Analyzing Uniform Endless Fin Via Dinesh Verma Transform

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***Abstract-* Usually, the distribution of temperature and hence the number of warmth convected from the fin surfaces has been determined via the calculus approach. The number of warmth convected from the fin surfaces has been determined by finding the overall equation cooling from the infinite fin via the calculus approach. The transferring of heat by fascinates quality of gradient and therefore the modes that transfer heat from single part of the way to a different square measure physical phenomenon, convection, and radiation. This paper is presenting the employment of a Dinesh Verma rework for the associate analysis of uniform infinite fin by finding the overall type equation of energy that describe the warmth dissipation from the surface of the medium and getting the distribution of temperature and thence the speed of warmth convected into the environment from an infinite uniform fin.**

***Indexed Terms-* Heat convected, Uniform Infinite Fin, Dinesh Verma Transform.**

I. INTRODUCTION

The Dinesh Verma Transform (DVT) useful in different fields of science, engineering and technology. The DVT is applicable in many fields and determines L.D.E. , O.D.E with variable & constant coefficient may be solved by the Dinesh Verma Transform (DVT) without finding their complimentary solution.[1],[2],[3],[4],[5],[6,[7],[8],[9], [10]. It also comes out to be very effective tool to analyze differential equations, Simultaneous differential equations, Integral equations etc. [11],[12],[13],[14], [15,16,17,18,19,20,21,22,23] wing tip are the enlarged surfaces projected from heat-conducting surfaces to improve the heat dissipation into the surroundings [1-3]. Fourier's law expressed as H $= -KA\frac{dt}{dy}$, is the basic law of conduction or dissipation of heat, where K is the thermal conductivity of the medium, A is the area of the cross-section of the medium, H is the rate of heat dissipated, $\frac{dt}{dy}$ is the temperature gradient and the negative sign shows that the heat is transfers in the direction in which the temperature is decreasing . Generally, the temperature distribution and hence the warmth convected from the infinite fin surface have been determined via the calculus approach [1-4]. This paper presents the study of uniform infinite fin to get the temperature distribution and hence the rate of heat convected into the surroundings by uniform infinite fin.

II. BASICS OF DVT

Dinesh Verma recently introduced a great transform and named it as DVT. Let f(t) is a well-defined function of real numbers t ≥ 0. The Dinesh Verma Transform (DVT) of f(t), denoted by $D\{$ {f(t)}, is defined as

$$D\{ \{f(t)\} = p^5 \int_0^\infty e^{-pt} f(t)dt = \bar{f}(p)$$

Provided that the integral exists, where p may be a real or complex parameter and D is the Dinesh Verma Transform (DVT) operator.

DVT OF ELEMENTARY FUNCTIONS

According to the definition of Dinesh Verma transform (DVT),

$$D\{t^n\} = p^5 \int_0^\infty e^{-pt}\, t^n dt$$

$$= p^5 \int_0^\infty e^{-z} \left(\frac{z}{p}\right)^n \frac{dz}{p} \quad , z = pt$$

$$= \frac{p^5}{p^{n+1}} \int_0^\infty e^{-z} (z)^n dz$$

Applying the definition of gamma function,

$$D\,\{y^n\} = \frac{p^5}{p^{n+1}} \lceil(n+1)$$

$$= \frac{1}{p^{n-4}} n!$$

$= \frac{n!}{p^{n-4}}$

Hence, $\boldsymbol{D\{t^n\} = \frac{n!}{p^{n-4}}}$

DVT of some elementary Functions

- $D\{t^n\} = \frac{n!}{p^{n-4}}$, $where\ n = 0,1,2, ..$
- $D\{e^{at}\} = \frac{p^5}{p-a}$,
- $D\{sinat\} = \frac{ap^5}{p^2+a^2}$,
- $D\{sinhat\} = \frac{ap^5}{p^2-a^2}$,
- $D\{coshat\} = \frac{p^6}{p^2-a^2}$.
- $D\{\delta(t)\} = p^5$

