Analysis Of Malaria Coupled Transmission Equation Using Lie Groups Method

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Abstract- The aim of this paper is to analyse malaria coupled transmission equation which incorporates both the susceptible and infectious class of humans and female Anopheles mosquitoes in order to obtain exact solution. We restructure the infectious class of humans and mosquito as a function of partial differential operator for malaria coupled transmission equation. We determine the prolongations infinitesimally for infectious classes of humans and female Anopheles mosquito for malaria coupled transmission equation. We further determine the infectious class of humans and female anopheles' mosquito in order to obtain exact solutions for malaria coupled transmission equation using Lie group's method

Indexed Terms- Malaria Coupled Transmission Equation, Lie Groups Method

I. INTRODUCTION

Malaria was termed as the ancient and deadliest infectious disease resulted to death of millions of people worldwide, about 40% of the global population in 104 countries have high risk of being infected by malaria [1]. There were 243 million (60%) malaria cases reported in which pregnant women accounts for 25% and children below five years accounts for 20 % of the global malaria cases [2]. In addition, malaria was also called massive killer because annually it results to deaths of nearly 750000 to 882000 in which children below five years accounts for 67% of malaria deaths. More so, many antimalarial drugs were becoming less effective against the parasite also with absence of specific malaria vaccine the parasites were still evolving drug resistance mechanism [3].

The transformation of the pathogen by the parasite depends on two interacting components, the human population and mosquito where female anopheles mosquito of class protozoan parasites of genus plasmodium with subclass; plasmodium vivax, plasmodium malariae, plasmodium ovale, plasmodium falciparum and plasmodium knowles. Therefore, plasmodium falciparum was mostly deadly among the five classes in which the parasite was transformed from infected mosquito to human, also non-infected mosquito gets infected by sucking blood from infected human thus the pathogen multiplies, develop in mouth and salivary glands result to fever, fatigue, headache, vomiting and death [4].

Malaria vaccine still unclear known though clinical trials were being conducted. The current malaria drugs have become ineffective against the parasite creating a siren that malaria disease was a threat to human beings, emerging and re-emerging of malaria in an eradicated

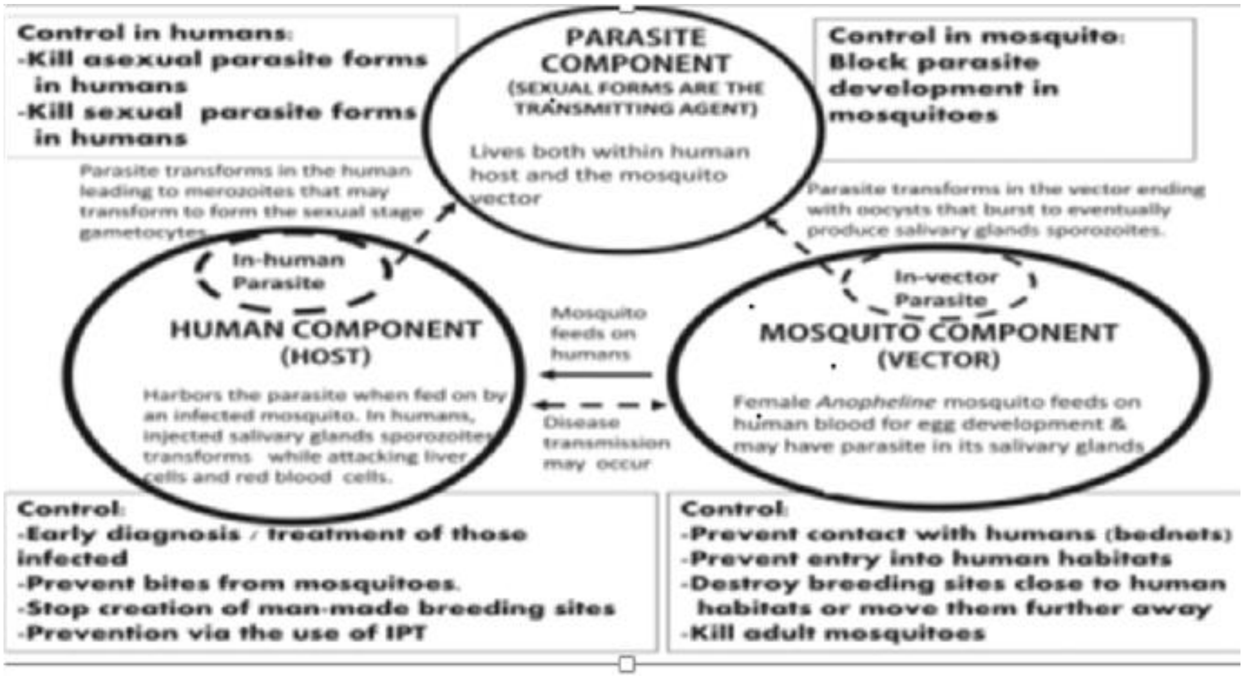


Figure 1.1: Shows the relationship btween the mosquito bites, human and parasite [5]

areas indicates the need to address the control for malaria transmission. Therefore, the use of ACT, chloroquine and malaria control programs historically have helped in lowering the number of malaria cases in an endemic areas, resulting to significant reduction of malaria worldwide. However, to understand the process of introduction, formation and transmission of new strain of malaria parasite to human population from the infected mosquito, there was need to speed up research on establishment of new drugs as a critical priority in war against malaria disease. Many of older and current anti-malarial drugs were losing effectiveness due to mutation by the parasite, spraying Dichlorodiphenyl trichloroethane, repellants and prevention like use of mosquito bed nets which was still the only hope [6]. Malaria have gained its prominence in an increase or fluctuation of temperature due to global warming.

Thus, climate change affects malaria prevalence that affects the vector and parasite life cycle[7].

Malaria was geographically spread in various regions of the world including Europe, middle East, Asia, South America and Africa. Major deaths caused by malaria occur in sub-Saharan Africa. In sub-Saharan Africa thirty countries including Kenya records about 90% of malaria deaths globally, sadly the disease claim lives of children below five years in every 30 seconds and over 2500 most children lost their lives as a result of malaria disease worldwide [8]. In addition, breeding sites for vectors to lay eggs in water reservoirs either in places due to human activities or natural, these breeding grounds have a significant role in the spread and transmission of malaria. Humans and mosquitoes have different lifestyle, this is essential since humans and vectors are able to transmit agents that cause malaria because they reside from different habitats with different lifestyle in terms of social-economic factors such as demographic increase, fertility rate, misdiagnosis, premature mortality and consideration of spatial dynamics of mosquito population was an essential aspects.

