Evaluation and Formulation of Raleigh- Ritz Based Peculiar Total Potential Energy Functional (TPEF) For Mono Symmetric Single (MSS) Cell Thin- Walled Box Column (TWBC) Cross- Section in Preparation for The Stability Analysis of TWBC

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Abstract- This research work is aimed at evaluating, as well as formulating the peculiar Total Potential Energy Functional (TPEF) for a Mono Symmetric Single (MSS) cell Thin -walled Box Column (TWBC). It is one of the peculiar TPEF which is a follow up of the formulation of the general/governing TPEF for TWBC by Nwachukwu and others (2017). For the cross – section under consideration, the cross –sectional properties are evaluated first to obtain the cross-sectional areas and moment of inertia. This is followed by the formulation of the TPEF for the cross- sections for different boundary conditions. The formulated Energy Functional Equations support the stability analysis of a MSS cell thin-walled box (closed) column cross-section using Raleigh - Ritz Method (RRM) .The Raleigh- Ritz based formulated TPEF equations are found suitable, handy and simple to be used in the Flexural(F) , Flexural- Torsional(FT) and Flexural- Torsional- Distortional(FTD) buckling/stability analysis of MSS cell TWBC cross-section where data obtained (critical bulking loads) will be compared with the works of other authors in subsequent works.

Indexed Terms- Mono Symmetric Single (MSS) Cell, Total Potential Energy Functional (TPEF), Thin - Walled Box Column (TWBC)or Thin-Walled Column (TWC), Raleigh- Ritz Method (RRM), Bulkling/ Stability Analysis

I. INTRODUCTION

According to Murray (1984), a thin-walled structure (TWS) is one which is made from thin plates joined along their edges. The plate thickness for the TWS however is small compared to other cross sectional dimensions which are in turn often small compared with the overall length of the member or structure. Thin-walled columns (TWC) as well as other TWS are very light compared with alternative structures and therefore, they are used extensively in long-span bridges and other structures where weight and cost are prime considerations.

MSS are common examples of TWC cross –sections .According to Simao and Simoes da silva (2004), it is a common knowledge that the use of very slender thin-walled cross-sections members have become increasingly in demand due to their high stiffness/weight ratio, in recent years. In general, according to Ezeh and Osadebe (2010), thin-walled structures consist of a wide and growing field of engineering application which seeks efficiency and effectiveness in strength and cost by minimizing material. For about a century many branches of the industry have sought stronger and at the same time lighter structural solutions which optimize the effectiveness and the cost of the structures (Andreassen, 2012). Such industries cut across civil, mechanical, naval, and aerospace industries.

This recent study is an attempt to evaluate and formulate the TPEF for MSS cross-sections. It is the follow up of the works by Nwachukwu and others (2017) and Nwachukwu and others (2021) where the governing equation for the TPEF for a TWBC applicable to RRM and peculiar TPEF for DSS cross – section were derived respectively . Before now, many researchers have carried out one form of analysis or the other on thin- walled box columns and related topics, but none has been able to address the present subject matter. For instant, Krolak and others (2009) presented a theoretical, numerical and experimental analysis of the stability and ultimate load of multi-cell thin-walled columns of rectangular and square cross-sections subjected to axial compression. Shanmugam and others (1989) presented a numerical method to investigate the ultimate strength behavior of thin-walled steel box columns subjected to axial loads and biaxial end moments. The work of Ezeh (2009) involved a theoretical formulation based on Vlasov's theory as modified by Varbanov, in analyzing flexural, flexural-torsional, and flexural-torsional-distortional buckling modes of thin-walled closed columns. Chidolue and Osadebe (2012), also used Vlasov's theory to carryout Torsional- Distortional analysis of thin- walled box girder bridges. Chidolue and Aginam (2012) investigated the effects of shape factor on the Flexural- Torsional-Distortional behavior of thin-walled box girder structures using Vlasov's Theory. Ezeh (2010) also investigated the buckling behavior of axially compressed multi- cell doubly symmetric thin-walled column using Vlasov's theory. The works of Osadebe and Chidolue (2012a), Osadebe and Chidolue (2012b), Osadebe and Ezeh (2009a), Osadebe and Ezeh (2009b) were also based on Vlasov's method. Nwachukwu and others (2017) and Nwachukwu and others (2021a) derived the RRM based governing TPEF equation for the TWBC applicable to RRM and evaluate and formulate the peculiar TPEF for DSS cross – section respectively . Finally, Nwachukwu and others (2021b) evaluated and formulated the TPEF for DSM and MSM cross section.

