Optimization of Compressive Strength of Steel Fibre Reinforced Concrete (SFRC) Using Scheffe's Third-Degree Regression Model

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Abstract- This research work is aimed at using Scheffe's Third Degree Regression Model for five component mixture, simply abbreviated as Scheffe's (5, 3) to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). One of the specific objective is to compare the results of the present work with the results of the previous work done on SFRC based on Scheffe's Second Degree Regression Model, Scheffe's (5, 2) by Nwachukwu and others (2022b). Through the use of Scheffe's Simplex optimization method, the compressive strengths of SFRC with respect to Scheffe's third degree model were obtained for different mix proportions. Control experiments were also carried out, and the compressive strengths evaluated. By using the Student's t-test statistics, the adequacy of the model was confirmed .The optimum attainable compressive strength of SFRC based on the Scheffe's (5, 3) model was 33.74 MPa which is higher than 27.81 MPa , being the maximum value obtained by Nwachukwu and others (2022b) for the previous work done on SFRC based on the Scheffe's (5,2) model. Although there is slight difference between the maximum compressive strength values obtained by the two models, however, the values from both models are higher than the minimum value specified by the American Concrete Institute (ACI), as 20 MPa . As a result of this, the SFRC compressive strength values based on both models can sustain construction of light-weight structures. Again, by considering its safety and economic advantages, SFRC can find applications as concrete flooring for parking lots, playgrounds, airport runways, taxiways, maintenance hangars, access roads, workshops, port pavements, container storage and handling areas, bulk storage warehouses, as well as military warehouses.

Indexed Terms- SFRC, Scheffe's (5,3) Regression Model, Optimization, Compressive Strength, Mixture Design

I. INTRODUCTION

The design of concrete mix according to (Shetty, 2006) has not being a simple task on the account of the widely varying properties of the constituent materials, the conditions that prevail at the site of work, in particular the exposure condition, and the conditions that are demanded for a particular work for which the mix is designed. This implies that the design of concrete mix requires complete knowledge of the various properties of these constituent materials, the implication in case of change on these conditions at the site, the impact of the properties of plastic concrete on the hardened concrete and the complicated inter-relationship between the variables. From the above criteria make the task of mix design more complex and laborious. Furthermore, Concrete Mix Design according to Jackson and Dhir (1996) has been defined as the procedure which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. In this context, the cost of any concrete includes, in addition to that of the materials themselves, the cost of the mix design, of batching, mixing and placing the concrete and of the site supervision. Following the above guidelines, the methods proposed by Hughes (1971), ACI- 211(1994) and DOE (1988) appears to be more complex and time consuming as they involve a lot of trial mixes and deep statistical calculations before the desired strength of the concrete can be actualized. Thus, optimization of the concrete mixture design remains the fastest method, best option and the most efficient way of selecting concrete mix /proportion for better efficiency and performance of concrete when compared with usual empirical methods listed above. A typical example of optimization model is Scheffe's Regression Models. Scheffe's Optimization model can be in the form of Scheffe's Second Degree model or Scheffe's Third Degree model. Although work has been previously done on SFRC based on Scheffe's Second Degree model, the knowledge acquired from the works of Obam (2006) , Nwachukwu and others (2022a) , Nwachukwu and others (2022e) and Nwachukwu and others (2022f) shows that the results from the third degree models usually have edges over that from the second degree models. Therefore, in this present study, Scheffe's Third Degree Regression for five components mixtures (namely cement, fine aggregate, coarse aggregate, water and steel fibre) will be presented.

According to Oyenuga (2008), concrete is a composite inert material comprising of a binder course (cement), mineral filter or aggregates and water. Concrete, being a homogeneous mixture of cement, sand, gravel and water is very strong in carrying compressive forces and hence is gaining increasing importance as building materials throughout the world (Syal and Goel, 2007). Concrete has remained an important material widely used in the construction industry since ancient time. Concrete, according to Neville (1990), plays an important part in all building structures owing to its numerous advantages which ranges from low built in fire resistance, high compressive strength to low maintenance. However, according to Shetty (2006) , plain concrete possesses a very low tensile strength, limited ductility and little resistance to cracking. To remedy this situation, attempts have been made in the past to impact improvement in tensile properties of concrete members by way of using conventional reinforced steel bars and also by applying restraining techniques. Although both these methods provide tensile strength to the concrete members, they however, do not increase the inherent tensile strength of concrete itself.. Based on several further researches and recent developments, it has been observed that the addition of small, closely spaced or uniformly dispersed fibres (either as glass fibre, polypropylene fibre, nylon fibre, steel fibre , plastic fibre, asbestor (mineral fibre), or carbon fibres , etc.) to concrete would act as crack arrester and would substantially improve its static and dynamic properties. This type of concrete is known as Fibre reinforced concrete (FRC). FRC is a composite material consisting of mixtures of cement, mortar or concrete and discontinuous, discrete, uniformly dispersed suitable fibres. Incorporation of the fibrous materials is aimed at increasing the concrete's durability and structural integrity at a reasonable cost. In general, all fibres reduce the concrete's need for steel reinforcements and also, since fibre reinforcement tends to be less expensive than steel bars, it makes FRC more cost-effective. Steel Fibre Reinforced Concrete (SFRC) is concrete mixture where the conventionally steel reinforcement in concrete production is replaced (wholly or partially) with steel fibre. Steel fibres are short discontinues strips of specially manufactured steel. A certain amount of steel fibre in concrete can cause qualitative changes in concrete's physical property, greatly increases resistance to cracking, impact, fatigue, and bending, tenacity, durability, and other properties. It is a well-established fact that one of the important properties of Steel Fibre Reinforced Concrete (SFRC) is its superior resistance to cracking. This property is likely attributed to the addition of steel fibres (SF).

According to Ettu (2001), the purpose of design is to ensure that the structure being designed will not reach a Limit State. Limit State here is of two types: Serviceability Limit State (SLS), which is connected with deflection, cracking, vibration etc, and Ultimate Limit State (ULS), which is generally connected with collapse. In all of the above, the concrete's compressive strength is one of the most important properties of concrete that require close examination because of its important role. Compressive strength of concrete is the Strength of hardened concrete measured by the compression test. It is a measure of the concrete's ability to resist loads which tend to compress it. It is measured by crushing cylindrical concrete specimens in a universal testing machine. The compressive strength of the concrete cube test also provides an idea about all the characteristics of concrete in question.

This recent work therefore examines the use of Scheffe's Third Degree Regression Model in optimizing the compressive strength of SFRC. Although many researchers have done related works on steel fibre, SFRC or optimization, none has been able to fully address the recent subject matter . For instance, Baros and others (2005) investigated the post – cracking behaviour of SFRC. Jean-Louis and Sana (2005) investigated the corrosion of SFRC from the crack. Lima and Oh (1999) carried out an experimental and theoretical investigation on the shear of SFRC beams. Similarly, Lau and Anson (2006) carried out research on the effect of high temperatures on high performance SFRC. The work of Lie and Kodar (1996) was on the study of thermal and mechanical properties of SFRC at elevated temperatures. Blaszczynski and Przybylska-Falek (2015) investigated the use of SFRC as a structural material. Huang and Zhao (1995) investigated the properties of SFRC containing larger coarse aggregate. Arube and others (2021) investigated the Effects of Steel Fibres in Concrete Paving Blocks. Again, Khaloo and others (2005) examined the flexural behaviour of small SFRC slabs. And Ghaffer and others (2014) investigated the use of steel fibres in structural concrete to enhance the mechanical properties of concrete. Recently in the area of optimization, many researchers have used Scheffe's method to carry out one form of optimization work or the other. For example, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere (2006) were also based on the use of Scheffe' mathematical model in the optimization of compressive strength of Perwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's (4,2) and Scheffe's (4,3). Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also, Nwachukwu and others (2022a) developed and used Scheffe's Third Degree Polynomial model, Scheffe's (5,3) to optimize the compressive strength of GFRC where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). Nwachukwu and others (2022b) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). Furthermore, Nwachukwu and others (2022e) used Scheffe's Third Degree Regression model, Scheffe's (5,3) to optimize the compressive strength of PFRC. And lastly, Nwachukwu and others (2022f) applied Modified Scheffe's Third Degree Polynomial model to optimize the compressive strength of NFRC. From the forgoing, it can be envisaged that no work has been done on the use of Scheffe's Third Degree method to optimize the compressive strength of SFRC. Henceforth, the need for this present research work, whose background is informed from the findings stated in the first paragraph of this introduction.

