Modeling Gases, Nutrients and Wastes in The Feto-Placental Circulation

SIRMA C. PURITY[1], BONIFACE O. KWACH[2], FRANKLINE TIREITO[3], JANE KABO[4]
[1, 2] *Mathematics Department, Kibabii University*
[3] *Mathematics Department, Masinde Muliro University of Science and Technology*
[4] *School of Nursing, Kibabii University*

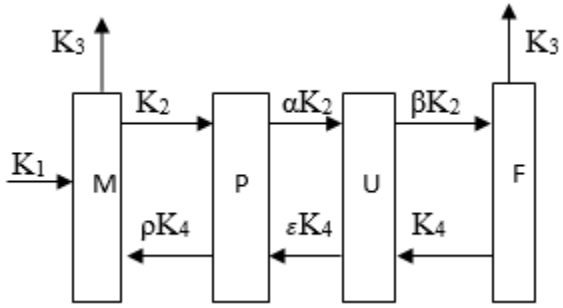
Abstract- ***The circulation between the mother and fetus is a very important process as it enables the fetus to receive all the necessary conditions for its growth, nourishment and the elimination of the waste products. For general growth of the fetus confined within uterus, proper exchange and transport of nutrients, gases and waste materials is extremely necessary. In the feto-placental circulation, the blood rich in oxygen and nutrients flows from the placenta through the umbilical vein while the deoxygenated and nutrient-depleted blood from the fetus flows through umbilical arteries back to the placenta. It's of great importance to Identify pregnancies with fetal compromise which could cause stillbirth and defects in fetus, through effective diagnosis in the umbilical arteries. We've developed a four compartmental model representing mother, placenta, umbilical cord and fetus.***

Indexed Terms- ***Fetus, placenta, umbilical cord***

I. INTRODUCTION

During pregnancy, the fetus depends entirely on the mother for nutrients, oxygen and elimination of wastes. There are two different systems of circulation for blood in the maternal and fetal circulation, that is, the utero-placental and the feto-placental blood circulation. There is an interface in the maternal and fetal circulation called the placenta, it receives blood supplies from both fetal and maternal circulation, and it allows the maternal and fetal blood to come so close for exchange without mixing. The fetus is connected with the placenta by umbilical cord, it has three blood vessels that transport all the necessities for growth and nourishment of the fetus. Umbilical cord abnormalities are a big risk factor for still births which according to research amounts for up to 19% of cases.

To develop a four compartmental model for nutrients, gases and wastes we included the umbilical cord as the fourth compartment. The following assumptions were made for umbilical cord abnormalities: the rate of transfer of gases and nutrients from placenta is not the same rate as it is received by the fetus; not all the wastes from the fetus is delivered to the placenta. The following model was developed;



$$\frac{dM}{dt} = K_1 + \rho K_4 P - (K_2 + K_3)M$$
$$\frac{dP}{dt} = K_2 M + \varepsilon k_4 U - \left(\alpha K_2 + \rho K_4\right) P$$

$$\frac{dU}{dt} = \alpha K_2 M + K_4 F - \left(\beta K_2 + \varepsilon K_4\right) U$$
$$\frac{dF}{dt} = \beta k_2 \mathrm{U} - (k_3 + k_4)\mathrm{F}$$

Where:
M = the total calories the mother has (calories)
P = the placenta (the interface)
U= Umbilical cord
F = the total calories the fetus has (calories)
K_1= the total amount of calories the pregnant woman takes in a given day (calories per day)
K_2= the rate of gases and calorie transfer from mother to fetus (per day)

αK_2 = the rate of gases and calorie transfer between mother and fetus both metabolized and partially consumed (per day)
βK_2 = the rate of gases and calorie transfer between mother and fetus through umbilical cord both partially consumed and metabolized (per day)
K_3 = the rate of calorie burn-off and waste (per day)
K_4 = the amount of waste product transferred from fetus to the mother
εK_4= the amount of waste product transferred from fetus to the mother through the umbilical cord
ρK_4= total amount of waste transferred from fetus to the mother through the placenta (per day)

ACKNOWLEDGMENT

Special thanks to members of Mathematics department, Kibabii University for their valuable inputs.

REFERENCES

[1] Damasceno, de Lima P.P., (2013). Wharton's jelly absence a possible cause of still birth. Autopsy case ReP *PMC free article [PubMed]*. 3(4):43-47.

[2] Fahmy M., (2018). Congenital Anomalies of the Umbilical cord}. Umbilicus and Umbilical cord. *springer Int. Pub.*ISBN 978-3-319-62383-2.

[3] Groome L.J., (1991). A theoretical analysis of the effect of placental metabolism on fetal oxygenation under conditions of limited oxygen availability. *Bio-systems* 26(1):45,56.

[4] Hill E.P., Power G.G. and Longo L.D... (1973). A mathematical model of carbon- dioxide transfer in the placenta and its interaction with oxygen. *AM, J. Physiol*,224(2):283-299

[5] Jason C., Lewis, M., Rassi, D. et al. 2019). A data-driven model to study utero-ovarian blood flow physiology during pregnancy. *Biomech Model Mechanobio*l18, 1155-1176.

[6] Joel Olielo Odongo, Omolo Ongati, Boniface Otieno Kwach. Model for Nutrient Exchange across the Placenta (2015) *International Journal of Engineering & Mathematical Sciences*, Volume 8, Issue – 1, pp.28 - 35ISSN (Print) – 2319 - 4537, (Online) – 2319 - 4545.

[7] Kirschbaum T.H. and Shapiro N.Z., (1969). A mathematical model of placental Oxygen transfer. *J Theor*, 25:380-402

[8] Moore L. Keith, Persaud T.V.N., Torchia Mark G., 2016. The Developing Human: Clinically Oriented Embryology (10th ed.). Philadelphia: *Elsevier.p*.58. ISBN978-0-323-31338-4

[9] Lardner T.J., (1975). A model for placental oxygen exchange. *J. Biomech*.8(2):131-134

[10] The Naked scientist, Science Articles., (2005). The Placenta. A mathematical Model. University of Cambridge. http://www.thenakedscientists.com. Accessed in May, 2014.

[11] Bowen R. (2000). http://www.vivo.colostate.edu/hbooks/pathphys/reprod/placenta/transport.html. Accesed on 17th July, 2015.

[12] Sam Zimmerman, (2011). Developing A Model for Mother-Fetal Nutrient Relations: An Anthology "Pregnancy." Mayo Clinic. Web.29 Apr. 2011.http://www.mayoclinic.com.

[13] Wilbur, Power, Lango, (1978). Modelled water and solute exchange in the human placenta. *Am J PhsiolRegul Inter Comp Phsiol*; 235: R181-199