Temperature and rainfall acts independently in maturation of mosquito larvae habitats but during rainfall, malaria prevalence increases thus temperature affects developmental stages of malaria parasites, change in rainfall and temperature influence dynamics of malaria hence warmer temperature leads to faster maturation of mosquito larvae. The optimum temperature for development of the parasite life cycle and malaria transmission was 25^0c.

The epidemiological structure on infectious disease and transmission in human population was based on fundamental process. Therefore, when the pathogen was identified in a certain community, the individuals were divided into different categories depending on the density of the parasite within them and type of infection [9]. These structures were given by the standard representation as S-E-I-R simplified as;

i. Susceptible (S) represent the fraction of human population,
ii. Exposed class (E) represent individuals infected by the pathogen but are incapable of transmitting the infection to others during latent period,
iii. Infectious class (I) represent the infected individuals due to interaction with the susceptible class,
iv. Recovered (R), those individuals who recovers from an infection. Therefore, variation of compartment structure depends with type of the infectious disease for instance the infected class (I) may not recover and die but recovered individuals (R) with temporary or permanent immunity were subdivide further the epidemiological components to eight classes as; S-I, S-I-S, S-E-I, S-E-I-S, S-I-R, S-I-R-S, S-E-I-R and S-E-I-R-S. These class S-E-I-R-S can recover from the disease but when the temporary immunity was decreased an individual becomes susceptible.

Therefore, first series of paper was published using mathematical model framework to study malaria transmission according to [10]. Thus, our focus was engineered by the eradication [11]. Mathematical models on malaria transmission dynamics provides important insight in understanding malaria disease.

$$\begin{aligned} \frac{dx}{d\epsilon} &= ae_1\left(\frac{N-x}{N}\right)y - d_1x \\ \frac{dy}{d\epsilon} &= ae_2\left(\frac{M-y}{N}\right)x - d_2y \end{aligned} \tag{1}$$

The model provides a basic idea on how transmission of malaria occurs. we can analyse the malaria coupled transmission equation by a new technique to get an exact solution that will assist in controlling the transmission of malaria. The malaria model have been formulated and can be analysed by Lie groups methods to obtain an exact solution.

Basic reproduction number was defined as the population of infected humans which a rise from a class of infected human in an otherwise fully susceptible population or the population of infected mosquitoes which a rise from a class infected mosquito after a generation of the parasite given by $R_0 = \frac{ae_1}{d_1}\frac{M}{N}\frac{ae_2}{d_2}$, which was explicitly computed from the model parameter to measure maximum potential of malaria transmission that if $R_0 > 0$ we have an outbreak but $R_0 < 0$ there's no outbreak which was biologically inaccurate [12]. Recent research incorporates other factors like latency for modification of the model that uses numerical analysis by descretization and probability approaches according to [13]. Malaria coupled transmission equation was termed as first order non-linear ordinary differential equation. Therefore, Lie groups method were used by first, discussing the applications of the method where systems of differential equations were not directly open to solutions. Analysis of differential equation can be of dynamical systems, numerical, and symmetry analysis. Application of Lie groups methods to simplify systems of differential equations had become an essential tool in analysing the non-linear ordinary differential problems. These new approach was introduced by Sophus Lie the Norwegian mathematician in [14] where he used Lie groups of transformations to study symmetry structure of differential equations with respect to their solutions.

II. ANALYSIS OF THE MALARIA COUPLED TRANSMISSION EQUATION MALARIA

coupled transmission equation represented as;

$$\begin{aligned} \frac{dx}{d\epsilon} &= ae_1 \quad \frac{N-x}{N}\Big) y - d_1x \\ \frac{dy}{d\epsilon} &= ae_2 \quad \frac{M-y}{N}\Big) x - d_2y \end{aligned}$$

where the following variables x, y, N, M, $N-x$, $M-y$, a, e_1, e_2, d_1, d_2 and $\epsilon > 0$

represented; dependent variable for population of infectious class of humans, dependent variable for population of infectious class of female anopheles mosquito, total population of human, total population of female Anopheles mosquitoes, population of susceptible humans that were children below five years and pregnant women, population of susceptible mosquitoes, constant number of mosquito bites, probability that a bite from infective mosquitoes will cause an infection of a susceptible human, probability that a bite from a susceptible mosquito on infective human will cause infection to non-infective mosquito, the natural death rate of infectious class of humans, the natural death rate of infectious class of mosquito and time respectively. Therefore, describing the original work of Ross-Macdonald model in which various researchers were modifying from original work.

III. INFINITESIMAL GENERATOR FOR MALARIA COUPLED TRANSMISSION EQUATION

The one parameter Lie group of infinitesimal transformation in *,x,y* and the corresponding infinitesimal generators for malaria coupled transmission equation;

$$G^{(0)} = \tau(\epsilon, x, y)\partial_\epsilon + \xi(\epsilon, x, y)\partial_x + \eta(\epsilon, x, y)\partial_y \cdot \tag{2}$$

IV. PROLONGATION OF INFINITESIMAL GENERATOR

The vector field $X \times U$ of the prolongation where we determine all the possible coefficients functions for τ,ξ and η so that the corresponding one-parameter group expansion of ε was a symmetry group of Malaria coupled transmission equation to the known first order prolongation[15]. Therefore, the prolongation of infinitesimal generators for malaria coupled transmission equation was of the partial differential equation form and when applied to ordinary differential equation it works similarly as in partial differential equation [16].

$$G^{(1)} = \tau\partial_\epsilon + \xi\partial_x + \eta\partial_y + \xi^{(1)}\partial_{x'} + \eta^{(1)}\partial_{y'} \tag{3}$$

V. DETERMINING EQUATIONS FOR MALARIA COUPLED TRANSMISSION EQUATION

The prolonged infinitesimal generator work similarly as partial differential equation of the product which was similar to ordinary differential equation using implicit differentiation, law of commutativity and differentiation by product rule [17][18]. Therefore, for τ,ξ,η were functions of *,x,y* that results to the set of determining equations which simplify to the following equations;

$$G^{(1)} = \tau\partial_\epsilon + \xi\partial_x + \eta\partial_y + \xi^{(1)}\partial_{x'} + \eta^{(1)}\partial_{y'} \cdot \{\frac{dx}{d\epsilon} = ae_1\left(\frac{N-x}{N}\right)y - d_1x\} = 0$$
$$G^{(1)} = \tau\partial_\epsilon + \xi\partial_x + \eta\partial_y + \xi^{(1)}\partial_{x'} + \eta^{(1)}\partial_{y'} \cdot \{\frac{dy}{d\epsilon} = ae_2\left(\frac{M-y}{N}\right)x - d_2y\} = 0$$
(4)