II. SCHEFFE'S (5, 3) REGRESSION EQUATION

According to Aggarwal (2002), a simplex lattice is a structural representation of lines joining the atoms of a mixture, whereas these atoms are constituent components of the mixture. For SFRC mixture, the constituent elements are these five components, water, cement, fine aggregate, coarse aggregate and steel fibre. Thus, a simplex of five-component mixture is a four-dimensional solid. An imaginary space showing a four dimensional factor space with respect to Scheffe's third degree model has been shown in the work of Nwachukwu and others (2022a). According to Obam (2009), mixture components are subject to the constraint that the sum of all the components must be equal to 1. That is:

$$X_1 + X_2 + X_3 + \dots + X_q = 1 \;; \quad \Rightarrow \sum_{i=1}^{q} X_i = 1 \quad (1)$$

where $X_i \geq 0$ and $i = 1, 2, 3 \dots q$, and q = the number of mixtures.

2.1 BACKGROUND INFORMATION ON THE SIMPLEX LATTICE DESIGN

The (q, m) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen polynomial equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains $^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \ldots, 1; \quad i = 1, 2, \ldots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degee, which in this present study is 3.

For example a (3, 2) lattice consists of $^{3+2-1}C_2$ i.e. $^4C_2$ = 6 points. Each X_i can take m+1 = 3 possible values; that is $x = 0, \frac{1}{2}, 1$ with which the possible design points are:

$$(1,0,0), (0,1,0), (0,0,1), \left(\frac{1}{2}, \frac{1}{2}, 0\right), \left(0, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, 0, \frac{1}{2}\right)$$

The general formula for evaluating the number of coefficients/terms/points required for a given lattice is given by:

$$k = \frac{(q+m-1)!}{(q-1)!\,.\,m!} \quad \text{Or} \quad ^{q+m-1}C_m \qquad 2(a\text{-}b)$$

Where k = number of coefficients/ terms / points

q = number of components = 5 in this study

m = number of deqree of polynomial = 3 in this present work

Using either of Eqn. (2), $k_{(5,3)} = 35$

Thus, the possible design points for Scheffe's (5,3) lattice can be as follows:

A_1 (1,0,0,0,0); A_2 (0,1,0,0,0); A_3 (0,0,1,0,0); A_4 (0,0,0,1,0), A_5 (0,0,0,0,1); A_{112} (2/3. 1/3, 0, 0, 0,); A_{122} = (1/2, 2/3, 0,0,0); A_{113} (2/3, 0, 1/3, 0,0); A_{113} (2/3, 0, 1/3, 0,0); A_{133} (1/3, 0, 0, 2/3, 0, 0); A_{114} (2/3, 0,0,1/3,0); A_{114} (1/3, 0, 0, 2/3, 0); A_{115}, (2/3, 0, 0, 0, 1/3); A_{115} (1/3, 0,0,0, 2/3); A_{223} (0, 2/3, 1/3, 0,0); A_{223} (O, 1/3, 0,0); A_{224} (0,0 2/3, 0, 1/3, 0); A_{224} (0, 1/3, 0, 2/3,0); A_{225} (0, 2/3, 0,0, 1/3); A_{255} (0, 1/3, 0, 0, 2/3); A_{334} (0,0, 2/3, 1/3, 0); A_{344} (0,0,1/3, 2/3,0), A355 (0,0,2/3,0, 1/3); A_{355} (0,0,1/3,0, 2/3); A_{445} (0,0,0, 2/3, 1/3); A_{445} (0,0,0, 1/3, 2/3); A_{123} (1/3, 1/3, 1/3, 0,0); A_{124} (1/3, 1,3, 0, 1/3, 0); A_{125} (1/3, 1/3, 0,0, 1/3); A_{134} (134 (1/3, 0, 1/3, 1/3, 0); A_{135} (1/3, 0, 1/3, 0, 1/3); A_{145} (1/3, 0, 0,1/3,1/3); A_{234} (0,1/3, 1/3,1/3, 0); A_{235} (0,1/3, 1/3, 0, 1/3); A_{245} (0, 1/3, 0, 1/3, 1/3); A_{345} (0,0,1/3,1/3, 1/3). (3)

According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable X_1, X_2, X_3, X_4 … X_q is given in the form of Eqn.(4)

$$N = b_0 + \sum b_i x_i + \sum b_{ij} x_j + \sum b_{ijk} x_j x_k + + \sum b_{i_{j_2}} + \ldots i_n x_{i_2} x_{i_n} \qquad (4)$$

where ($1 \le i \le q$, $1 \le i \le j \le k \le q$, $1 \le i_1 \le i_2 \le \ldots \le i_n \le q$ respectively) , b = constant coefficients and N is the response which represents the property under investigation, which ,in this case is the compressive strength.

As this research work is based on the Scheffe's (5, 3) simplex , the actual form of Eqn. (4) for five component mixture , degree three (5, 3) has been developed by Nwachukwu and others (2022a) and will be applied subsequently.

2.2 PSEUDO AND ACTUAL COMPONENTS.

In Scheffe's mix design, there exist a relationship between the pseudo components and the actual components which has been established as Eqn.(5):

$$Z = A * X \quad (5)$$

where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging Eqn. (5) yields:

$$X = A^{-1} * Z \quad (6)$$

2.3. FORMULATION OF SFRC REGRESSION EQUATION FOR SCHEFFE'S (5, 3) LATTICE

The Regression equation by Scheffe (1958), otherwise known as response is given in Eqn.(4) .Hence, for Scheffe's (5,3) simplex lattice, the regression equation for five component mixtures has been formulated from Eqn.(4) by Nwachukwu and others (2022a) and is given as follows:

$$N = b_0 + b_1 X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11}X_1^2 + b_{12}X_1X_2 + b_{13}X_1X_3$$
$$+ b_{14}X_1X_4 + b_{15}X_1X_5 + b_{111}X_1^3 + b_{112} X_1^2X_2 + b_{113}X_1^2X_3 + b_{114}X_1^2X_4 + b_{115}X_1^2X_5 + b_{22}X_2^2 + b_{23}X_2X_3$$