The Inverse DVT of some of the functions are given by

- $D^{-1}\left\{\frac{1}{p^{n-4}}\right\} = \frac{t^n}{n!}$, $where\ n = 0,1,2, ..$
- $D^{-1}\left\{\frac{p^5}{p-a}\right\} = e^{at}$, (i)
- $D^{-1}\left\{\frac{p^5}{p^2+a^2}\right\} = \frac{sinat}{a}$, (ii)
- $D^{-1}\left\{\frac{p^6}{p^2+a^2}\right\} = cosat$,
- $D^{-1}\left\{\frac{p^5}{p^2-a^2}\right\} = \frac{sinhat}{a}$,
- $D^{-1}\left\{\frac{p^6}{p^2-a^2}\right\} = coshat$,
- $D^{-1}\{p^5\} = \delta(t)$

DINESH VERMA TRANSFORM (DVT) OF DERIVATIVES

- $D\{f'(t)\} = p\bar{f}(p) - p^5 f(0)$
- $D\{f''(t)\} = p^2\bar{f}(p) - p^6 f(0) - p^5 f'(0)$
- $D\{f'''(y)\} = p^3\bar{f}(p) - p^7 f(0) - p^6 f'(0) - p^5 f''(0)$ And so on.
- $D\{tf(t)\} = \frac{5}{p}\bar{f}(p) - \frac{d\bar{f}(p)}{dp}$,

  $D\{tf'(t)\} = \frac{5}{p}\left[p\bar{f}(p) - p^5 f(0)\right] - \frac{d}{dp}\left[p\bar{f}(p) - p^5 f(0)\right]$ and
- $D\{tf''(t)\} = \frac{5}{p}\left[p^2\bar{x}(p) - p^6 x(0) - p^5 x'(0)\right] - \frac{d}{dp}\left[p^2\bar{x}(p) - p^6 x(0) - p^5 x'(0)\right]$ And so on.

III. FORMULATION

By the uniform infinite fin, The differential equation which describes the heat dissipated given by $u''(y)$- $\frac{\sigma P}{\mathcal{KA}}[t(y) - t_s] = 0$ ……… (I), where Let us consider that the one end of the fin is jointed to a heat source at y = 0 and the other end at y = ∞ is free for giving up heat into the surroundings. The source of heat is maintained at fixed temperature 'T' and t_s is the temperature of the surroundings of the infinite fin and is kept constant.

For convenience, let $(\frac{\sigma P}{\mathcal{KA}})^{\frac{1}{2}} = z$……. (II)

And $t(y) - t_s = u(y)$ ……… (III) known as the excess temperature at the length 'y' of the infinite fin. Then equation (I) can be rewritten as

$u''(y)$ - $z^2\, u(y) = 0$ ……….. (iv)

Equations (I) and (iv) are the basic form of energy equations for one-dimensional heat dissipation from the surface of the infinite fin. In equation (II), z is a constant provided that σ is constant over the entire surface the infinite fin and k is constant within the range of temperature considered.

The necessary initial conditions are

$t(0)$ = T. In terms of excess temperature, at y = 0, $t - t_s$ = T- t_s or $\varphi(0) = \varphi_0$… (e)

$t(\infty) = t_s$ .In terms of excess temperature, at y = ∞, $\varphi(\infty) = 0$

Taking Dinesh Verma Transform of equation (iv), we get

$q^2\bar{u}(q) - q^6 u(0) - q^5 u'(0) - z^2\bar{u}(q) = 0$... (v)

Applying boundary condition: $u(0) = u_0$,

$q^2\bar{u}(q) - u - q^5 u'(0) - z^2 u(q) = 0$

Or

$q^2\bar{u}(q) - z^2\bar{u}(q) = q^5 u'(0) + q^6 u_0$….. (vi)

In this equation, $u'(0)$ is some constant.