Similarly;

$$G^{(1)} = \tau\partial_\epsilon + \xi\partial_x + \eta\partial_y + \xi^{(1)}\partial_{x'} + \eta^{(1)}\partial_{y'} \cdot \{x' - ae_1y + ae_1\frac{xy}{N} + d_1x\} = 0$$
$$G^{(1)} = \tau\partial_\epsilon + \xi\partial_x + \eta\partial_y + \xi^{(1)}\partial_{x'} + \eta^{(1)}\partial_{y'} \cdot \{y' - ae_2x\frac{M}{N} + ae_2\frac{xy}{N} + d_2y\} = 0$$
(5)

The first expansion;

$$\tau\partial_\epsilon \cdot \{x' - ae_1y + ae_1\frac{xy}{N} + d_1x\} = \tau\{x''\}$$
$$\xi\partial_x \cdot \{x' - ae_1y + ae_1\frac{xy}{N} + d_1x\} = \xi\{x'' + ae_1\frac{y}{N} + d_1\}$$
$$\eta\partial_y \cdot \{x' - ae_1y + ae_1\frac{xy}{N} + d_1x\} = \eta\{-ae_1 + ae_1\frac{x}{N}\}$$
$$\xi^{(1)}\partial_{x'} \cdot \{x' - ae_1y + ae_1\frac{xy}{N} + d_1x\} = \xi^{(1)}\{1\}$$
$$\eta^{(1)}\partial_{y'} \cdot \{x' - ae_1y + ae_1\frac{xy}{N} + d_1x\} = \eta^{(1)} \cdot \{0\}$$

The second expansion;

$$\tau\partial_\epsilon \cdot \{y' - ae_2x\frac{M}{N} + ae_2\frac{xy}{N} + d_2y\} = \tau \cdot \{y''\}$$
$$\xi\partial_x \cdot \{y' - ae_2x\frac{M}{N} + ae_2\frac{xy}{N} + d_2y\} = \xi \cdot \{-ae_2\frac{M}{N} + ae_2\frac{y}{N}\}$$
$$\eta\partial_y \cdot \{y' - ae_2x\frac{M}{N} + ae_2\frac{xy}{N} + d_2y\} = \eta \cdot \{y'' + ae_2\frac{x}{N} + d_2\}$$
$$\xi^{(1)}\partial_{x'} \cdot \{y' - ae_2x\frac{M}{N} + ae_2\frac{xy}{N} + d_2y\} = \xi^{(1)} \cdot \{0\}$$
$$\eta^{(1)}\partial_{y'} \cdot \{y' - ae_2x\frac{M}{N} + ae_2\frac{xy}{N} + d_2y\} = \eta^{(1)} \cdot \{1\}$$
(7)

Summing the first equation;

$$\tau \cdot \{x''\} + \xi \cdot \{x'' + ae_1\frac{y}{N} + d_1\} + \eta \cdot \{-ae_1 + ae_1\frac{x}{N}\} + \xi^{(1)} \cdot \{1\} + \eta^{(1)} \cdot \{0\} = 0 \tag{8}$$

Summing the second equation;

$$\tau \cdot \{y''\} + \xi \cdot \{-ae_2\frac{M}{N} + ae_2\frac{y}{N}\} + \eta \cdot \{y'' + ae_2\frac{x}{N} + d_2\} + \xi^{(1)} \cdot \{0\} + \eta^{(1)} \cdot \{1\} = 0 \tag{9}$$

Combining the first and second equations, we get;

$$\tau \cdot \{x''\} + \xi \cdot \{x'' + ae_1\frac{y}{N} + d_1\} + \eta \cdot \{-ae_1 + ae_1\frac{x}{N}\} + \xi^{(1)} \cdot \{1\} + \eta^{(1)} \cdot \{0\} + \tau \cdot \{y''\} + \xi \cdot \{-ae_2\frac{M}{N} + ae_2\frac{y}{N}\} + \eta \cdot \{y'' + ae_2\frac{x}{N} + d_2\} + \xi^{(1)} \cdot \{0\} + \eta^{(1)} \cdot \{1\} = 0 \tag{10}$$

Substituting the total derivatives $\xi^{(1)}$ and $\eta^{(1)}$ in the above equations,

$$\xi^{(1)} = \xi_\epsilon x'\xi_x + y'\xi_y - x'(\tau_\epsilon + x'\tau_x + y'\tau_\epsilon) \tag{11}$$

$$\eta^{(1)} = (\xi_\epsilon)^2 + 2y'\xi_\epsilon\xi_y + x''\xi_x + y''\xi_y - (y'\xi_y)^2 - 2x''\tau_\epsilon + (x')^2\{(\xi_y)^2 - 2(\tau_\epsilon\tau_x + y'\tau_x\tau_y)\} - 2y'x''\tau_y - x'\{-2\xi_\epsilon\xi_x - 2y'\xi_x\xi_y + (\tau_\epsilon)^2 + 2y'\tau_x\tau_y + 3x''\tau_x + y''\tau_y + (x'\tau_\epsilon)^2\} \tag{12}$$

We obtain the following expansion;

$$\tau \cdot \{x''\} + \xi \cdot \{x'' + ae_1\frac{y}{N} + d_1\} + \eta \cdot \{-ae_1 + ae_1\frac{x}{N}\} + \xi_\epsilon x'\xi_x + y'\xi_y - x'(\tau_\epsilon + x'\tau_x + y'\tau_\epsilon) \cdot \{1\} + (\xi_\epsilon)^2 + 2y'\xi_\epsilon\xi_y + x''\xi_x + y''\xi_y - (y'\xi_y)^2 - 2x''\tau_\epsilon + (x')^2\{(\xi_y)^2 - 2(\tau_\epsilon\tau_x + y'\tau_x\tau_y)\} - 2y'x''\tau_y - x'\{-2\xi_\epsilon\xi_x - 2y'\xi_x\xi_y + (\tau_\epsilon)^2 + 2y'\tau_x\tau_y + 3x''\tau_x + y''\tau_y + (x'\tau_\epsilon)^2\} \cdot \{0\} + \tau \cdot \{y''\} + \xi \cdot \{-ae_2\frac{M}{N} + ae_2\frac{y}{N}\} + \eta \cdot \{y'' + ae_2\frac{x}{N} + d_2\} + \xi_\epsilon x'\xi_x + y'\xi_y - x'(\tau_\epsilon + x'\tau_x + y'\tau_\epsilon) \cdot \{0\} + (\xi_\epsilon)^2 + 2y'\xi_\epsilon\xi_y + x''\xi_x + y''\xi_y - (y'\xi_y)^2 - 2x''\tau_\epsilon + (x')^2\{(\xi_y)^2 - 2(\tau_\epsilon\tau_x + y'\tau_x\tau_y)\} - 2y'x''\tau_y - x'\{-2\xi_\epsilon\xi_x - 2y'\xi_x\xi_y + (\tau_\epsilon)^2 + 2y'\tau_x\tau_y + 3x''\tau_x + y''\tau_y + (x'\tau_\epsilon)^2\} \cdot \{1\} = 0 \tag{13}$$