$$+ b_{24}X_2X_4 + b_{25}X_2X_5 + b_{222} X_2^3 + b_{223}X_2^2X_3 + b_{224}X_2^2X_4 + b_{225}X_2^2X_5 + b_{33}X_3^2 + b_{34}X_3X_4$$
$$+ b_{35}X_3X_5 + b_{333}X_3^3 + b_{334}X_3^2X_4 + b_{335}X_3^2X_5 + b_{44}X_4^2 + b_{45}X_4X_5 + b_{444}X_4^3 + b_{445}X_4^2X_5 + b_{55}X_5^2 + b_{555}X_5^3 \quad (7)$$
$$= b_0X_1 + b_0X_2 + b_0X_3 + b_0X_4 + b_0X_5 + b_1 X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11} X_1 - b_{11}X_1X_2 - b_{11}X_1X_3$$
$$- b_{11}X_1X_4 - b_{11}X_1X_5 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{15}X_1X_5 + b_{111}X_1^3 + b_{112} X_1 X_2 - b_{112} X_1 X_2^2$$
$$- b_{112} X_1 X_2X_3 - b_{112} X_1 X_2X_4 - b_{112} X_1 X_2X_5 + b_{113}X_1 X_3 - b_{113}X_1X_2 X_3 - b_{113}X_1 X_3^2 - b_{113}X_1 X_3X_4 - b_{113}X_1 X_3X_5$$
$$+ b_{114}X_1 X_4 - b_{114}X_1X_2 X_4 - b_{114}X_1X_3 X_4 - b_{114}X_1 X_4^2 - b_{114}X_1 X_4X_5 + b_{115} X_1 X_5 - b_{115}X_1X_2 X_5$$
$$- b_{115}X_1X_3 X_5 - b_{115}X_1X_4 X_5 - b_{115}X_1 X_5^2 + b_{22}X_2 - b_{22}X_1X_2 - b_{22}X_2X_3 - b_{22}X_2X_4 - b_{22}X_2X_5$$
$$+ b_{23}X_2X_3 + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{222} X_2^3 + b_{223}X_2 X_3 - b_{223}X_1X_2 X_3 - b_{223}X_2 X_3^2 - b_{223}X_2 X_3X_4$$
$$- b_{223}X_2 X_3X_5 + b_{224} X_2 X_4 - b_{224} X_1X_2 X_4 - b_{224} X_2X_3 X_4 - b_{224} X_2 X_4^2 - b_{224} X_2 X_4X_5 + b_{225}X_2 X_5$$
$$- b_{225}X_1X_2 X_5 - b_{225}X_2X_3 X_5 - b_{225}X_2X_4 X_5 - b_{225}X_2 X_5^2 + b_{33}X_3 - b_{33}X_1X_3 - b_{33}X_2X_3 - b_{33}X_3X_4 - b_{33}X_3X_5$$
$$+ b_{34}X_3X_4 + b_{35}X_3X_5 + b_{333}X_3^3 + b_{334}X_3 X_4 - b_{334}X_1X_3 X_4 - b_{334}X_2X_3 X_4 - b_{334}X_3 X_4^2 - b_{334}X_3 X_4X_5$$
$$+ b_{335}X_3 X_5 - b_{335}X_1X_3 X_5 - b_{335}X_2X_3 X_5 - b_{335}X_3X_4 X_5 - b_{335}X_3 X_5^2 + b_{44}X_4 - b_{44}X_1X_4 - b_{44}X_2X_4 - b_{44}X_3X_4$$
$$- b_{44}X_4X_5 + b_{45}X_4X_5 + b_{444}X_4^3 + b_{445}X_4 X_5 - b_{445}X_1X_4 X_5 - b_{445}X_2X_4 X_5 - b_{445}X_3X_4 X_5 - b_{445}X_4 X_5^2$$
$$+ b_{55} X_5 - b_{55} X_1X_5 - b_{55} X_2X_5 - b_{55} X_3X_5 - b_{55} X_4X_5 + b_{555}X_5^3 \quad (8)$$

$$N = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14}X_1 X_4 + \beta_{15}X_1 X_5 + \beta_{23} X_2 X_3 + \beta_{24}X_2 X_4$$
$$+ \beta_{25}X_2 X_5 + \beta_{34} X_3 X_4 + \beta_{35}X_3 X_5 + \beta_{45}X_4 X_5 + \gamma_{12}X_1 X_2^2 + \gamma_{13} X_1 X_3^2 + \gamma_{14} X_1 X_4^2 + \gamma_{15} X_1 X_5^2$$
$$+ \gamma_{23} X_2 X_3^2 + \gamma_{24}X_2 X_4^2 + \gamma_{25} X_2X_5^2 + \gamma_{34} X_3 X_4^2 + \gamma_{35} X_3 X_5^2 + \gamma_{45} X_4 X_5^2 + \beta_{123} X_1 X_2 X_3 + \beta_{124} X_1 X_2 X_4$$
$$+ \beta_{125}X_1 X_2 X_5 + \beta_{134}X_1 X_3 X_4 + \beta_{135}X_1 X_3 X_5 + \beta_{145}X_1 X_4 X_5 + \beta_{234}X_2 X_3 X_4 + \beta_{235}X_2 X_3 X_5$$
$$+ \beta_{245} X_2 X_4 X_5 + \beta_{345}X_3 X_4 X_5 \quad (9)$$

Where

$\beta_1 = [b_0 + b_1 + b_{11}]$; $\beta_2 = [b_0 + b_2 + b_{22}]$; $\beta_3 = [b_0 + b_3 + b_{33}]$; $\beta_4 = [b_0 + b_4 + b_{44}]$; $\beta_5 = [b_0 + b_5 + b_{55}]$;
$\beta_{12} = [b_{12} - b_{11} - b_{22} + b_{112}]$.; $\beta_{13} = [b_{13} - b_{11} - b_{33} + b_{113}]$; $\beta_{14} = [b_{14} - b_{14} - b_{11} - b_{44} + b_{114}]$;
$\beta_{15} = [b_{15} - b_{11} - b_{55} + b_{115}]$; $\gamma_{12} = [- b_{112}]$; $\gamma_{13} = [- b_{113}]$; $\gamma_{14} = [- b_{114}]$; $\gamma_{15} = [- b_{115}]$;
$\beta_{123} = [-b_{112} - b_{113} - b_{223}]$; $\beta_{124} = [-b_{112} - b_{114} - b_{224}]$; $\beta_{125} = [-b_{112} - b_{115} - b_{225}]$; $\beta_{134} = [-b_{113} - b_{114} - b_{334}]$;
$\beta_{135} = [-b_{113} - b_{115} - b_{335}]$; $\beta_{145} = [-b_{113} - b_{115} - b_{445}]$; $\beta_{23} = [b_{23} - b_{22} - b_{33} + b_{223}]$; $\beta_{24} = [b_{24} - b_{22} - b_{44} + b_{224}]$;
$\beta_{25} = [b_{25} - b_{22} - b_{55} + b_{225}]$; $\gamma_{23} = [- b_{223}]$; $\gamma_{24} = [- b_{224}]$; $\gamma_{25} = [- b_{225}]$; $\beta_{234} = [-b_{223} - b_{224} - b_{334}]$;
$\beta_{235} = [-b_{223} - b_{225} - b_{335}]$; $\beta_{245} = [-b_{224} - b_{225} - b_{445}]$; $\beta_{34} = [b_{34} - b_{33} - b_{44} + b_{334}]$; $\beta_{35} = [b_{35} - b_{33} - b_{55} + b_{335}]$;
$\gamma_{34} = [- b_{334}]$; $\gamma_{35} = [- b_{335}]$; $\beta_{345} = [-b_{334} - b_{335} - b_{445}]$; β_{45}; $= [b_{45} - b_{44} - b_{55} + b_{445}]$; $\gamma_{45} = [- b_{445}]$ (10)

Equation (9) is the polynomial equation for Scheffe's (5, 3) simplex

2.4 . COEFFICIENTS OF THE SCHEFFE'S (5, 3) POLYNOMIAL

From the work of Nwachukwu and others (2022a), the coefficients of the Scheffe's (5, 3) polynomial have been determined as under. :

$\beta_1 = N_1$; $\beta_2 = N_2$; $\beta_3 = N_3$; $\beta_4 = N_4$; and $\beta_5 = N_5$ 11(a-e)

$\beta_{12} = 9/4(N_{112} + N_{122} - N_1 - N_2)$; $\beta_{13} = 9/4 (N_{113} + N_{133} - N_1 - N_3)$; $\beta_{14} = 9/4 (N_{114} + N_{144} - N_1 - N_4)$; 12(a-c)

