n+k)} = T^{2(n+k-1)}\, (T^{*m}T)^2 CC = T^{2(n+k-1)}\, T^{*2m}T^{2}CC = T^{2(n+k-1)}\, T^{2}T^{*2m}CC = T^{*2m}T^{2(n+k)} = (T^{*m}T^{n+k})^2$

Hence T is n+k-power class (Q).

REFERENCES

[1] Eiman H. Abood and Mustafa A. Al-loz., On some generalizations of (n,m)-normal powers operators on Hilbert space , Journal of progressive research in mathematics , Vol. 7(3) (2016) , 2395 -0218.

[2] Jibril, A.A.S.., On Operators for which T ∗2(T)2 = (T ∗T)2, international mathematical forum, vol. 5(46) ,2255-2262.

[3] S. Paramesh, D. Hemalatha and V.J. Nirmala., A study on n-power class (Q) operators, international research journal of engineering and technology, vol.6(1), (2019) , 2395-0056.

[4] wanjala Victor and A.M. Nyongesa., On (α, β)-class (Q) Operators, international journal of mathematics and its applications, vol. 9(2) (2021), 111-113.

[5] Wanjala Victor and Beatrice AdhiamboObiero., On almost class (Q) and class (M,n) operators ,international journal of mathematics and its applications ,vol . 9(2) (2021), 115-118.

[6] Wanjala Victor and Beatrice AdhiamboObiero., On class (BQ) operators, Global Journal of advanced research, vol. 8 (4) (2021), 118-120.

[7] Wanjala Victor and Peter KiptooRutto., K* Quasi-n- Class (Q) Operators, international journal of mathematics and its applications, vol. 9 (2) (2021), 189-193.