Similarly;

$$\tau \cdot \{x''\} + \xi \cdot \{x'' + ae_1\frac{y}{N} + d_1\} + \eta \cdot \{-ae_1 + ae_1\frac{x}{N}\} + \xi_\epsilon x'\xi_x + y'\xi_y - x'(\tau_\epsilon + x'\tau_x + y'\tau_\epsilon) + (\xi_\epsilon)^2 + 2y'\xi_\epsilon\xi_y + x''\xi_x + y''\xi_y - (y'\xi_y)^2 - 2x''\tau_\epsilon + (x')^2\{(\xi_y)^2 - 2(\tau_\epsilon\tau_x + y'\tau_x\tau_y)\} - 2y'x''\tau_y + \tau \cdot \{y''\} + \xi \cdot \{-ae_2\frac{M}{N} + ae_2\frac{y}{N}\} + \eta \cdot \{y'' + ae_2\frac{x}{N} + d_2\} + \xi_\epsilon x'\xi_x + y'\xi_y + (\xi_\epsilon)^2 + 2y'\xi_\epsilon\xi_y + x''\xi_x + y''\xi_y - (y'\xi_y)^2 - 2x''\tau_\epsilon + (x')^2\{(\xi_y)^2 - 2(\tau_\epsilon\tau_x + y'\tau_x\tau_y)\} - 2y'x''\tau_y - x'\{-2\xi_\epsilon\xi_x - 2y'\xi_x\xi_y + (\tau_\epsilon)^2 + 2y'\tau_x\tau_y + 3x''\tau_x + y''\tau_y + (x'\tau_\epsilon)^2\} = 0 \tag{14}$$

Again;

$$\tau \cdot \{x''\} + \xi\{x'' + ae_1\frac{y}{N} + d_1\} + \eta \cdot \{-ae_1 + ae_1\frac{x}{N}\} + x'\{\left(\frac{\partial \xi}{\partial \epsilon}\right)\left(\frac{\partial \xi}{\partial x}\right)\} + y'\left(\frac{\partial \xi}{\partial y}\right) - x'\left(\frac{\partial \tau}{\partial \epsilon}\right) - x'x'\left(\frac{\partial \tau}{\partial x}\right) - x'y'\left(\frac{\partial \tau}{\partial y}\right) + \tau\{y''\} + \eta \cdot \{y'' + ae_2\frac{x}{N} + d_2\} + \left(\frac{\partial \xi}{\partial \epsilon}\right)^2 + 2y'\left(\frac{\partial \xi}{\partial \epsilon}\right)\left(\frac{\partial \xi}{\partial y}\right) + x''\left(\frac{\partial \xi}{\partial x}\right) + y''\left(\frac{\partial \xi}{\partial y+}\right) + \left(\frac{y'\partial \xi}{\partial y}\right)^2 - 2x''\left(\frac{\partial \tau}{\partial \epsilon}\right) + (x')^2\left(\frac{\partial \tau}{\partial \epsilon}\right) - 2(x')^2\left(\frac{\partial \tau}{\partial \epsilon}\right)\left(\frac{\partial \tau}{\partial x}\right) - 2(x')^2y'\left(\frac{\partial \tau}{\partial x}\right)\left(\frac{\partial \tau}{\partial y}\right) - 2x''y'\left(\frac{\partial \tau}{\partial y}\right) + 2x'\left(\frac{\partial \xi}{\partial \epsilon}\right)\left(\frac{\partial \xi}{\partial x}\right) + 2x'y'\left(\frac{\partial \xi}{\partial x}\right)\left(\frac{\partial \xi}{\partial y}\right) - x'\left(\frac{\partial \tau}{\partial \epsilon}\right)^2 - 2x'y'\left(\frac{\partial \tau}{\partial \epsilon}\right)\left(\frac{\partial \tau}{\partial y}\right) - 3x'x'\left(\frac{\partial \tau}{\partial x}\right) - x'y''\left(\frac{\partial \tau}{\partial y}\right) - x'\left(\frac{y'\partial \tau}{\partial \epsilon}\right)^2 = 0 \tag{15}$$

VI. SOLUTIONS OF THE DETERMINING EQUATIONS

Computing the solutions of the determining equations for malaria coupled transmission equation by equating to zero, the coefficients of various monomials. The monomials were functions of partial derivatives or ordinary derivatives of the dependent variables equated to zero.

$$y'^2 \rightarrow \left(\frac{\partial \xi}{\partial\ y}\right)^2 = \left(\frac{\partial \tau}{\partial\ \epsilon}\right)^2 = 0$$

$$y'^1 \rightarrow \left(\frac{\partial \xi}{\partial\ y}\right) = 2\left(\frac{\partial \xi}{\partial\ \epsilon}\right)\left(\frac{\partial \xi}{\partial\ y}\right) = 0$$

(16)

Therefore, (16) was linear thus solvable using linear partial differential equation theory

$$
\begin{gathered}
y^{'0} \rightarrow \left(\frac{\partial \xi}{\partial \epsilon}\right)^2 = 0 \\
y^{''} \rightarrow \left(\frac{\partial \xi}{\partial y}\right)^2 = 0 \\
x^{'2} \rightarrow \left(\frac{\partial \xi}{\partial x}\right)^2 = 2\left(\frac{\partial \tau}{\partial \epsilon}\right)\left(\frac{\partial \tau}{\partial x}\right) = 0 \\
x^{''} \rightarrow 2\left(\frac{\partial \tau}{\partial \epsilon}\right) = \left(\frac{\partial \xi}{\partial x}\right) = 0 \\
x^{'1} \rightarrow \left(\frac{\partial \xi}{\partial \epsilon}\right)\left(\frac{\partial \xi}{\partial x}\right) = \left(\frac{\partial \tau}{\partial x}\right) = 2\left(\frac{\partial \xi}{\partial \epsilon}\right)\left(\frac{\partial \xi}{\partial x}\right) = \left(\frac{\partial \tau}{\partial \epsilon}\right)^2 = 0 \\
x^{'0} \rightarrow \left(\frac{\partial \xi}{\partial \epsilon}\right) = 0 \\
x^{'}x^{'} \rightarrow \left(\frac{\partial \tau}{\partial x}\right) = 0 \\
x^{'}x^{''} \rightarrow 3\left(\frac{\partial \tau}{\partial x}\right) = 0 \\
x^{'}y^{'} \rightarrow \left(\frac{\partial \tau}{\partial y}\right) = 2\left(\frac{\partial \xi}{\partial x}\right)\left(\frac{\partial \xi}{\partial y}\right) = 2\left(\frac{\partial \tau}{\partial \epsilon}\right)\left(\frac{\partial \tau}{\partial y}\right) = 0 \\
x^{''}y^{'} \rightarrow 2\left(\frac{\partial \tau}{\partial y}\right) = 0 \\
x^{'}y^{''} \rightarrow \left(\frac{\partial \tau}{\partial y}\right) = 0 \\
x^{'2}y^{'} \rightarrow 2\left(\frac{\partial \tau}{\partial x}\right)\left(\frac{\partial \tau}{\partial y}\right) = 0 \\
x^{'}y^{'2} \rightarrow \left(\frac{\partial \tau}{\partial \epsilon}\right)^2 = 0
\end{gathered}
\tag{17}
$$