$\beta_{15} = 9/4 (N_{115} + N_{155} - N_1 - N_5)$; $\beta_{23} = 9/4 (N_{223} + N_{233} - N_2 - N_3)$; $\beta_{24} = 9/4 (N_{224} + N_{244} - N_2 - N_4)$ 13(a-c)

$\beta_{25} = 9/4(N_{225} + N_{255} - N_2 - N_5)$; $\beta_{34} = 9/4(N_{334} + N_{344} - N_3 - N_4)$; $\beta_{35} = 9/4(N_{335} + N_{355} - N_3 - N_5)$ 14(a-c)

$\beta_{45} = 9/4(N_{445} + N_{455} - N_4 - N_5)$; $\gamma_{12} = 9/4(3N_{112} + 3N_{122} - N_1 + N_2)$; $\gamma_{13} = 9/4(3N_{113} + 3N_{133} - N_1 + N_3)$ 15(a-c)

$\gamma_{14} = 9/4(3N_{114} + 3N_{144} - N_1 + N_4)$; $\gamma_{15} = 9/4(3N_{115} + 3N_{115} - N_1 + N_5)$; $\gamma_{23} = 9/4(3N_{223} + 3N_{233} - N_2 + N_3)$ 16(a-c)

$\gamma_{24} = 9/4 (3 N_{224} + 3 N_{244} - N_2 + N_4)$; $\gamma_{25} = 9/4(3N_{225} + 3N_{255} - N_2 + N_5)$; $\gamma_{34} = 9/4(3N_{334} + 3N_{344} - N_3 + N_4)$ 17(a-c)

$\gamma_{35} = 9/4(3N_{335} + 3N_{355} - N_3 + N_5)$; $\gamma_{45} = 9/4(3N_{445} + 3N_{455} - N_4 + N_5)$ 18(a-b)

$$\beta_{123} = 27N_{123} - 27/4(N_{112} + N_{122} + N_{113} + N_{133} + N_{223} + N_{233}) + 9/4(N_1 + N_2 + N_3) \quad (19)$$
$$\beta_{124} = 27N_{124} - 27/4(N_{112} + N_{122} + N_{114} + N_{144} + N_{224} + N_{244}) + 9/4(N_1 + N_2 + N_4) \quad (20)$$
$$\beta_{125} = 27N_{125} - 27/4(N_{112} + N_{122} + N_{115} + N_{155} + N_{225} + N_{255}) + 9/4(N_1 + N_2 + N_5) \quad (21)$$
$$\beta_{134} = 27N_{134} - 27/4(N_{113} + N_{133} + N_{114} + N_{144} + N_{334} + N_{344}) + 9/4(N_1 + N_3 + N_4) \quad (22)$$
$$\beta_{135} = 27N_{135} - 27/4(N_{113} + N_{133} + N_{115} + N_{155} + N_{335} + N_{355}) + 9/4(N_1 + N_3 + N_5) \quad (23)$$
$$\beta_{145} = 27N_{145} - 27/4(N_{114} + N_{144} + N_{115} + N_{155} + N_{445} + N_{455}) + 9/4(N_1 + N_4 + N_5) \quad (24)$$

$\beta_{234} = 27N_{234} - 27/4(N_{223}+N_{233}+N_{224}+N_{244}+N_{334}+N_{344}) + 9/4(N_2+N_3+N_4)$ (25)

$\beta_{235} = 27N_{235} - 27/4(N_{223}+N_{233}+N_{225}+N_{255}+N_{335}+N_{355}) + 9/4(N_2+N_3+N_5)$ (26)

$\beta_{245} = 27N_{245} - 27/4(N_{224}+N_{244}+N_{225}+N_{255}+N_{445}+N_{455}) + 9/4(N_2+N_4+N_5)$ (27)

$\beta_{345} = 27N_{345} - 27/4(N_{334}+N_{344}+N_{335}+N_{355}+N_{445}+N_{455}) + 9/4(N_3+N_4+N_5)$ (28)

Where N_i = Response Function (Compressive Strength) for the pure component, i

2.5. SCHEFFE'S (5, 3) MIXTURE DESIGN MODEL

Substituting Eqns. (11)-(28) into Eqn. (9), yields the mixture design model for the SFRC Scheffe's (5, 3) lattice.

2.6. PSEUDO AND ACTUAL MIX PROPORTIONS OF SCHEFFE'S (5, 3) DESIGN LATTICE

The requirement of simplex lattice design based on Eqn. (1) criteria makes it impossible to use the conventional mix ratios such as 1:2:4, 1:3:6, etc., at a given water/cement ratio for the actual mix ratio. This necessitates the transformation of the actual components proportions to meet the above criterion. Based on experience and previous knowledge from literature, the following arbitrary prescribed mix ratios are always chosen for the five vertices of Scheffe's (5, 3) lattice. See the works of Nwachukwu and others (2022a), for the figure showing the vertices of a Scheffe's (5, 3) lattice for both actual and pseudo mix ratios.

A_1 (0.67:1: 1.7: 2:0.5); A_2 (0.56:1:1.6:1.8:0.8); A_3 (0.5:1:1.2:1.7:1); A_4 (0.7:1:1:1.8:1.2) and A_5 (0.75:1:1.3:1.2:1.5), which represent water/cement ratio, cement, fine aggregate, coarse aggregate and steel fibre respectively.

For the pseudo mix ratio, the following corresponding mix ratios at the vertices for five component mixtures are always chosen: A_1(1:0:0:0:0), A_2(0:1:0:0: 0), A_3(0:0:1:0:0), A_4(0:0:0:1:0), and A_5(0:0:0:0:1)

For the transformation of the actual component, Z to pseudo component, X, and vice versa, Eqns. (5) and (6) are used.

Substituting the mix ratios from point A_1 into Eqn. (5) yields:

$$\begin{pmatrix} 0.67 \\ 1 \\ 1.7 \\ 2 \\ 0.5 \end{pmatrix} = \begin{pmatrix} A_{111} & A_{112} & A_{113} & A_{114} & A_{115} \\ A_{221} & A_{222} & A_{223} & A_{224} & A_{225} \\ A_{331} & A_{332} & A_{333} & A_{334} & A_{335} \\ A_{441} & A_{442} & A_{443} & A_{444} & A_{445} \\ A_{551} & A_{552} & A_{553} & A_{554} & A_{555} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

(29)

Transforming the R.H.S matrix and solving, we obtain

A_{111}= 0.67; A_{221}= 1; A_{331}= 1.7; A_{441}= 2; A_{551}= 0.5

The same approach is used to obtain the remaining values as shown in Eqn. (30)

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.7 & 1.6 & 1.2 & 1.0 & 1.3 \\ 2.0 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1.0 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix}$$

(30)

Considering mix ratios at the mid points from Eqn.(3) and substituting these pseudo mix ratios in turn into Eqn.(30) will yield the corresponding actual mix ratios.

For instance, considering point A_{112} we have:

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.7 & 1.6 & 1.2 & 1.0 & 1.3 \\ 2.0 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1.0 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.67 \\ 0.33 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0.63 \\ 1 \\ 1.67 \\ 1.90 \\ 1.60 \end{Bmatrix}$$

(31)

Solving, Z_1 = 0.63; Z_2 = 1.00; Z_3 = 1.67' Z_4 = 1.90; Z_5 = 1.60

The same approach goes for the remaining mid-point mix ratios.

Hence, to generate the polynomial coefficients, thirty-five (35) experimental tests will be carried out and the corresponding mix ratios are depicted in Table 1.

Table 1: Pseudo (X) and Actual (Z) Mix Ratio for SFRC based on Scheffe's (5.3) Lattice.