Thus in the area of stability analysis of thin-walled box (closed) columns, little or no effort has been done to use RRM to evaluate and formulate the peculiar TPEF for MSS cross section. Henceforth, the need for this recent research work. The formulated energy functional will now be used to analyze a MSS thin-walled box (closed) columns of different boundary conditions in subsequent works.

II. THEORITICAL BACKGROUND

Before now, the prominent total potential energy functional (TPEF), π for the TWBC is based on Vlasov' s theory and is given by Eqn. (1)

$$\Pi = \frac{1}{2}\int_L \int_S \left[\left\{ \frac{\delta^2}{E} + \frac{\tau^2_{(x,s)}}{G} \right\} t_{(s)} + \frac{M^2}{EI} - P\,(v^I)^2 \right] dxds \quad (1)$$

Eqn.(1) was used by Ezeh (2009) and Chidolue and Osadede (2012) to analyse a thin- walled box column and a thin- walled box girder bridge respectively using Vlasov' s theory. However, Eqn.(1) has been transformed by Nwachukwu and others (2017) in such a way that the Rayleigh- Ritz method can be applied. The transformed governing formulation is as stated in Eqn.(2).

$$\pi = k_1 \int_L v^2\, x^2 (2L-x)^2 dx + k_2 \int_L (v^I)^2\, dx + k_3 \int_L (v^{II})^2\, dx - k_4 \int_L (v^I)^2\, dx. \quad (2)$$

$$k_1 = \frac{Ap^2}{8EI^2}; \quad k_2 = \frac{AG}{2}; \quad k_3 = \frac{EI}{2}; \; and \;\; k_4 = \frac{P}{2} \quad 3(a\text{-}d)$$

Where P is critical buckling load, A is Cross sectional area, E is young modulus of elasticity, G is shear modulus, I is moment of inertia, and L is length of the column.

Here, v = the displacement function, which is a function of polynomial shape function, ϕ

According to Raleigh- Ritz Theory

$$v = \sum_i^n c_i\, \phi_i = c_1\phi_1 + c_2\phi_2 + c_3\phi_3 + \ldots\ldots + c_n\phi_n \quad (4)$$

Where ϕ = Polynomial shape function
c = undetermined coefficient / unknown constant.

It is noteworthy to note that Nwachukwu and others (2021a) have generated the Polynomial shape function used in the formulation and have also demonstrated the efficacy of Eqn. (2) by using it to formulate the peculiar TPEF for Doubly Symmetric Single (DSS) cell TWBC for different boundary conditions. Nwachukwu and others (2021b) have also formulated the peculiar TPFF for Doubly Symmetric Multi (DSM) cell TWBC and Mono Symmetric Multi (MSM) cell TWBC for different boundary conditions from the general/governing equation stated in Eqn. (2).

As a way of illustration, let us recall the peculiar TPEF formulated by Nwachukwu and others (2021a) for Fixed-Fixed or Clamped- Clamped (C-C) boundary condition as given under:

$$\pi_{DSS}^{C-C} = k_1{}^{DSS}\,\varphi_1{}^{C-C} + k_2{}^{DSS}\,\varphi_2{}^{C-C} + k_3{}^{DSS}\varphi_3{}^{C-C} - k_4{}^{DSS}\varphi_4{}^{C-C}. \quad (5)$$

