On N Quasi (m, p+k)-Power D-Operator Operators

WANJALA VICTOR[1], WANJALA WILBERFORCE[2]
[1, 2] Maasai Mara University, Narok-Kenya.

Abstract- In this paper, we introduce the class of (p+k)-D-Operator acting on the usual Hilbert space H over the complex plane. An operator T is said to be an (p+k)-D-Operator if $T(T^{*2}(T^D)^{2(p+k)}) = N(T^*(T^D)^{p+k)})^{2}T$ for positive integers p and k and for N which is a bounded operator on H. We investigate the basic behavior of this class of operator.

Indexed Terms- Normal operators, D-Operator, Almost Class (Q), quasi -class (Q) operators, N quasi D-operator.

I. INTRODUCTION

H denotes the separable complex Hilbert space in this paper, while B (H) is the usual Banach algebra of all bounded linear operators on H. Let T ∈ B(H), Drazin inverse of T is an operator T^D∈ B(H), such that $TT^D = T^DT$, $T^D = T^DTT^D$ and $T^{k+1}T^D = T^k$ provided it exists. An operator T ∈ B(H) is said to be D-Operator if $T^{*2}(T^D)^2 = (T^*T^D)^2$ (1), class (Q) if $T^{*2}T^2 = (T^*T)^2$ (4), M Quasi class (Q) if $T(T^{*2}T^2) = M(T^*T)^2T$ (5), Quasi class (Q) if $T(T^{*2}T^2) = (T^*T)^2T$, N quasi-D- Operator if $T(T^{*2}(T^D)^2) = N(T^*T^D)^2T$, for a bounded linear operator N. Let T = ξ + iζ, with ξ = Re(T) = $\frac{T^D + T^*}{2}$ and ζ = Im(T) = $\frac{T^D - T^*}{2i}$. We shall simply denote $U^2 = (T^*T^D)^2$ and $V^2 = T^{*2}(T^D)^2$ where C and V are non-negative definite.

II. MAIN RESULTS

Definition 1. Let T ∈ B(H) be Drazin invertible, an operator T is called (m, p+k)-D-Operator if T (T ∗2m (T D)2(p+k)) = N (T ∗m (T D) p+k)2T for positive integers p and k and N which is a bounded operator on H.

Theorem 2. Let T ∈ B(H) and let V commute with ξ and ζ such that V 2T = NU 2T,
it follows that T is an (m, p+k)-D-Operator.
Proof. We recall that T = ξ + iζ, with ξ = Re(T) = (T D+ T *)/2 and ζ = Im(T) = (T D - T *)/2i and
U 2 = (T ∗m (T D) p+k)2 and V 2 = T ∗2m(T D)2(p+k).
Since V ξ = ξV and Uζ = ζU, we have;
V 2ξ = ξV 2 and U 2ζ = ζU 2, thus
V 2T + V 2(T)∗ = TV 2 + (T)∗V 2
V 2T − V 2(T)∗ = TV 2 − (T)∗V 2 implies;
TV 2 = V 2T. Hence;
T (T ∗2m(T D)2(p+k)) = ((T ∗m (T ∗m (T D) p+k) (T D) p+k) T
= (T ∗m (T D) p+k)2T.
TU 2 = NU 2T implies;
T (T ∗2m(T D)2(p+k)) = N ((T ∗m (T ∗m (T D) p+k) (T D) p+k) T
T (T ∗2m(T D)2(p+k)) = N (T ∗m (T D) p+k)2T
Hence T is an (m, p+k)-D-Operator.

Proposition 3. Let T ∈ B(H) be a (m, p+k)-D-operator where V 2ξ = 1/NξV 2 and V 2ζ = 1/NζV 2,
then T is an (m, p+k)-D-Operator.
Proof. V 2ξ = 1/NξV 2 and V 2ζ = 1/NζV 2 implies
V 2(ξ + iζ) = 1/N(ξ + iζ) V 2
V 2T = 1/N TV 2
(T ∗m (T ∗m (T D) p+k) (T D) p+k) T = 1/NT (T ∗m (T ∗m (T D) p+k) (T D) p+k)

T (T ∗m (T ∗m (T D) p+k) (T D) p+k) = N (T ∗m (T ∗m (T D) p+k) (T D) p+k) T

= N (T ∗m (T D) p+k)2 (Since T is a (m,p+k)- D-operator).
Hence T is an (m,p+k)-D-Operator.

Theorem 4. Let Tα and Tβ be two N Quasi- (m,p+k)-D-Operators from B (H, H) such that (Tα D) p+k Tβ ∗2m = (Tβ D) p+k Tα ∗2m = Tα ∗2m(Tβ D)2(p+k) = Tβ ∗2m(Tα D)2(p+k) = 0, then Tα + Tβ is an N Quasi-(p+k)-D-Operator.
Proof. Since Tα and Tβ are N Quasi- (p+k)-D-Operator, we have ;
(Tα + Tβ)[(Tα + Tβ)∗2m(Tα D + Tβ D)2(p+k)] = (Tα + Tβ)[(Tα ∗2m + Tβ ∗2m)((Tα D)2(p+k) + (Tβ D)2(p+k))