Points	Pseudo Component					Response Symbol	Actual Component				
	X_1	X_2	X_3	X_4	X_5		Z_1	Z_2	Z_3	Z_4	Z_5
1	1	0	0	0	0	N_1	0.67	1.00	1.70	2.0	0.5
2	0	1	0	0	0	N_2	0.56	1.00	1.60	1.8	0.8
3	0	0	1	0	0	N_3	0.50	1.00	1.20	1.7	1.0
4	0	0	0	1	0	N_4	0.70	1.00	1.00	1.8	1.2
5	0	0	0	0	1	N_5	0.75	1.00	1.30	1.2	1.5
112	0.67	0.33	0	0	0	N_{112}	0.63	1.00	1.67	1.9	1.6
122	0.33	0.67	0	0	0	N_{122}	0.60	1.00	1.63	1.8	0.7
113	0.67	0	0.33	0	0	N_{113}	0.61	1.00	1.54	1.9	0.6
133	0.33	0	0.67	0	0	N_{133}	0.56	1.00	1.37	1.8	0.8
114	0.67	0	0	0.33	0	N_{114}	0.68	1.00	1.47	1.9	0.7
144	0.33	0	0	0.67	0	N_{144}	0.69	1.00	1.23	1.8	0.9
115	0.67	0	0	0	0.33	N_{115}	0.70	1.00	1.57	1.7	0.8
155	0.33	0	0	0	0.67	N_{115}	0.72	1.00	1.43	1.4	1.1
223	0	0.67	0.33	0	0	N_{223}	0.55	1.00	1.40	1.7	0.8
233	0	0.33	0.67	0	0	N_{233}	0.52	1.00	1.20	1.7	0.9
224	0	0.67	0	0.33	0	N_{224}	0.61	1.00	1.67	1.8	0.9
244	0	0.33	0	0.67	0	N_{244}	0.66	1.00	1.73	1.8	1.0
225	0	0.67	0	0	0.33	N_{225}	0.63	1.00	1.50	1.6	0.7
255	0	0.33	0	0	0.67	N_{255}	0.69	1.00	1.40	1.4	0.6
334	0	0	0.67	0.33	0	N_{334}	0.57	1.00	1.13	1.7	1.0
344	0	0	0.33	0.67	0	N_{344}	0.64	1.00	1.07	1.7	1.1
335	0	0	0.67	0	0.33	N_{355}	0.58	1.00	1.23	1.5	1.1
355	0	0	0.33	0	0.67	N_{335}	0.67	1.00	1.27	1.3	1.3
445	0	0.33	0	0	0.67	N_{445}	0.72	1.00	1.10	1.6	1.3
455	0	0	0	0.67	0.33	N_{445}	0.73	1.00	1.20	1.4	1.4
123	0.33	0.33	0.33	0	0	N_{123}	0.57	1.00	1.49	1.8	0.7
124	0.33	0.33	0	0.33	0	N_{124}	0.64	1.00	1.09	1.8	0.8
125	0.33	0.33	0	0	0.33	N_{125}	0.66	1.00	1.52	1.6	0.9
134	0.33	0.33	0	0.33	0	N_{134}	0.62	1.00	1.29	1.8	0.8
135	0.33	0	0.33	0	0.33	N_{135}	0.63	1.00	1.39	1.6	0.9
145	0.33	0	0	0.33	0.33	N_{145}	0.70	1.00	1.32	1.6	1.0
234	0	0.33	0.33	0.33	0	N_{234}	0.58	1.00	1.25	1.7	0.9
235	0	0.33	0.33	0	0.33	N_{235}	0.60	1.00	1.32	1.5	1.0
245	0	0.33	0	0.33	0.33	N_{245}	0.67	1.00	1.29	1.5	1.1
345	0	0	0.33	0.33	0.33	N_{345}	0.64	1.00	1.6	1.5	1.2

2.7. THE CONTROL POINTS

Thirty five (35) different controls were predicted which according to Scheffe's (1958) ,their summation should not be greater than one. The same approach for component transformation adopted for the initial experimental points are also adopted for the control points and the results are shown in Table 2.

Table 2 : Actual and Pseudo Component of SFRC Based on Scheffe (5,3) Lattice for Control Points

Points	Pseudo Component					Control Points	Actual Component				
	X_1	X_2	X_3	X_4	X_5		Z_1	Z_2	Z_3	Z_4	Z_5
1	0.25	0.25	0.25	0.25	0	C_1	0.61	1	1.38	1.83	0.5
2	0.25	0.25	0.25	0	0.25	C_2	0.62	1	1.45	1.68	0.8
3	0.25	0.25	0	0.25	0.25	C_3	0.67	1	1.40	1.70	1
4	0.25	0	0.25	0.25	0.25	C_4	0.66	1	1.30	1.68	1.2
5	0	0.25	0.25	0.25	0.25	C_5	0.63	1	1.28	1.63	1.5
112	0.20	0.20	0.2	0.20	0.20	C_{112}	0.64	1	1.36	1.70	0.65
122	0.30	0.30	0.30	0.10	0	C_{122}	0.59	1	1.45	1.83	0.75
113	0.30	0.30	0.30	0	0.10	C_{113}	0.59	1	1.48	1.77	0.85
133	0.30	0.30	0	0.30	0.10	C_{133}	0.65	1	1.42	1.80	1
114	0.30	0	0.30	0.30	0.10	C_{114}	0.64	1	1.30	1.77	0.9
144	0	0.30	0.30	0.30	0.10	C_{144}	0.60	1	1.27	1.71	1
115	0.10	0.30	0.30	0.30	0	C_{115}	0.60	1	1.31	1.79	1.55
155	0.30	0.10	0.30	0.30	0	C_{155}	0.62	1	1.33	1.83	1.1
223	0.30	0.10	0.30	0.30	0	C_{223}	0.63	1	1.41	1.85	1.25
233	0.10	0.20	0.30	0.40	0	C_{233}	0.61	1	1.25	1.79	1.35
224	0.30	0.20	0.10	0.40	0	C_{224}	0.64	1	1.35	1.85	0.89
244	0.20	0.20	0.10	0.10	0.40	C_{244}	1.40	1	1.04	1.59	1.08
225	0.30	0.10	0.30	0.20	0.10	C_{225}	0.62	1	1.36	1.77	0.92
255	0.25	0.25	0.15	0.15	0.20	C_{255}	0.61	1	1.51	3.16	0.91
334	0.30	0.30	0.20	0.10	0.10	C_{334}	0.68	1	1.56	1.96	0.98
344	0.10	0.30	0.30	0.30	0	C_{344}	1.30	1	1.31	1.79	0.95
335	0.25	0.15	0.20	0.20	0.20	C_{335}	0.65	1	0.96	1.05	0.97
355	0.15	0.25	0.20	0.20	0.20	C_{355}	0.64	1	1.37	1.71	0.79
445	0.10	0.20	0.30	0.40	0	C_{445}	0.61	1	1.25	1.79	0.99
455	0.30	0.10	0.20	0.30	0.10	C_{455}	0.61	1	1.31	1.72	1.03
123	0.25	0.10	0.40	0	0.25	C_{123}	0.61	1	1.39	1.66	0.98
124	0.30	0.20	0.40	0.10	0	C_{124}	0.58	1	1.41	1.82	0.83
125	0.15	0.15	0.20	0.10	0.40	C_{125}	0.65	1	1.36	1.57	1.11
134	0.10	0.30	0	0.30	0.30	C_{134}	0.67	1	1.34	1.65	1.10
135	0.25	0.20	0.20	0.20	0.15	C_{135}	0.74	1	1.38	2.08	0.88
145	0.10	0.10	0.10	0.30	0.40	C_{145}	0.68	1	1.27	1.57	1.19
234	0.40	0.20	0.10	0.10	0.20	C_{234}	0.73	1	1.61	1.87	1.03
235	0.25	0.25	0.15	0.25	0.10	C_{235}	0.63	1	1.39	1.78	0.93
245	0.15	0.20	0.10	0.25	0.30	C_{245}	0.66	1	1.34	1.64	1.09
345	0.30	0.10	0.20	0.25	0.15	C_{345}	0.64	1	1.34	1.75	0.96

The actual component as transformed from Eqn. (30) , Table (1) and (2) were used to measure out the quantities of water/cement ratio (Z_1), cement (Z_2), fine aggregate (Z_3), coarse aggregate (Z_4) and steel fibre

(Z_5) in their respective ratios for the concrete cube strength test.