$$= k_1{}^{DSS}\,[360{c_1}^2 L^{12} - 1575{c_1}^2L^{13} + 2870{c_1}^2L^{14} - 2772{c_1}^2L^{15} + \frac{16380{c_1}^2L^{16}}{11} - 420{c_1}^2L^{17} + \frac{630{c_1}^2L^{18}}{13} - \frac{72c_1c_2\sqrt{53900}\,L^{12}}{7} + 63c_1c_2\sqrt{53900}\,L^{13} - 162c_1c_2\sqrt{53900}\,L^{14} + \frac{2028c_1c_2\sqrt{53900}\,L^{15}}{10} - \frac{1836c_1c_2\sqrt{53900}\,L^{16}}{11} + 87c_1c_2\sqrt{53900}\,L^{17} - 24c_1c_2\sqrt{53900}\,L^{18} + \frac{36c_1c_2\sqrt{53900}\,L^{19}}{14} + 3960{c_2}^2L^{12} - 31185{c_2}^2L^{13} + 105490{c_2}^2L^{14} - 1995840{c_2}^2L^{15} + 230580{c_2}^2L^{16} - 166320{c_2}^2L^{17} + \frac{949410{c_1}^2L^{18}}{13} - 17820{c_2}^2L^{19} + 1848{c_2}^2L^{20}]$$

$$+k_2{}^{DSS}\,[840{c_1}^2L^2 - 3780{c_1}^2L^3 + 6552{c_1}^2L^4 - 5040{c_1}^2L^5 + 1440{c_1}^2L^6 - 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240{c_2}^2L^2 - 83160{c_2}^2L^3 + 310464{c_2}^2L^4 - 600600{c_2}^2L^5 + 633600{c_2}^2L^6 - 346500{c_2}^2L^7 + 77000{c_2}^2L^8]$$

$$+ k_3{}^{DSS}\,[\frac{2520{c_1}^2}{L^2} - \frac{8820{c_1}^2}{L} + 40320{c_1}^2 - 45360{c_1}^2L + 18144{c_1}^2L^2 - \frac{72c_1c_2\sqrt{53900}}{L^2} + \frac{648c_1c_2\sqrt{53900}}{L} - 2592c_1c_2\sqrt{53900} + 4896c_1c_2\sqrt{53900}\,L - 4320c_1c_2\sqrt{53900}L^2 + 1440c_1c_2\sqrt{53900}L^3 + \frac{27720{c_2}^2}{L^2} - \frac{332640{c_2}^2}{L} + 1884960{c_2}^2 - 5266800{c_2}^2L + 7650720{c_2}^2L^2 - 5544000{c_2}^2L^3 + 1584000{c_2}^2L^4]$$

$$-k_4{}^{DSS}\,[840{c_1}^2L^2 - 3780{c_1}^2L^3 + 6552{c_1}^2L^4 - 5040{c_1}^2L^5 + 1440{c_1}^2L^6 - 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240{c_2}^2L^2 - 83160{c_2}^2L^3 + 310464{c_2}^2L^4 - 600600{c_2}^2L^5 + 633600{c_2}^2L^6 - 346500{c_2}^2L^7 + 77000{c_2}^2L^8] \quad (6)$$

Where k_1, k_2, k_3 and k_4 are all defined in Eqns.3 (a-d) respectively, but in terms of DSS cross-section. With Eqns.(5) and (6) in focus, the next section comprises of evaluation of MSS cross-section and the eventual formulation of the MSS peculiar TPEF.

III. EVALUATION AND FORMULATION OF TPEF FOR MSS TWBC CROSS-SECTION

3.1 EVALUATION OF MSS CROSS- SECTIONAL PROPERTIES

Our interest in evaluating the cross-sectional properties are to determine the Cross- Sectional Area for MSS, A^{MSS} and its Moment of Inertia, I^{MSS}. An MSS thin walled box column cross section is shown in Fig.1.

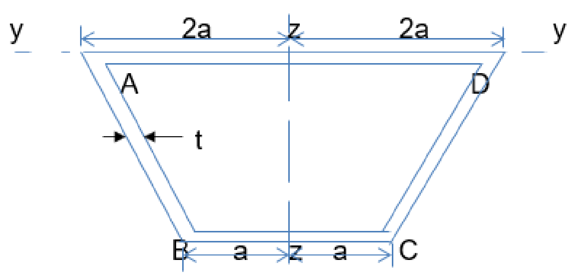

Fig.1: An MSS thin- walled column cross section

(a) CROSS- SECTIONAL AREA, A^{MSS} AND CENTROID.

