On POSI-Class (Q) Operators

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Abstract- In this paper, we introduce the class of Posi-Class (Q) operator acting on a complex Hilbert space H to H. An operator $T \epsilon L(H)$ is said to be Posi-Class (Q) if $T^{*2}PT^2 = (T^*T)^2$ for a positive operator $P \epsilon L(H)$. We look at some nice properties that are associated with this class

Indexed Terms- Class (Q), Almost-Class (Q) operators, Posi-Class (Q) and Posinormal operators.

I. INTRODUCTION

H denotes the usual Hilbert space over the complex eld throughout this paper while L(H) the Banach algebra of all bounded linear algebra on an in nite dimensional separable Hilbert space H . The class of (Q) operators was covered by Jibril in (1) , while Wanjala Victor and Beatrice Adhiambo introduced and studied Almost-class (Q) operators in (3). In this paper, we introduce and study the class of Posi-class (Q) operators and look at the relation between this class and other classes such as almost class (Q) and posinormal operators.

Definition 1. An operator's $T \epsilon B(H)$ is said to be:

1) Class (Q) if $T^{*2}T^2 = (T^* T)^2$ (1)
2) posinormal if $T^* P T = T T^*$ for $P \epsilon L(H)$ (2).
3) Almost class (Q) if $T^{*2}T^2 (T^* T)^2$
4) Posi-class (Q) if $T^{*2}P T^2 = (T^* T)^2$ for a positive operator $P \epsilon L(H)$
5) 2- posinormal if $T^* P T^2 = T^{*2}T$ for $P \epsilon L(H)$.

$T \epsilon B(H)$ is co-Posi-class (Q) if T is Posi-class (Q), that is $T^{*2}T^2 = (T^* P T)^2$ for a positive operator $P \epsilon L(H)$.

II. MAIN RESULTS

Theorem 2. If $T \epsilon L(H)$ is posinormal, then T is Posi-class (Q).

Proof. T being posinormal implies; $T T^* = T^* P T$. Multiplying both sides on the left by T and on the right by T we obtain; $T^* T T^* T = T^* T^* P T T = (T^* T) 2 = T^{*2}P T^2$ as required.

Theorem 3. Let $T \epsilon B(H)$ be such that it's a Posi-class (Q) and it has a dense range ,then the interuptor P is unique.

Proof. Let P_q and P_r be interruptors for T . T being Posi-class (Q) we have; $T^{*2}P_q T^2 = (T^* T)^2 = T^{*2}P_r T^2$. Hence $T^{*2}(P_q P_r)T^2 = 0$. T having a dense range implies T is one

to one , thus ; $T^* T^* (P_q P_r)T^2 = T^* (P_q P_r)T^2 = (P_q P_r)T^2 = 0$. Similarly T having a dense range implies; $(P_q P_r)T T = (P_q P_r)T = P_q P_r = 0$

Theorem 4. If $T \epsilon L(H)$ is a Posi-class (Q) operator, then $\varphi^2 T^{*2}T^2 \geq (T^* T)^2$ for $\varphi^2 = 0$

Proof. Proof is trivial.

corollary 5. Every Almost-class (Q) operator is Posi-class (Q).

Proof. Proof follows directly from Theorem 4 for $\varphi = 1$.

Proposition 6. Let $T \epsilon L(H)$ be both 2-Posinormal and Posi-class (Q) , it follows T is Posinormal with interuptor P being self-adjoint .

Proof. $(T T^* PT)^* (TT^* - T^* PT) = (TT^* - T^* T^* - PT))(TT^* - T^* PT)$

$= (TT^*)^2 - TT^{*2}PT – T^* PT^2T^* + (T^* PT)^2$

$= (TT^*)^2 - {}^2PT^2 \; T \; T^{*2}P T^2 + (T P T)^2)$ Since T is 2-Posinormal

T

$= (TT^*)^2 - {}^2PT^2$ Since T is Posi-class (Q)

$= 0$

Hence

$T T^* - T^* P T = 0$ implying T is posinormal .

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