III. MATERIALS AND METHODS

3.1 MATERIALS

Cement, water, fine and coarse aggregates and steel fibre are the materials under investigation in this research work. The cement is Dangote cement, a brand of Ordinary Portland Cement, conforming to British Standard Institution BS 12 (1978). The fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite of 20mm size was obtained from a local stone market and was downgraded to 4.75mm. The same size and nature of steel fibre used previously by Nwachukwu and others (2022b) is the same as the one being used in this present work. Also, potable water from the clean water source was used in this experimental investigation.

3.2. METHOD

3.2.1. SPECIMEN PREPARATION / BATCHING/ CURING

The specimens for the compressive strength were concrete cubes. They were cast in steel mould measuring 15cm*15cm*15cm. The mould and its base were damped together during concrete casting to prevent leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 70 mix ratios were to be used to produce 140 prototype concrete cubes. Thirty-five (35) out of the 70 mix ratios were as control mix ratios to produce 70 cubes for the conformation of the adequacy of the mixture design given by Eqn. (9), whose coefficients are given in Eqns. (11) – (28). Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28days of curing the specimens were taken out of the curing tank.

3.2.2. COMPRESSIVE STRENGTH TEST

Compressive strength testing was done in accordance with BS 1881 – part 116 (1983) - Method of determination of compressive strength of concrete cube and ACI (1989) guideline. Two samples were crushed for each mix ratio and in each case, the compressive strength was then calculated using Eqn. (32)

$$\text{Compressive Strength} = \frac{\text{Average failure Load,P (N)}}{\text{Cross– sectional Area,A (mm2)}}$$

(32)

IV. RESULTS AND DISCUSSION

4.1. COMPRESSIVE STRENGTH RESULTS FOR THE INITIAL EXPERIMENTAL TESTS.

The results of the compressive strength ($R_{esponse,}$ N_i) based on a 28-days strength is presented in Table 3. These are calculated from Eqn.(32)

Table 3: 28th Day Compressive Strength Test Results for SFRC Based on Scheffe's (5, 3) Model for the Initial Experimental Tests.

Points	Experimental Number	Response, N_i, MPa	Response Symbol	$\sum N_i$	Average Response N, MPa
1	1A 1B	28.54 28.43	N_1	56.97	28.49
2	2A 2B	20.44 21.06	N_2	41.50	20.75
3	3A 3B	26.65 26.32	N_3	52.97	26.49
4	4A 4B	22.56 21.98	N_4	44.54	22.27

5	5A 5B	19.22 19.42	N_5	38.64	19.32
112	6A 6B	23.43 23.38	N_{112}	46.81	23.41
122	7A 7B	25.08 24.86	N_{122}	49.94	24.97
113	8A 8B	24.66 24.62	N_{113}	49.28	24.64
133	9A 9B	27.78 27.76	N_{133}	55.54	27.77
114	10A 10B	26.98 26.78	N_{114}	53.76	26.88
144	11A 11B	33.75 33.73	N_{144}	67.48	33.74
115	12A 12B	29.11 29.23	N_{115}	58.34	29.17
155	13A 13B	21.32 21.43	N_{155}	42.75	21.38
223	14A 14B	23.21 24.00	N_{223}	47.21	23.61
233	15A 15B	30.23 29.96	N_{233}	60.19	30.10
224	16A 16B	26.48 26.48	N_{224}	52.96	26.48
244	17A 17B	21.80 22.02	N_{244}	43.82	21.91
225	18A 18B	18.98 18.88	N_{225}	37.86	18.93
255	19A 19B	23.54 23.58	N_{255}	47.12	23.56
334	20A 20B	20.77 20.87	N_{334}	41.64	20.82
344	21A 21B	27.46 27.43	N_{344}	54.89	27.45
335	22A 22B	22.33 22.35	N_{335}	44.68	22.34
355	23A 23B	31.87 32.01	N_{355}	63.88	31.94
445	24A 24B	28.86 28.69	N_{445}	57.55	28.78
455	25A 25B	24.54 24.66	N_{455}	49.20	24.60
123	26A 26B	25.54 25.48	N_{123}	51.02	25.51
124	27A 27B	20.86 21.21	N_{124}	42.07	21.04
125	28A 28B	23.76 24.22	N_{125}	47.98	23.99

134	29A 29B	19.88 20.06	N_{134}	39.94	19.97
135	30A 30B	29.18 28.88	N_{135}	58.06	29.03
145	31A 31B	22.65 22.39	N_{145}	45.04	22.52
234	32A 32B	23.34 22.94	N_{234}	46.28	23.14
235	33A 33B	18.64 18.62	N_{235}	37.26	18.63
245	34A 34B	22.23 22.45	N_{245}	44.68	22.34
345	35A 35B	20.11 19.86	N_{345}	39.97	19.99

4.2 COMPRESSIVE STRENGTH RESULTS FOR THE EXPERIMENTAL (CONTROL) TEST.

Table 4 shows the 28th day Compressive strength results for the Experimental (Control) Test

Table 4: 28TH Day Compressive Strength Results for SFRC Based on Scheffe's (5, 3) Model for the Experimental (Control) Tests.

Control Points	Experimental Number	Response, MPa	Average Response, MPa
C_1	1A 1B	27.67 26.98	27.33
C_2	2A 2B	21.24 20.66	20.95
C_3	3A 3B	27.43 27.54	27.49
C_4	4A 4B	22.00 22.08	22.04
C_5	5A 5B	20.33 20.45	20.33
C_{112}	6A 6B	22.98 22.76	22.87
C_{122}	7A 7B	25.22 25.32	25.27
C_{113}	8A 8B	23.88 23.76	23.82
C_{133}	9A 9B	28.08 27.98	28.08
C_{114}	10A 10B	26..09 26.15	26.12
C_{144}	11A 11B	32.04 32.45	32.25
C_{115}	12A 12B	28.96 29.11	29.04
C_{155}	13A	22.23	22.34

	13B	22.44	
C_{223}	14A 14B	24.08 24.21	24.15
C_{233}	15A 15B	29.87 29.54	29.71
C_{224}	16A 16B	25.44 25.18	25.31
C_{244}	17A 17B	22.16 22.21	22.19
C_{225}	18A 18B	20.02 19.86	19.94
C_{255}	19A 19B	23.06 23.22	23.14
C_{334}	20A 20B	21.26 20.98	21.12
C_{344}	21A 21B	26.44 27..08	26.76
C_{335}	22A 22B	22.45 22.48	22.47
C_{355}	23A 23B	30.98 31.12	31.05
C_{445}	24A 24B	28.05 28.28	28.17
C_{455}	25A 25B	25.24 24.88	25.06
C_{123}	26A 26B	25.04 25.32	25.18
C_{124}	27A 27B	21.54 21.48	21.51
C_{125}	28A 28B	24.21 24.33	24.27
C_{134}	29A 29B	19.66 19.43	19.55
C_{135}	30A 30B	28.76 28.43	28.60
C_{145}	31A 31B	21.87 22.12	22.00
C_{234}	32A 32B	22.76 22.88	22.82
C_{235}	33A 33B	18.76 18.08	18.42
C_{245}	34A 34B	21.23 22.04	21.64
C_{345}	35A 35B	20.32 20.11	20.22