Applying the thin- walled assumptions, we have in Fig.2 as follows:

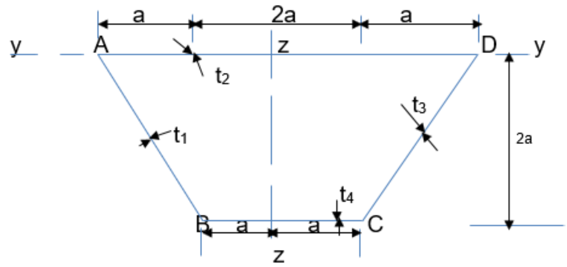

Fig.2: Thin – Walled Assumption for MSS

From Fig.2,

$$AB = CD = \sqrt{a^2 + (2a)^2} = \sqrt{5a^2} = a\sqrt{5} \quad (7)$$

The evaluation of the centroid and the cross- sectional area, are validated using Table 1

Table 1: Centroid and Cross- Sectional Area For MSS

Section	$\bar{y}_i$	$\bar{z}_i$	A_i
1	0	$-\frac{a\sqrt{5}}{2}$	$at\sqrt{5}$
2	2a	0	4at
3	0	$-\frac{a\sqrt{5}}{2}$	$at\sqrt{5}$
4	a	0	2at
SUM	$\sum \bar{y}_i = 3a$	$\sum \bar{z}_i = -a\sqrt{5}$	$\sum A_i = 6at + 2at\sqrt{5}$

Note $t_1 = t_2 = t_3 = t_4 = t$ (8)

Thus $A^{MSS} = \sum A_i = 6at + 2at\sqrt{5} = 10.472at$ (9)

$\bar{y}_G = 3a$ (10)

And $\bar{z}_G = -a\sqrt{5} = -2.236a$ (11)

(b) MOMENT OF INERTIA, I^{MSS}

Evaluation process for moment of inertia I^{MSS} is shown in Table 2.

Since the cross-section is mono-symmetric about Z-Z, we concentrate on finding the moment of inertia about Z-Z as illustrated in Table 2.

Table 2: Moment of Inertia for MSS.

Section	1	2	3	4
I_{Zci}	$\frac{(t)^3 a\sqrt{5}}{12}$	$\frac{(t)^3 4a}{12}$	$\frac{(t)^3 a\sqrt{5}}{12}$	$\frac{(t)^3 2a}{12}$

Thus, $I_{ZZ} = 2 I_{zz1} + I_{zz2} + I_{zz4}$ (12)

Where

$I_{ZZ1} = I_{Zc1} + A_1[-(\bar{y}_G)]^2$

$= \frac{at^3\sqrt{5}}{12} + at\sqrt{5}\,[3a]^2 = 0.186at^3 + 20.125a^3 t$

$\therefore 2 I_{ZZ1} = 0.372at^3 + 40.25a^3 t$ (13)

Similarly,

$I_{ZZ2} = I_{Zc2} + A_2[\bar{y}_2 - \bar{y}_G]^2 = 0.333at^3 + 4a^3 t$ (14)

Again

$I_{ZZ4} = I_{Zc4} + A_4[\bar{y}_4 - \bar{y}_G]^2 = 0.167at^3 + 8a^3 t$ (15)

Thus, $I^{MSS} = I_{ZZ} = 0.872at^3 + 52.25a^3 t$ (16)

3.2 FORMULATION OF TPEF FOR MSS DIFFERENT BOUNDARY CONDITION CASES

(a) CASE 1: PINNED-PINNED(S-S)- MSS THIN-WALLED COLUMN.

The total potential energy functional (TPEF) for MSS-[S-S] thin-walled box column, π_{MSS}^{S-S} can be evaluated as follows:

$$\pi_{MSS}^{S-S} = k_1^{MSS}\varphi_1^{S-S} + k_2^{MSS}\varphi_2^{S-S} + k_3^{MSS}\varphi_3^{S-S} - k_4^{MSS}\varphi_4^{S-S} \quad (17)$$