=(Tα + Tβ)[Tβ ∗2m(Tα D)2(p+k) + Tβ ∗2m(Tβ D)2(p+k) + Tα ∗2m(Tα D)2(p+k) + Tα ∗2m(Tβ D)2(p+k)]
= (Tα + Tβ)[Tβ ∗2m(Tβ D)2(p+k) + Tα ∗2m(Tα D)2(p+k)] since Tβ ∗2m(Tα D)2(p+k) = Tα ∗2m(Tα D)2(p+k) = 0
= (Tα + Tβ)[Tβ ∗2m(Tβ D)2(p+k) + Tα ∗2m(Tα D)2(p+k)]
= TαTα ∗2m(Tα D)2(p+k) + TβTβ ∗2m(Tβ D)2(p+k) since TαTβ ∗2m(Tβ D)2(p+k) = TβTα ∗2m(Tα D)2(p+k) = 0
= N(Tα ∗2m(Tα D)2(p+k))Tα + N(Tβ ∗2m(Tβ D)2(p+k))Tβ
= N(Tα ∗m (Tα D)p+k)2Tα + N(Tβ ∗m (Tβ D)p+k)2Tβ
Thus Tα + Tβ is an (m, p+k)-D-Operator.

Theorem 5. Let Tα and Tβ be two N Quasi- (p+k)-D-Operator from B (H, H) such that (Tα D) p+kTβ ∗2m = (T β D) p+k
Tα ∗2m = Tα ∗2m(Tβ D)2(p+k) = Tβ ∗2m(Tα D)2(p+k) = 0, then Tα - Tβ is an N Quasi-(m, p+k)-D-Operator.
Proof. The proof follows from Theorem 4 above.

Theorem 6. Let Tα and Tβ be two N Quasi -(p+k)-D-Operators, then Tα Tβ is an N Quasi
-(p+k)-D-Operator provided Tα Tβ = Tβ Tα and (Tα D)2(p+k) Tβ ∗2m = Tβ ∗2m(Tα D)2(p+k).

Proof. Since Tα and Tβ are N Quasi-(p+k)-D-Operator, we have ;
(TαTβ)[(TαTβ)∗2m((TαTβ)D)2(p+k)]
=(TαTβ)[(Tα ∗2mTβ ∗2m)(Tα DTβ D)2(p+k)]
= (TαTβ)[(Tβ ∗2mTα ∗2m)(Tα DTβ D)2(p+k)]
= Tα(TβTα ∗2m)(Tβ ∗2m(Tα D)2(p+k))(Tβ D)2(p+k)
= Tα(Tα ∗2mTβ)(Tβ ∗2m(Tα D)2(p+k))(Tβ D)2(p+k)
= TαTα ∗2mTβ(Tα D)2(p+k)Tβ ∗2m(Tβ D)2(p+k)
= TαTα ∗2m(Tα D)2(p+k)TβTβ ∗2m(Tβ D)2(p+k)
= N(Tα ∗2m(Tα D)2(p+k))TαN(Tβ ∗2m(Tβ D)2(p+k))Tβ
= N(Tα ∗2m((Tα D)2(p+k)Tα)(Tβ ∗2m(Tβ D)2(p+k)))Tβ)
=N(Tα ∗2m(Tα D)2(p+k)Tβ ∗2mTα(Tβ D)2(p+k)Tβ)
=N(Tα ∗2mTβ ∗2m(Tα D)2(p+k)(Tβ D)2(p+k)TαTβ)
=N[(TαTβ)∗2m(Tα DTβ D)2(p+k)(TαTβ)]
=N[(TαTβ)∗2m((TαTβ)D))2(p+k)(TαTβ)]
=N[(TαTβ)∗m ((TαTβ)D)p+k]2(TαTβ)

Thus Tα Tβ is N Quasi -(m, p+k)-D-Operator.

Theorem 7. Power of N Quasi D-operator is similarly N Quasi- (m, p+k)-D-Operator.
Proof. We first show that the result holds for some p = 1, then we have;
T (T ∗2m(T D)2(p+k)) = N (T ∗m (T D) p+k)2T……………………. (0.1)
Suppose the result holds for p=n, we have;
[T (T ∗2m(T D)2(p+k))] n = (N (T ∗m (T D) p+k)2T) n …………………. (0.2)
we then prove that the result is true for p=n+1. We have;
[T (T ∗2m(T D)2(p+k))] n+1 = (N (T ∗m (T D) p+k)2T) n+1 ………………… (0.3)
[T (T ∗2m(T D)2(p+k))] n+1 = [NT (T ∗2m(T D)2(p+k))] n [NT (T ∗2m(T D)2(p+k))] ………. (0.4)
= [N (T ∗m (T D)p+k)2T] n [N (T ∗m (T D)p+k)2T] by (0.1) and (0.2)
[T (T ∗2m(T D)2(p+k))] n+1 = [N (T ∗m (T D)p+k)2T] n+1 ………………………. (0.5)
Hence the proof as required.

REFERENCES

[1] Abood and Kadhim. Some properties of D-operator. Iraqi Journal of science, vol.61(12), (2020), 3366-3371.
[2] Campbell, S.R. and Meyer, C.D. 1991. Generalized inverses of linear transformations, pitman, New York.
[3] Dana, M and Yousef, R.., on the classes of D-normal operators and D-quasi normal operators on Hilbert space, operators and matrices, vol.12 (2) (2018), 465-487.
[4] Jibril, A.A.S.., On Operators for which T ∗2(T)2 = (T ∗T)2, international mathematical forum, vol. 5(46) ,2255-2262.
[5] V. Revathi and P. Maheswari Naik., A study on properties of M quasi- class (Q) operator, international Journal of advance research, ideas and innovations in technology, vol. 5 (2).