4.3 SCHEFFE’S (5,3) REGRESSION MODEL FOR COMPRESSIVE STRENGTH OF SFRC

Substituting the values of the compressive strengths (responses) from Table 3 into Eqns. (11) through (28), we obtain the coefficients (in MPa) of the Scheffe's third degree polynomial as follows:

β_1 = 28.49; β_2 = 20.75; β_3 = 26.49; β_4 =22.27; β_5 = 19.32; β_{12} = -1.94; β_{13} = -2.36; β_{14} = 22.19; β_{15} = 6.21; β_{23} = 14.56; β_{24} = 12.08; β_{25} = 5.45; β_{34} = -1.10; β_{35} = 19.06; β_{45} = 26.53; γ_{12} = 309.15; γ_{13} = 349.27; γ_{14} =175.64; γ_{15} = 142.54; γ_{23} =375.46; γ_{24} =330.05; γ_{25} = 283.59; γ_{34} =140.59; γ_{35} =350.26; γ_{45} = 353.68; β_{123} = -183.72; β_{124} = -333.40; β_{125} = -152.73; β_{134} = -375.78; β_{135} = -110.53; β_{145} = -262.84; β_{234} = -158.63; β_{235} = -287.73; β_{245} = -230.31; β_{345} =.-359.62 (33)

Substituting the values of these coefficients in Eqn.(33) into Eqn. (9), we obtain the regression model for the optimization of the compressive strength of SFRC based on Scheffe's (5,3) lattice as given in Eqn.(34).

$$N= 28.49X_1 + 20.75X_2 + 26.49X_3 + 22.27X_4 + 19.32X_5 - 1.94X_1X_2 - 2.36X_1X_3 + 22.19X_1X_4 + 6.21X_1X_5 + 14.56X_2X_3 + 12.08X_2X_4 + 5.45X_2X_5 - 1.10X_3X_4 + 19.06X_3X_5 + 26.53X_4X_5 + 309.15X_1X_2^2 + 349.27X_1X_3^2 + 175.64X_1X_4^2 + 142.54X_1X_5^2 + 375.46X_2X_3^2 + 330.05X_2X_4^2 + 283.59X_2X_5^2 + 140.59X_3X_4^2 + 350.26X_3X_5^2 + 353.68X_4X_5^2 - 183.72X_1X_2X_3 - 333.40X_1X_2X_4 - 152.73X_1X_2X_5 - 375.78X_1X_3X_4 - 110.53X_1X_3X_5 - 262.84X_1X_4X_5 - 158.63X_2X_3X_4 - 287.73X_2X_3X_5 - 230.31X_2X_4X_5 - 359.62X_3X_4X_5 \quad (34)$$

4.4. SCHEFFE'S (5,3) MODEL RESPONSES FOR SFRC AT CONTROL POINTS

By substituting the pseudo mix ratio of points c_1, c_2, c_3, c_4, c_5, … ….. c_{345} of Table 2 into Eqn.(34) , we obtain the third degree model responses for the control points.

4.5 VALIDATION AND TEST OF ADEQUACY OF THE SCHEFFE'S (5,3) MODEL

The major objective of performing the test of adequacy is to check if there is any significant difference between the lab responses (compressive strength results) given in Table 4 and model responses from the control points based on Eqn.(34). Here, the Student's – T - test is adopted as the means of validating the Sheffe's model. Note that the procedures for using the Student's – T - test have been explained by Nwachukwu and others (2022 c). The result of the test shows that there is no significant difference between the experimental results and model results. Therefore, the model is very adequate for predicting the compressive strength of SFRC based on Scheffe's (5,3) lattice.

4.6. DISCUSSION OF RESULTS

The maximum compressive strength of SFRC based on Scheffe's (5,3) lattice is 33.74MPa which corresponds to mix ratio of 0.70:1.00:1.00:1.80:1.20 for water/cement ratio, cement, fine aggregate, coarse aggregate and steel fibre respectively.. Similarly, the lowest compressive strength of 18.63 MPa was obtained through the Scheffe's third degree lattice which corresponds to mix ratio of 0.60:1.00:1.32:1.50:1.00. The highest strength value from the model was found to be greater than the minimum value specified by the American Concrete Institute for the compressive strength of good concrete. Therefore, using the model, compressive strength of SFRC of all points (1 - 345) in the simplex can be evaluated based on Scheffe's third degree model.

CONCLUSION

In this work, Scheffe's Third Degree Regression Model, Scheffe's (5, 3) was presented and used to predict the mix proportions as well as a model for predicting the compressive strength of SFRC cubes. By using Scheffe's (5, 3) simplex model, the values of the compressive strength were obtained for SFRC at all 35 points. The outcome of the student's t-test confirmed that there exists a good correlation between the strengths predicted by the models and the corresponding experimentally observed results. The optimum attainable compressive strength of SFRC predicted by the Scheffe's (5, 3) model at the 28th day was 33.74MPa. This value is higher than the highest value (27.81 MPa) obtained by Nwachukwu and others (2022b) for SFRC based on Scheffe's (5,2) model. However, both optimum values meet the minimum standard requirement (of 20 MPa) stipulated by American Concrete Institute (ACI) for the compressive strength of good concrete. Thus, with the Scheffe's (5,3) model, any desired strength of Steel Fibre Reinforced Concrete, given any mix proportions can be easily predicted and evaluated and vice versa. Finally, by the utilization of this Scheffe's optimization model, the problem of having to go through vigorous, time-consuming and laborious mix-design procedures to obtain a desiring strength of SFRC has been reduced.

REFERENCES

[1] ACI Committee 211. (1994): "Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete (ACI 211.1R-91)"; American Concrete Institute, ACI Manual of Concrete Practice, Detroit, Michigan, USA.

[2] ACI Committee 544. (1982): "State-of-the-Report on Fibre Reinforced Concrete, (ACI 544.1R-82)"; *Concrete International: Design and Construction*. Vol. 4, No. 5: Pp. 9-30, American Concrete Institute, Detroit, Michigan, USA.

[3] ACI Committee 544. (1989): "Measurement of Properties of Fibre Reinforced Concrete, (ACI 544.2R-89)"; American Concrete Institute, Detroit, Michigan, USA.

[4] Aggarwal, M.L. (2002): "Mixture Experiments: Design Workshop Lecture Notes", University of Delhi, India.

[5] Arube, G.E, Akusobo, I.S and Damini, R.G. (2021): "Analyzing the Effects of Steel Fibres in Concrete Paving Blocks" International Journal of Civil Engineering Construction and Estate Managemen, Vol.9, No.2, Pp.38- 51.

[6] Baros, J.A.O.., Cunha, V.M., Ribeiro, A.F. and Antunes, J.A.B. (2005):"Post- Cracking Behaviour of Steel Fibre Reinforced Concrete"; Journal of Materials and Structures , Vol. 38. Pp 47-56.

[7] Blaszczynski, T, and Przybylska-Falek, M. (2015):"Steel Fibre Reinforced Concrete As a Structural Material "; Procedia Engineering, Science Direct, Vol. 122. Pp 282-289.

[8] British Standards Institution, BS 12 (1978): Ordinary and Rapid – Hardening Portland cement; London.

[9] British Standards Institution, BS 1881-Part 116 (1983). Methods of Determination of Compressive Strength of Concrete Cube, London.,

[10] Department of Environment, DOE(1988): Design of Normal Concrete Mixes , HMSO, London.

[11] Ettu, L.O. (2001): Design Of Reinforced Concrete Elements To BS 8110 (1997), Charismatic Forum Publication Ltd, Oweeri, Nigeria

[12] Ezeh, J.C.& Ibearugbulam, O.M. (2009):"Application of Scheffe's Model in Optimization of Compressive Cube Strength of River Stone Aggregate Concrete";*International Journal of Natural and Appllied Sciences;*Vol. 5, No. 4,Pp 303 – 308 .