$$= k_1^{MSS}\Big[24c_1^2L^{10} - 60c_1^2L^{11} + \frac{390c_1^2L^{12}}{7} - 10c_1^2L^{13} + \frac{10c_1^2L^{14}}{3} - \frac{8c_1c_2\sqrt{6300}\,L^{10}}{5} + \frac{20c_1c_2\sqrt{6300}\,L^{11}}{3} - \frac{74c_1c_2\sqrt{6300}\,L^{12}}{7} + 8c_1c_2\sqrt{6300}\,L^{13} - \frac{26c_1c_2\sqrt{6300}\,L^{14}}{9} + \frac{2c_1c_2\sqrt{6300}\,L^{15}}{5} + 168c_2^2L^{10} - 980c_2^2L^{11} + 2310c_2^2L^{12} - \frac{5565c_2^2L^{13}}{2} + \frac{5390c_2^2L^{14}}{3} - 588c_2^2L^{15} + \frac{840c_2^2L^{16}}{11}\Big]$$

$$+ k_2^{MSS}[30c_1^2 - 60c_1^2L + 40c_1^2L^2 - 2c_1c_2\sqrt{6300}$$

$$+8\,c_1c_2\,\sqrt{6300}\,L - 12c_1c_2\,\sqrt{6300}\,L^2 + 6\,c_1c_2\,\sqrt{6300}\,L^3 + 210{c_2}^2 - 1260{c_2}^2L + 3360{c_2}^2L^2 - 3780{c_2}^2L^3 + 1512{c_2}^2L^4\,]$$

$$+ {k_3}^{MSS}\,[\frac{120{c_1}^2}{L^2} - \frac{24\,c_1c_2\sqrt{6300}}{L^2} + \frac{24\,c_1c_2\,\sqrt{6300}}{L} + \frac{7560{c_2}^2}{L^2} - \frac{15120{c_2}^2}{L} + 10080{c_2}^2]$$

$$-{k_4}^{MSS}\,[30{c_1}^2 - 60{c_1}^2\,L + 40{c_1}^2L^2 - 2\,c_1c_2\sqrt{6300} +8\,c_1c_2\,\sqrt{6300}\,L - 12c_1c_2\,\sqrt{6300}\,L^2 + 6\,c_1c_2\,\sqrt{6300}\,L^3 + 210{c_2}^2 - 1260{c_2}^2L + 3360{c_2}^2L^2 - 3780{c_2}^2L^3 + 1512{c_2}^2L^4\,] \quad (18)$$

Where

$${k_1}^{MSS} = \frac{A^{MSS}p^2}{8EI^{2(MSS)}},\quad {k_2}^{MSS} = \frac{A^{MSS}\mathrm{G}}{2},\ {k_3}^{MSS} = \frac{E\,I^{MSS}}{2}\ \&\ \ {k_4}^{MSS} = \frac{P}{2} \quad 19(a\text{-}d)$$

A^{MSS} and I^{MSS} are defined in Eqns.(11) and (16) respectively.

(b) CASE 2: FIXED-FIXED[C-C]- MSS THIN-WALLED COLUMN

The total potential energy functional for MSS-[C-C] thin-walled box column can be obtained as follows:

$$\pi_{MSS}^{C-C} = {k_1}^{MSS}\,{\varphi_1}^{C-C} + {k_2}^{MSS}\,{\varphi_2}^{C-C} + {k_3}^{MSS}{\varphi_3}^{C-C} - {k_4}^{MSS}{\varphi_4}^{C-C} \quad (20)$$

$$= {k_1}^{MSS}\,[360{c_1}^2\,L^{12} - 1575{c_1}^2L^{13} + 2870{c_1}^2L^{14} - 2772{c_1}^2L^{15} + \frac{16380{c_1}^2L^{16}}{11} - 420{c_1}^2L^{17} + \frac{630{c_1}^2L^{18}}{13} - \frac{72c_1c_2\sqrt{53900}\,L^{12}}{7} + 63c_1c_2\sqrt{53900}\,L^{13} - 162c_1c_2\sqrt{53900}\,L^{14} + \frac{2028c_1c_2\sqrt{53900}\,L^{15}}{10} - \frac{1836c_1c_2\sqrt{53900}\,L^{16}}{11} + 87c_1c_2\sqrt{53900}\,L^{17} - 24c_1c_2\sqrt{53900}\,L^{18} + \frac{36c_1c_2\sqrt{53900}\,L^{19}}{14} + 3960{c_2}^2\,L^{12} - 31185{c_2}^2L^{13} + 105490{c_2}^2\,L^{14} - 1995840{c_2}^2L^{15} + 230580{c_2}^2\,L^{16} - 166320{c_2}^2L^{17} + \frac{949410{c_1}^2L^{18}}{13} - 17820{c_2}^2L^{19} + 1848{c_2}^2L^{20}$$