Let us substitute $u'(0) = r$,

Equation (vi) becomes

$q^2\bar{u}(q) - z^2\bar{u}(q) = q^5\omega + q^6 u_0$

Or

$\bar{u}(q) = \frac{q^5 r}{(q^2-r^2)} + \frac{q^6 u_0}{(q^2-z^2)}$ ……. (vii)

Taking inverse Dinesh Verma Transform of above equation, we get

$u(y) = \frac{r}{z} sinhzy + u_0 \cos hzy$

Or

$u(y) = \frac{r}{2z}\left[e^{zy} - e^{-zy}\right] + u_o\left[\frac{e^{zy} + e^{-zy}}{2}\right]$… (viii)

Determination of constant r:

Applying initial condition: $u_o(\infty) = 0$, we can write

$$\frac{r}{2z}\left[e^{z(\infty)} - e^{-z(\infty)}\right] + u_o\left[\frac{e^{z(\infty)} + e^{-z(\infty)}}{2}\right] = 0$$

Or

$$\frac{r}{2z}\left[e^{z(\infty)}-0\right]+\varphi_o\left[\frac{e^{z(\infty)}+0}{2}\right]=0$$

Or

$$\left[\frac{\omega}{2z}+\frac{u_o}{2}\right]e^{z(\infty)}=0$$

As $e^{z(\infty)} \neq 0$, therefore,

$$\left[\frac{r}{2z}+\frac{u_o}{2}\right]=0$$

Or

$r=-zu_o$ …….. (ix)

Put the value of , we get

$u(\mathrm{y})=\frac{-u_o}{2z}\left[e^{zy}-e^{-zy}\right]+u_o\left[\frac{e^{zy}+e^{-zy}}{2}\right]$

Or

$u(\mathrm{y})=\frac{-u_o}{2}\left[e^{zy}-e^{-zy}\right]+u_o\left[\frac{e^{zy}+e^{-zy}}{2}\right]$

Or

$u(\mathrm{y})=\frac{u_o}{2}\left[e^{zy}+e^{-zy}-e^{zy}+e^{-zy}\right]$

Or

$u(\mathrm{y})=u_o e^{-zy}$ …….. (x)

Equation (x) provides the distribution of temperature along the length of the infinite fin and confirmsthat the temperature of the infinite fin decreases along its length with the increase in distance from the heat source maintained at the temperature$T$.

The quantity of heat convected from the surface of the infinite fin can be obtained

$$\mathrm{H}_f = -KA\left[D_y t(y)\right]_{y=0}$$

Or

$\mathrm{H}_f = -KA\left[D_y\varphi(y)\right]_{y=0}$…… (xi)

Now since$u'(y) = -zue^{-zy}$,

Therefore,

$[u'(y)]_{y=0}=-zu_o$….. (xii)

Using (xii) , we get

$\mathcal{H}_f = KAzu_o$

Or

$\mathcal{H}_f = KAz\,(\mathrm{T}-t_s)$….. (xiv)

Put the value of z , we get

$$\mathcal{H}_f = KA\left(\frac{\sigma \mathrm{P}}{\mathcal{KA}}\right)^{\frac{1}{2}}(\mathrm{T}-t_s)$$

Or

$\mathcal{H}_f = (KA\sigma\mathrm{P})^{\frac{1}{2}}(\mathrm{T}-t_s)$ .... (xv)

This equation (xv) gives the rate of heat convected from the surface of the infinite fin into its surroundings and confirms that the rate of convection of heat can be increased by increasing the surface area of the fin.

CONCLUSION

This paper, DVT Transform is represented for the analysis of uniform infinite fin for settling the temperature distribution by the side of the infinite fin, and the quantity of heat convected from its surface into the environs. We have proved that the temperature of the infinite fin decreases with the increase in its length from the heat source, and the rate of heat convected from the infinite fin surface into the environs can be better by increasing the surface area of the infinite fin.

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