[13] Ezeh, J.C., Ibearugbulam, O.M.& Anyaogu, L. (2010a):"Optimization of Compressive Strength of Cement- Sawdust Ash Sandcrete Block using Scheffe's Mathematical Model"; *Journal of Applied Engineering Research.* Vol.4, No.4, Pp 487–494.

[14] Ezeh, J.C., Ibearugbulam, O.M.&Anya, U. C (2010b):"Optimization of aggregate composition of laterite/sand hollow Block Using Scheffe's Simplex Method";*International Journal of Engineering.* Vol.4, No.4, Pp 471 – 478.

[15] Ghaffar,, .A.., Chavhan, A.S., and Tatwawadi, R.S. (2014):"Steel Fibre Reinforced Concrete"; International Journal of Engineering Trends And Technology (IJETT), Vol..9., No. 15, Pp 791-797.

[16] Huang, C., Cunha, and Zhao, G. (1995):"Properties of Steel Fibre Reinforced Concrete Containing Larger Coarse Aggregate"; Science Direct.

[17] Hughes, B.P. (197I): The Economic Utilization of Concrete Materials, Proceedings of the Symposium on Advances in Concrete , Concrete Society, London

[18] Ibearugbulam, O.M. (2006):"Mathematical Model for Optimization of Compressive Strength of Periwinkle Shell-Granite Aggregate Concrete";*M.Eng.. Thesis,* Federal University of Technology, Owerri, Nigeria.

[19] Jackson,N. and Dhir, R.K. (1996): Civil Engineering Materials, 5th Edition, Palgrave, New York.

[20] Jean-Louis, G. and Sana, U.B. (2005):"Corrosion of Steel Fibre Reinforced Concrete From The Crack", Journal of Cement and Concrete Research , Vol. 35. Pp 572 -577.

[21] khaloo, A. R. and Afshari, M. (2005):" Flexural Behaviour of Small Steel Fibre Reinforced

Concrete Slabs "; Science Direct, Vol. 27, No.1. Pp 141 -149.

[22] Lau, A.and Anson, M. (2006):"Effect of High Temperatures on High Performance Steel Fibre Reinforced Concrete"; Science Direct, Vol. 36, No.9, Pp 1698 -1707.

[23] Lie, T.T. and Kodur, V.K.R. (1996):"Thermal and Mechanical Properties of Steel Fibre Reinforced Concrete At Elevated Temperatures ";Canadian Journal of Civil Engineering , Vol. 23, No. 4, Pp 511 -517.

[24] Lima, D.H. and Oh, B.H. (1999):"Experimental and Theoretical Investigation On The Shear Of Steel Fibre Reinforced Concrete Beam "; Journal of Engineering Structures , Vol. 21, No. 10, Pp 937 -944.

[25] Neville, A.M. (1990): Properties of Concrete; 3rd edition, Pitman, England.

[26] Nwachukwu,, K. C., Njoku, ,K.O., Okorie ,P. O, .Akosubo,, I.S. , Uzoukwu , C. S.,Ihemegbulem, E.O. and .Igbojiaku , A.U (2022a):" Comparison Between Scheffe's Second Degree (5,2) And Third Degree (5,3) Polynomial Models In The Optimization Of Compressive Strength Of Glass Fibre Reinforced Concrete (GFRC)"; American Journal of Computing and Engineering (AJCE), Vol. 5, No. 1, Pp 1– 23.

[27] Nwachukwu, K.C., Okafor, M. ,Thomas, B., Oputa, A.A., Okodugha,D.A. and Osaigbovo, M.E. (2017): "An Improved Model For The Optimization Of The Compressive Strength Of Glass Fibre Reinforced Concrete (GFRC) Using Scheffe's Second Degree Polynomials"; Researchjournal's Journal of Civil Engineering, Vol. 3, No. 2

[28] Nwachukwu, K.C.,Okodugha, D.A., Uzoukwu , C. S.,Igbojiaku, A.U. and Okafor, M. (2022b): "Application Of Scheffe's Second Degree Mathematical Model For The Optimization Of Compressive Strength Of Steel Fibre Reinforced Concrete (SFRC)"; International Journal of Advances in Engineering and Management (IJAEM) , Vol. 4, No. 3.

[29] Nwachukwu, K.C., Okorie, P.O.. Ihemegbulem , E.O., Kemebiye Dimaro , Uzoukwu , C. S., and Maduagwu, J.C.. (2022c): "The Use Of Scheffe's Model For The Optimization Of Compressive Strength Of Polypropylene Fibre Reinforced Concrete (PFRC)"; International Journal of Innovative Science and Research Technology (IJISRT), Vol. 7, No. 1

[30] Nwachukwu, K.C.,Ozioko, H.O., Okorie, P.O, and Uzoukwu , C. S. (2022d): "Application Of Scheffe's Mathematical Model For The Optimization Of Compressive Strength Of Nylon Fibre Reinforced Concrete (NFRC)"; International Journal of Advances in Engineering and Management (IJAEM) , Vol. 4, No. 2, Pp 850-862

[31] Nwachukwu, K. C., Okodugha, D.A., Igbojiaku , A.U. , Attah, U.G. And Ihemegbulem, E.O. (2022e): Application Of Scheffe's Third Degree Regression Model For The Optimization Of Compressive Strength Of Polypropylene Fibre Reinforced Concrete (PFRC); International Journal Of Advance Research And Innovative Ideas In Education (IJARIIE) , Vol. 8, No. 2.

[32] Nwachukwu, K. C., Okodugha,, D.A., Edike, O., Okafor, M.. And Attah, U.G. (2022f): Application Of Modified Scheffe's Third Degree Polynomial Model For The Optimization Of Compressive Strength Of Nylon Fibre Reinforced Concrete (NFRC); International Journal of Trend In Scientific Research And Development (IJTSRD) , Vol. 6, No. 3.

[33] Nwakonobi, T.U and Osadebe, N.N (2008):"Optimization Model for Mix Proportioning of Clay- Ricehusk- Cement Mixture for Animal Buildings"; *Agricultural Engineering International:the CIGR Ejournal,Manuscript BC 08 007*, Vol x

[34] Obam, S.O. (2006). The Accuracy of Scheffe's Third Degree over Second Degree Optimization Regression Polynomials, Nigerian Journal of Technology, Vol. 2, No.25, Pp 1 – 10.

[35] Obam, S.O.(2009):"A Mathematical model for Optimization of Strength of concrete : A case study for shear modulus of Rice Husk Ash Concrete." *Journal of Industrial Engineering International l;* Vol.5, No.9,Pp 76 – 84.

[36] Okere, C.E., (2006):"Mathematical Models for Optimization of the Modulus of rupture of concrete"; M.Eng. Thesis, Civil Engineering Department, Federal University of Technology, Owerri.

[37] Onwuka,D.O, Okere, C.E.,Arimanwa, J.I. and Onwuka, S.U.(2011):"Prediction of Concrete

Mix ratios using Modified Regression Theory, *Computer Methods in Civil Engineering*, Vol. 2. No.1 Pp.95- 107.

[38] Oyenuga, V.O. (2008): Simplified Reinforced Concrete Design, 2nd Edition, Asros Ltd, Lagos, Nigeria.

[39] Scheffe, H. (1958): Experiment with Mixtures"; *International Journal of Royal Statistics Society*, Series B, Vol.20, Pp. 344-360.

[40] Shetty, M.S. (2006): Concrete Technology (Theory and Practice), S. Chand and Company Ltd, India.

[41] Syal, I.C. and Goel, A.K. (2007): Reinforced Concrete Structures, S.Chand , India.