$$+{k_2}^{MSS}\,[840{c_1}^2L^2 - 3780{c_1}^2L^3 + 6552{c_1}^2L^4 - 5040{c_1}^2L^5 + 1440{c_1}^2L^6 - 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240{c_2}^2L^2 - 83160{c_2}^2L^3 + 310464{c_2}^2L^4 - 600600{c_2}^2L^5 + 633600{c_2}^2L^6 - 346500{c_2}^2L^7 + 77000{c_2}^2L^8]$$

$$+ {k_3}^{MSS}\,[\frac{2520{c_1}^2}{L^2} - \frac{8820{c_1}^2}{L} + 40320{c_1}^2 - 45360{c_1}^2L + 18144{c_1}^2L^2 - \frac{72c_1c_2\,\sqrt{53900}}{L^2} + \frac{648c_1c_2\,\sqrt{53900}}{L} - 2592c_1c_2\,\sqrt{53900} + 4896c_1c_2\,\sqrt{53900}\,\mathrm{L} - 4320c_1c_2\,\sqrt{53900}L^2 + 1440c_1c_2\,\sqrt{53900}L^3 + \frac{27720{c_2}^2}{L^2} - \frac{332640{c_2}^2}{L} + 1884960{c_2}^2 - 5266800{c_2}^2L + 7650720{c_2}^2L^2 - 5544000{c_2}^2L^3 + 1584000{c_2}^2L^4\,]$$

$$-{k_4}^{MSS}\,[840{c_1}^2L^2 - 3780{c_1}^2L^3 + 6552{c_1}^2L^4 - 5040{c_1}^2L^5 + 1440{c_1}^2L^6 - 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240{c_2}^2L^2 - 83160{c_2}^2L^3 + 310464{c_2}^2L^4 - 600600{c_2}^2L^5 + 633600{c_2}^2L^6 - 346500{c_2}^2L^7 + 77000{c_2}^2L^8\,] \quad (21)$$

Where ${k_1}^{MSS}$, ${k_2}^{MSS}$, ${k_3}^{MSS}$ and ${k_4}^{MSS}$ are defined in Eqns. 19(a-d) respectively.

(c) CASE 3: FIXED-PINNED[C-S]- MSS THIN- WALLED COLUMN.

$$\pi_{MSS}^{C-S} = {k_1}^{MSS}\,{\varphi_1}^{C-S} + {k_2}^{MSS}\,{\varphi_2}^{C-S} + {k_3}^{MSS}{\varphi_3}^{C-S} - {k_4}^{MSS}{\varphi_4}^{C-S} \quad (22)$$