Through inspection, we get the τ from above monomials but η was undetermined then we choose to be linear in ,*x*,*y* that is $\eta(\epsilon, x, y)$.

$$\tau = k_0 \tag{18}$$

$$\eta = k_o\epsilon + k_1x + k_2y + k_3 \tag{19}$$

The ξ is non characteristics, undetermined coefficient and computed by trial error method

as,

$$\left(\frac{\partial \xi}{\partial y}\right) - 2\left(\frac{\partial \xi}{\partial \epsilon}\right)\left(\frac{\partial \xi}{\partial y}\right) = 0 \tag{20}$$

Let

$$F(D, D') = F(x, y) = F(\epsilon, y) \tag{21}$$

Therefore, (26) is non-homogeneous linear equation;

$$F(D, D') = F(\epsilon, y) \tag{22}$$

Its solution was given by;

Z = C.F + P.I

$$
\begin{gathered}
Z = e^{ax}(k(y + mx)) \\
\left(\frac{\partial \xi}{\partial y}\right) - 2\left(\frac{\partial \xi}{\partial \epsilon}\right)\left(\frac{\partial \xi}{\partial y}\right) = 0 \\
\left(\frac{\partial \xi}{\partial y}\right) = D', \left(\frac{\partial \xi}{\partial \epsilon}\right) = D \\
\left(\frac{\partial}{\partial y}\right) - 2\left(\frac{\partial}{\partial \epsilon}\right)\left(\frac{\partial}{\partial y}\right)\xi = 0 \\
D' - 2D'D = 0, D = m, D' = 1
\end{gathered}
\tag{23}
$$

Thus, m=1/2

$$\xi = e^{ax}(k_1(y + m\epsilon))$$

$$\xi = e^{ax}(k_1(y + 1/2\epsilon)) = C.F,$$

$$P.I = 0, \tag{24}$$

$$\xi = e^{ax}(k_1(y + 1/2\epsilon)) + k_0 \tag{25}$$

Hence

,*a* = 0

$$\xi = e^{0x}(k_1(y + 1/2\epsilon)) + k_0 = C.F,$$

$$\xi = e^{0}(k_1(y + 1/2\epsilon)) + k_0 = C.F,$$

$$\xi = k_1(y + 1/2\epsilon) + k_0 \tag{26}$$

Therefore, we obtain the following;

$$
\begin{gathered}
\tau = k_0 \\
\xi = k_1(y + 1/2\epsilon) + k_0 \\
\eta = k_0\epsilon + k_1x + k_2y + k_3
\end{gathered}
\tag{27}
$$

VII. INFINITESIMAL TRANSFORMATIONS

Substituting equations (4.75), (4.76) and (4.77) in the infinitesimal generator and do some alignment to obtain the infinitesimal transformation, the one parameter groups G_j admitted by infinitesimal generators.

$$G^{(1)} = \tau\partial_\epsilon + \xi\partial_x + \eta\partial_y + \xi^{(1)}\partial_{x'} + \eta^{(1)}\partial_{y'}$$
$$G^{(1)} = \tau\partial_\epsilon + \xi\partial_x + \eta\partial_y \qquad (28)$$

Then through special alignment and inspection, we get the infinitesimal transformation as;

$$G_1 = \partial_\epsilon + \partial_x + \epsilon\partial_y$$
$$G_2 = (y + 1/2\epsilon)\partial_x + x\partial_y \qquad (29)$$

$G_3 = y\partial_y$
$G4 = \partial y$

The new generators G_5 and G_6 were obtained by Lie brackets

$$G_5 = \partial_\epsilon + \partial_y$$
$$G6 = \partial x + \partial y \qquad (30)$$

VIII. THE LIE BRACKETS (COMMUTATORS) FOR MALARIA COUPLED TRANSMISSION EQUATION

Let G_i and G_j be represented as $[G_i,G_j]$ where $i = j$ as $i = 1,2,3,...n$

$$[G_1, G_1] = (\partial_\epsilon + \epsilon\partial_y)(\partial_\epsilon + \epsilon\partial_y) - (\partial_\epsilon + \epsilon\partial_y)(\partial_\epsilon + \epsilon\partial_y) = \partial_\epsilon + \epsilon\partial_y = G_6$$
$$[G_1, G_2] = (\partial_\epsilon + \epsilon\partial_y)((y + 1/2\epsilon)\partial_x + x\partial_y) - ((y + 1/2\epsilon)\partial_x + x\partial_y)(\partial_\epsilon + \epsilon\partial_y) = \partial_y = G_4 \quad (4.71)$$
$$[G_1, G_3] = (\partial_\epsilon + \epsilon\partial_y)(y\partial_y) - (y\partial_y)(\partial_\epsilon + \epsilon\partial_y) = \partial_y = G_4$$
$$[G_1, G_4] = (\partial_\epsilon + \epsilon\partial_y)(\partial_y) - (\partial_y)(\partial_\epsilon + \epsilon\partial_y) = \partial_y = G_4$$
$$[G_1, G_5] = (\partial_\epsilon + \epsilon\partial_y)(\partial_\epsilon + \partial_y) - (\partial_\epsilon + \partial_y)(\partial_\epsilon + \epsilon\partial_y) = \partial_\epsilon + \partial_y = G$$
$$[G_1, G_6] = (\partial_\epsilon + \partial_x + \epsilon\partial_y)(\partial_x + \partial_y) - (\partial_x + \partial_y)(\partial_\epsilon + \partial_x + \epsilon\partial_y) = \partial_y = G_4$$
$$[G_2, G_1] = ((y + 1/2\epsilon)\partial_x + x\partial_y)(\partial_\epsilon + \epsilon\partial_y + \partial_x) - (\partial_\epsilon + \epsilon\partial_y + \partial_x)((y + 1/2\epsilon)\partial_x + x\partial_y) \quad (4.76)$$
$$= \partial_y = G_4$$
$$[G_2, G_2]$$
$$= ((y+1/2\epsilon)\partial_x + x\partial_y)((y+1/2\epsilon)\partial_x + x\partial_y) - ((y+1/2\epsilon)\partial_x + x\partial_y)((y+1/2\epsilon)\partial_x + x\partial_y) \quad (4.79$$
$$= \partial_x + \partial_y = G_6$$
$$[G_2, G_3] = ((y + 1/2\epsilon)\partial_x + x\partial_y)(y\partial_y) - (y\partial_y)((y + 1/2\epsilon)\partial_x + x\partial_y$$
$$= \partial_y = G_4$$
$$[G_2, G_4] = ((y + 1/2\epsilon)\partial_x + x\partial_y)(\partial_y) - (\partial_y)((y + 1/2\epsilon)\partial_x + x\partial_y \qquad (4.83)$$
$$= \partial_y = G_4$$
$$[G_2, G_6] = ((y + 1/2\epsilon)\partial_x + x\partial_y)(\partial_x + \partial_y) - (\partial_x + \partial_y)((y + 1/2\epsilon)\partial_x + x\partial_y)$$
$$= \partial_x + \partial_y = G_6$$
$$[G_3, G_1] = \partial_y = G_4$$
$$[G_3, G_2] = \partial_y = G_4$$
$$[G_3, G_3] = \partial_y = G_4$$
$$[G_3, G_4] = \partial_y = G_4$$
$$[G_3, G_5] = \partial_y = G_4$$
$$[G_3, G_6] = \partial_y = G_4$$
$$[G_4, G_1] = \partial_y = G_4$$
$$[G_4, G_2] = \partial_y = G_4$$
$$[G_4, G_3] = \partial_y = G_4$$
$$[G_4, G_4] = \partial_y = G_4$$
$$[G_4, G_5] = \partial_y = G_4$$
$$[G_4, G_6] = \partial_y = G_4$$
$$[G_5, G_1] = \partial_\epsilon + \partial_y = G_5$$
$$[G_5, G_2] = \partial_y = G_4$$
$$[G_5, G_3] = \partial_y = G_4$$
$$[G_5, G_4] = \partial_y = G_4$$
$$[G_5, G_5] = \partial_\epsilon + \partial_y = G_5$$
$$[G_5, G_6] = \partial_y = G_4$$
$$[G_6, G_1] = \partial_x + \partial_y = G_6$$
$$[G_6, G_2] = \partial_x + \partial_y = G_6$$
$$[G_6, G_3] = \partial_y = G_4$$
$$[G_6, G_4] = \partial_y = G_4$$
$$[G_6, G_5] = \partial_y = G_4$$
$$[G_6, G_6] = \partial_y = G_4 \qquad (31)$$

IX. THE LIE BRACKETS (COMMUTATORS) TABLE FOR MALARIA COUPLED TRANSMISSION EQUATION

$[G_i,G_j]$	G_1	G_2	G_3	G_4	G_5	G_6
G_1	G_5	G_4	G_4	G_4	G_5	G_4
G_2	G_4	G_6	G_4	G_4	G_4	G_6
G_3	G_4	G_4	G_4	G_4	G_4	G_4
G_4	G_4	G_4	G_4	0	G_4	G_4
G_5	G_5	G_4	G_4	G_4	G_5	G_4
G_6	G_6	G_6	G_4	G_4	G_4	G_6

X. INVARIANTS SOLUTION FOR MALARIA COUPLED TRANSMISSION EQUATION

Invariant solution is where a group transforms and maps a solution to itself

$$G = \tau\partial_\epsilon + \xi\partial_x + \eta\partial_y$$

$$G = \tau(\epsilon, x, y)\partial_\epsilon + \xi(\epsilon, x, y)\partial_x + \eta(\epsilon, x, y)\partial_y \quad (32)$$

The infinitesimal transformation and group transformation that a rise from all the generators for malaria coupled transmission equation were used to find the invariants solutions by method of characteristics.

$$G = \frac{d\epsilon}{\tau(\epsilon, x, y)} = \frac{dx}{\xi(\epsilon, x, y)} = \frac{dy}{\eta(\epsilon, x, y)} \quad (33)$$

Case 1: Invariant solution under transformation generated by the generator $G_1 = \partial_\epsilon + \partial_x + \epsilon\partial_y$,

have system of characteristics

$$\frac{d\epsilon}{1} = \frac{dx}{1} = \frac{dy}{\epsilon}$$

$$\int dx = \int d\epsilon$$

$$x + c_0 = \epsilon + c_1 \Rightarrow x = \epsilon + c_2 \quad (34)$$

Also;

$$\int dy = \int \epsilon dx$$

$$y + c_3 = \epsilon x + c_4 \Rightarrow y = \epsilon x + c_5$$
$$x = \epsilon + c_2, y = \epsilon x + c_5 \quad (35)$$

Case 2: Invariant solution under transformation generated by the generator $G_2 = (y + 1/2\epsilon)\partial_x + x\partial_y$, have system of characteristics

$$xdx - (y + 1/2\epsilon)dy = 0 ....(i) \quad (36)$$

Then by exactness;

$$\frac{\partial M}{\partial y} = x.......(ii)$$
$$\frac{\partial N}{\partial x} = (y + 1/2\epsilon).......(iii) \quad (37)$$

$$\frac{\partial M}{\partial y} = 0$$
$$\frac{\partial N}{\partial x} = 0$$

We integrate w.r.t.x by keeping the y constant for equation (iii)

$$xy + x\frac{\epsilon}{2} + \phi(y).........(iv) \quad (38)$$

Differentiating w.r.t.y and equate the results to equation (ii) for satisfaction

$x + \varphi 0(y) = x \Rightarrow \varphi 0(y) = 0 \Rightarrow \quad \varphi 0(y) = 0dy$

$\varphi(y) = c_6$...............(v) (39)

Then we substitute (v) into equation (iv) to obtain x and y

$$x = \frac{-c_6}{y + \frac{\epsilon}{2}}$$
$$y = -\frac{c_6}{x} - \frac{\epsilon}{2} \quad (40)$$

Case 3: Invariant solution under transformation generated by the generator $G_3 = y\partial_y$, have system of characteristics

$$\frac{d\epsilon}{0} = \frac{dx}{0} = \frac{dy}{y}$$

$$\epsilon = \Phi, x = c_7, y = c_8 \quad (41)$$

Case 4: Invariant solution under transformation generated by the generator $G_4 = \partial_y$, have system of characteristics

$$\frac{d\epsilon}{0} = \frac{dx}{0} = \frac{dy}{1}$$
$$\epsilon = \Phi(x, y), x = c_9, y = \Psi(\epsilon, x) \quad (42)$$

Case 5: Invariant solution under transformation generated by the generator $G_5 = \partial_\epsilon + \partial_y$ have system of characteristics $\frac{d\epsilon}{1} = \frac{dx}{0} = \frac{dy}{1}$

$$\epsilon = c_{10}, x = c_{11}, y = c_{12} \quad (43)$$

Case 6: Invariant solution under transformation generated by the generator $G_6 = \partial_x + \partial_y$, have system of characteristics

$$\frac{d\epsilon}{0} = \frac{dx}{1} = \frac{dy}{1}$$

$$\epsilon = c_{13}, x = \Phi(\epsilon, y), y = \Psi(\epsilon, x) \qquad (44)$$

Where $c_1, c_2, c_3, \ldots, c_{13}$ were arbitrary constants.