$$
\begin{aligned}
= {k_1}^{MSS} \Big[& \frac{22680{c_1}^2 L^{12}}{133} - \frac{98280{c_1}^2 L^{13}}{152} + \frac{174510{c_1}^2 L^{14}}{171} - \frac{162540{c_1}^2 L^{15}}{190} + \frac{83790{c_1}^2 L^{16}}{209} - \\
& \frac{22680{c_1}^2 L^{17}}{228} + \frac{2520{c_1}^2 L^{18}}{247} - \frac{216}{7} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{12} + \frac{10728}{8} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{13} - \\
& \frac{39078}{9} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{14} + \frac{58500}{10} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{15} - \frac{44640}{11} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{16} \\
& + \frac{18000}{12} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{17} - \frac{3546}{13} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{18} + \frac{252}{14} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{19} + \frac{27720{c_2}^2 L^{12}}{1729} \\
& - \frac{2633400{c_2}^2 L^{13}}{1976} + \frac{66784410{c_2}^2 L^{14}}{2223} - \frac{203312340{c_2}^2 L^{15}}{2470} + \frac{250637310{c_2}^2 L^{16}}{2717} \\
& - \frac{151295760{c_2}^2 L^{17}}{2964} + \frac{45952830{c_2}^2 L^{18}}{3211} - \frac{6500340{c_2}^2 L^{19}}{3458} + \frac{339570{c_2}^2 L^{20}}{3705} \Big] \\
+ {k_2}^{MSS} \Big[& \frac{22680{c_1}^2 L^{2}}{57} - \frac{113400{c_1}^2 L^{3}}{76)} + \frac{202230{c_1}^2 L^{4}}{95} - \frac{151200{c_1}^2 L^{5}}{114} + \frac{40320{c_1}^2 L^{6}}{133} - \\
& \frac{216}{3} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{2} + \frac{15768}{4} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{3} - \frac{61254}{5} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{4} + \\
& \frac{81324}{6} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{5} - \frac{39978}{7} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{6} + \frac{5040}{8} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{7} \\
& + \frac{27720{c_2}^2 L^{2}}{741} - \frac{3908520{c_2}^2 L^{3}}{988} + \frac{143497970{c_2}^2 L^{4}}{1235} - \frac{415273320{c_2}^2 L^{5}}{1482} \\
& + \frac{379861020{c_2}^2 L^{6}}{1729} - \frac{102841200{c_2}^2 L^{7}}{1976} + \frac{8489250{c_2}^2 L^{8}}{2223} \Big] \\
+ {k_3}^{MSS} \Big[& \frac{22680{c_1}^2}{19L^2} - \frac{226800{c_1}^2}{38L} + \frac{748440{c_1}^2}{57} - \frac{907200{c_1}^2 L}{76} + \frac{362880{c_1}^2 L^{2}}{95} \\
& - \frac{216}{L^2} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} + \frac{31536}{2L} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} - \frac{221832}{3} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} + \frac{480384}{4} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L - \\
& \frac{350352}{5} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{2} + \frac{60480}{6} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{3} \\
& + \frac{24640{c_2}^2}{247L^2} - \frac{7817040{c_2}^2}{494L} + \frac{568731240{c_2}^2}{741} - \frac{2489699520{c_2}^2 L}{988} \\
& + \frac{3350350080{c_2}^2 L^{2}}{1235} - \frac{123094400{c_2}^2 L^{3}}{1482} + \frac{135828000{c_2}^2 L^{4}}{1729} \Big] \\
- {k_4}^{MSS} \Big[& \frac{22680{c_1}^2 L^{2}}{57} - \frac{113400{c_1}^2 L^{3}}{76)} + \frac{202230{c_1}^2 L^{4}}{95} - \frac{151200{c_1}^2 L^{5}}{114} + \frac{40320{c_1}^2 L^{6}}{133} - \\
& \frac{216}{3} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{2} + \frac{15768}{4} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{3} - \frac{61254}{5} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{4} + \\
& \frac{81324}{6} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{5} - \frac{39978}{7} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{6} + \frac{5040}{8} c_1 c_2 \frac{\sqrt{53900}}{\sqrt{4693}} L^{7} \\
& + \frac{27720{c_2}^2 L^{2}}{741} - \frac{3908520{c_2}^2 L^{3}}{988} + \frac{143497970{c_2}^2 L^{4}}{1235} - \frac{415273320{c_2}^2 L^{5}}{1482} \\
& + \frac{379861020{c_2}^2 L^{6}}{1729} - \frac{102841200{c_2}^2 L^{7}}{1976} + \frac{8489250{c_2}^2 L^{8}}{2223} \Big] \qquad (23)
\end{aligned}
$$

Where ${k_1}^{MSS}$, ${k_2}^{MSS}$, ${k_3}^{MSS}$ and ${k_4}^{MSS}$ are defined in Eqns.19 (a-d) respectively.

CONCLUSION

The study was able to evaluate cross sectional properties of MSS cross sections, which are moment of Inertia and cross-sectional area, given in Eqns. (9) and (16). The study also formulated peculiar TPEF for MSS cross-section. The formulated Raleigh- Ritz based MSS TPEF given in Eqns. (18), (21) and (23) are found handy and convenient to be used in the bulking/stability analysis of MSS cross- section. The developed expressions will now be used to formulate series of stability matrices in subsequent works where the critical buckling load for MSS TWBC cross – section will be evaluated.

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