XI. TABLE OF INVARIANTS SOLUTION FOR MALARIA COUPLED TRANSMISSION EQUATION

Generator G_j	Invariants solution
G_1	$x = \epsilon + c_2, y = \epsilon x + c_5$
G_2	$x = \frac{-c_6}{y+\frac{\epsilon}{2}}, y = -\frac{c_6}{x} - \frac{\epsilon}{2}$
G_3	$x = c_7, y = c_8$
G_4	$x = c_9, y = \Psi(\epsilon, x)$
G_5	$x = c11, y = c12$
G_6	$x = \Phi(\epsilon, y), y = \Psi(\epsilon, x)$

XII. LIE GROUPS ADMITTED BY MALARIA COUPLED TRANSMISSION EQUATION

The one-parameter G_j admitted by infinitesimal generators G_1, G_2, G_3, G_4, G_5 and G_6 are determined by solving the corresponding Lie equation which yields the groups below;

$$G_1 = \partial_\epsilon + \partial_x + \epsilon\partial_y; G_1 = X_1(\epsilon, x, y) \to (\epsilon + \varepsilon, x + \varepsilon, y + \epsilon\varepsilon)$$
$$G_2 = (y + 1/2\epsilon)\partial_x + x\partial_y; G_2 = X_2(\epsilon, x, y) \to X_2(\epsilon, x + (y + 1/2\epsilon)\varepsilon, y + x\varepsilon)$$
$$G_3 = y\partial_y; G_3 = X_3(\epsilon, x, y) \to X_3(\epsilon, x, y + y\varepsilon)$$
$$G_4 = \partial_y; G_4 = X_4(\epsilon, x, y) \to X_4(\epsilon, x, y + \varepsilon)$$
$$G_5 = \partial_\epsilon + \partial_y; G_5 = X_5(\epsilon, x, y) \to X_5(\epsilon + \varepsilon, x, y + \varepsilon)$$
$$G_6 = \partial_x + \partial_y; G_6 = X_6(\epsilon, x, y) \to X_6(\epsilon, x + \varepsilon, y + \varepsilon)$$
(45)

XIII. GROUP TRANSFORMATION OF SOLUTIONS FOR MALARIA COUPLED TRANSMISSION EQUATION

Let G_j be a symmetry group say $X = f(\epsilon, x, y) = X^\alpha(\epsilon, x, y)$ was a solution of the generalised

Malaria coupled transmission equation in which these functions were also solutions

$$X^1 = f(\epsilon - \varepsilon, x - \varepsilon, y - \epsilon\varepsilon)$$
$$X^2 = f(\epsilon - \varepsilon, x - (y + 1/2\epsilon)\varepsilon, y - x\varepsilon)$$
$$X^3 = f(\epsilon, x, ye^{-\varepsilon})$$
$$X^4 = f(\epsilon, x, y - \varepsilon)$$
$$X^5 = f(\epsilon - \varepsilon, x, y - \varepsilon)$$
$$X^6 = f(\epsilon, x - \varepsilon, y - \varepsilon)$$
$$X^\alpha = f(\epsilon, x, y) + \varepsilon \cdot \alpha(\epsilon, x, y) \qquad (46)$$

Therefore, $X = f(\epsilon, x, y)$ and that groups G_1, G_3, G_4, G_5 and G_6 were merely the translations and scaling hence trivial, it's only G_2 that was non-trivial groups. Transformation groups of solutions of the system with consideration of symmetry group inversion (inverse mapping) theory. Symmetry inversion theory requires that for each symmetry group G_j for $x = \Psi(\epsilon, x, y)$ and $y = \Phi(\epsilon, x, y)$ be two different solutions of malaria coupled transmission equation. Then the functions x_i and y_j were also solutions since symmetry transformations changes known solutions to new solutions as shown below;

$$\widehat{x}_1 = \Psi(\epsilon, x)e^{-\varepsilon}, \widehat{y}_1 = \Phi(\epsilon, y)e^{-\epsilon\varepsilon}$$
$$\widehat{x}_2 = \Psi(\epsilon, x - (y + \frac{1}{2}\epsilon))\varepsilon, \widehat{y}_2 = \Phi(\epsilon, x, ye^{-x\varepsilon})$$
$$\widehat{x}_3 = \Psi(\epsilon, x), \widehat{y}_3 = \Phi(\epsilon, y)e^{-\varepsilon}$$
$$\widehat{x}_4 = \Psi(\epsilon, x), \widehat{y}_4 = \Phi(\epsilon, y)e^{-\varepsilon}$$
$$\widehat{x}_5 = \Psi(\epsilon - \varepsilon, x), \widehat{y}_5 = \Phi(\epsilon, y)e^{-\varepsilon}$$
$$\widehat{x}_6 = \Psi(\epsilon, xe^{-\varepsilon}) = \Psi(\epsilon, x)e^{-\varepsilon}, \widehat{y}_6 = \Phi(\epsilon, ye^{-\varepsilon}) = \Phi(\epsilon, y)e^{-\varepsilon}$$
(47)

XIV. SYMMETRY OF SOLUTIONS FOR MALARIA COUPLED TRANSMISSION EQUATION

The G_1 was a translation $G_1 = \partial_\epsilon + \partial_x + \epsilon\partial_y; G_1 = X_1(\epsilon, x, y) \to (\epsilon - \varepsilon, x - \varepsilon, y - \epsilon\varepsilon$) while

G_2 was non-trivial hence Galilean Boost to a moving coordinate given as;

$$G_2 = (y + 1/2\epsilon)\partial_x + x\partial_y; G_2 = X_2(\epsilon, x, y) \to X_2(\epsilon, x - (y + 1/2\epsilon)\varepsilon, y - x\epsilon)$$
(48)

Therefore, $x = \xi(\epsilon, x, y)$ and $y = \eta(\epsilon, x, y)$ forms different solutions of malaria coupled transmission equation, then obtaining the specific new solutions of malaria coupled transmission by taking into an account each of the invariant solutions and substituting them into the symmetry solutions as follows;

Case 1: New symmetry solutions for malaria coupled transmission equation for infinitesimal generator G_1

$G_1 = \partial_\epsilon + \partial_x + \epsilon\partial_y; G_1 = X_1(\epsilon, x, y) \rightarrow X_1(\epsilon - \varepsilon, x - \varepsilon, y - \epsilon\varepsilon)$

$$X = c_7 e^{-\varepsilon}, Y = c_8 e^{-\epsilon\varepsilon} \quad (49)$$

Case 2: New symmetry solutions for malaria coupled transmission equation for infinitesimal generator G_2

$G_2 = (y + 1/2\epsilon)\partial_x + x\partial_y; G_2 = X_2(\epsilon, x, y) \rightarrow X_2(\epsilon, x + (y + 1/2\epsilon)\varepsilon, y + x\epsilon)$

$$X_2(\epsilon, x, y) \rightarrow X_2(\epsilon - \varepsilon, x - (y + 1/2\epsilon)\varepsilon, y - x\varepsilon)$$

$$X = (\epsilon + c_2)e^{-\varepsilon}, Y = (\epsilon x + c_5)e^{-\epsilon\varepsilon} \quad (50)$$

XV. TABLE OF SYMMETRY SOLUTION FOR MALARIA COUPLED TRANSMISSION EQUATION

Generator G_j	New Symmetry solutions
G_1	$X = c_7 e^{-\varepsilon}$, $Y = c_8 e^{-\epsilon\varepsilon}$
G_2	$X = (\epsilon + c_2)e^{-\varepsilon}$, $Y = (\epsilon x + c_5)e^{-\epsilon\varepsilon}$

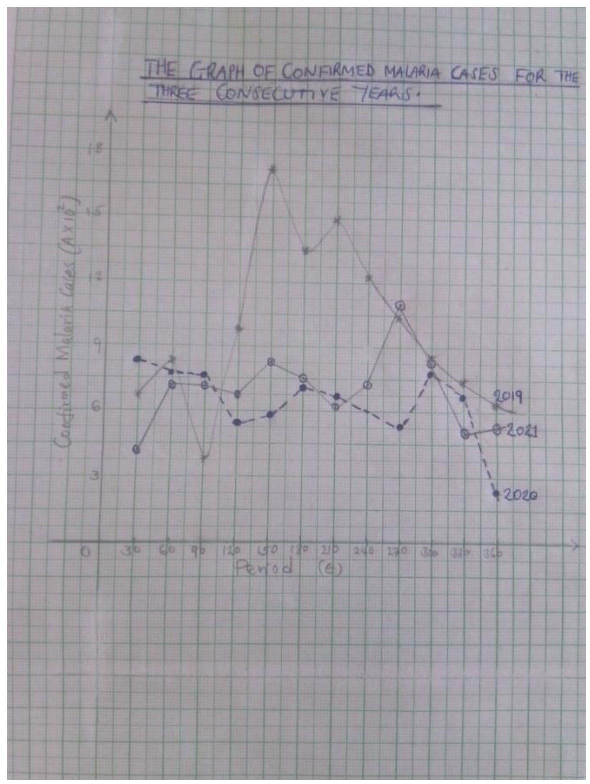
XVI. INTERPRETATION OF RESULTS AND SOLUTION OBTAINED

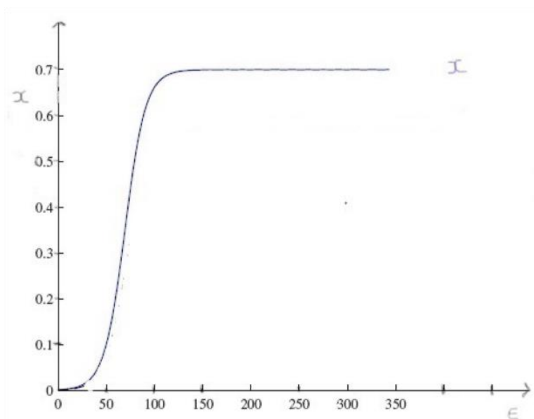
The infectious class of female anopheles' mosquito and humans obtained as $X = c_7 e^{-\varepsilon}$ and $Y = c_8 e^{-\epsilon\varepsilon}$ which assist in obtaining the solutions for both the infectious class of female anopheles mosquito and infectious class of humans since $\varepsilon[0, 1]$, $\epsilon > 0$, c_7 and c_8 were arbitrary constant as follows;

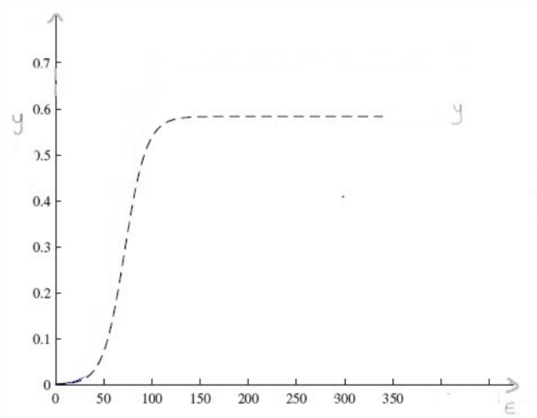
$X = 7, Y = 8$
$X = 6, Y = 6$
$X = 5, Y = 4$
$X = 5, Y = 2$
$X = 4, Y = 1$
$X = 4, Y = 0.4$
$X = 0.1, Y = 0.12$
$X = 0.03, Y = 0.03$
$X = 0.005, Y = 0.006$
$X = 0.0009, Y = 0.0009$
$X = 0, Y = 0 \quad (51)$

XVII. DISCUSSION

i. In 2019 malaria cases rises from January to February then March to may as the malaria cases drops from August to December,
ii. In 2020 malaria was reportedly high from January to February and drops from October to December,
iii. In 2021 malaria cases rises rapidly January and February then from July, August and September then drops from October to November and rises slowly due to covid-19 effect







CONCLUSION

Our main objective was to analyse the malaria coupled transmission equation using Lie groups method in order to obtain exact solution. we have been able to come up with a modified generator for malaria coupled transmission equation. Therefore, using Lie groups method we were able to develop an infinitesimal generator for malaria coupled transmission equation, we then prolonged the infinitesimal generator which act on our model which enabled us to obtain large overdetermined called determining equations. Then, after solving the determining equations by inspection we obtained the infinitesimal transformation and Lie brackets. we then constructed the Lie groups admitted by Malaria coupled transmission equation and obtain the group transformation of solutions for malaria coupled transmission equation. Therefore, we develop the characteristics equation and found the invariants solutions which enabled us to develop new symmetry solutions for our equations. The results obtained may be used to show how malaria transmission can be control by knowing the class of human infected with malaria and class of female anopheles mosquito which transmit malaria on known class of human with exponential growth.

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