Applications of Dinesh Verma Transformation to An Electromagnetic Device

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Abstract- This paper, integral transform is known as Dinesh Verma transform is produced to the study of a moving-coil galvanometer. A moving-coil galvanometer is an electromagnetic device it means that it is utilization to calculate little values of electric currents. When a quantity of current is passed at some stage in the moving-coil galvanometer, its coil may suffer a few back-and-forth oscillations about its final mean position before coming to rest. The moving-coil galvanometer and its mathematical study is ordinarily done by an ordinary calculus approach. This paper extends the useful of Dinesh Verma transform for study of a moving-coil galvanometer and hence, for getting its response. The response get gives the deflection of the coil of the moving-coil galvanometer from its mean situation. In this paper, the response of a moving-coil galvanometer is get as a demonstration of the application of the new integral transform called Dinesh Verma transform.

Indexed Terms- Dinesh Verma Transform; Response; moving-coil galvanometer.

I. INTRODUCTION

In different areas of science, engineering and technology has been applied The Dinesh Verma Transform (DVT). [1], [2], [3] [4], [5], [6], [7]. The Dinesh Verma Transform (DVT) is applicable in different fields and successfully solving L.D.E, O.D.E with constant coefficient and variable coefficient can be simply explain by the Dinesh Verma Transform (DVT) without finding their complementary solutions. It also comes out to be extremely useful tool to analyze differential equations, Simultaneous differential equations, Integral equations etc. [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18] [19], [20]. A moving-coil galvanometer is an electromagnetic device that is used to measure small values of electric currents. It consists of a coil wrapped over a non-metallic frame which has a soft iron core, permanent horse-shoe magnets, pivoted spring, scale, and pointer. When some current is passed through the moving-coil galvanometer, its coil may suffer a few back and forth oscillations about its final mean position before coming to rest. As the coil suffers deflection, it moves in a permanent magnetic field and therefore, an e.m.f. is induced in it, which opposes the motion of the coil. The electromagnetic damping, responsible for the damping of coil, can be increased by winding the coil on a metallic frame. When the coil rotates, eddy currents are produced in the frame moving along with the coil, which tends to damp its motion and hence the coil soon comes to rest [1-5] [21]. The Dinesh Verma transform is a new integral transform which has been recently put forward by the author Dinesh Verma. It has been applied in science and engineering to solve most of the initial value problems [6],[24],[25],[26]. The analysis of a moving-coil galvanometer is usually done by ordinary calculus approach [1-5] [22],[23]. This paper proves the applicability of Dinesh Verma Transform for obtaining the response of the moving-coil galvanometer and concludes that ; Dinesh Verma transform like other methods or approaches is an effective and simple tool for obtaining the response of the moving-coil galvanometer.

II. DEFINITIONS

DEFINITION OF DINESH VERMA TRANSFORM (DVT)

Dr. Dinesh Verma recently introduced a novel transform and named it as Dinesh Verma Transform (DVT). Let f(t) is a well-defined function of real numbers t ≥ 0. The Dinesh Verma Transform (DVT) of f(t), denoted by $D\{f(t)\}$, is defined as

$$D\{f(t)\} = p^5 \int_0^\infty e^{-pt} f(t) dt = \tilde{f}(p)$$

Provided that the integral is convergent, where p may
be a real or complex parameter and D is the Dinesh Verma Transform (DVT) operator.

DINESH VERMA TRANSFORM OF ELEMENTARY FUNCTIONS

According to the definition of Dinesh Verma transform (DVT),

\[ D(t^n) = p^5 \int_0^\infty e^{-pt} t^n dt \]
\[ = p^5 \int_0^\infty e^{-z} (\frac{z}{p})^n dz, \quad z = pt \]
\[ = \frac{p^5}{p^{n+1}} e^{-z} (z)^n dz \]

Applying the definition of gamma function,

\[ D\{y^n\} = \frac{n!}{p^{n+1}}, \text{ where } n = 0,1,2, \ldots \]
\[ D\{e^{at}\} = \frac{p^5}{p-a} \]
\[ D\{sin\} = \frac{ap^5}{p+a^2} \]
\[ D\{cos\} = \frac{p^5}{p+a^2} \]
\[ D\{sinh\} = \frac{ap^5}{p-a^2} \]
\[ D\{cosh\} = \frac{p^5}{p-a^2} \]
\[ D\{\delta(t)\} = p^5 \]

The Inverse Dinesh Verma Transform (DVT) of some of the functions are given by

\[ D^{-1}\left(\frac{1}{p^n}\right) = \frac{t^n}{n!}, \text{ where } n = 0,1,2, \ldots \]
\[ D^{-1}\left(\frac{p^5}{p-a}\right) = e^{at} \]
\[ D^{-1}\left(\frac{p^5}{p+a^2}\right) = \frac{sinh}{a} \]
\[ D^{-1}\left(\frac{p^5}{p-a^2}\right) = \frac{cosh}{a} \]
\[ D^{-1}\left(\frac{p^5}{p^2+a^2}\right) = \frac{sinh}{a} \]
\[ D^{-1}\left(\frac{p^5}{p^2-a^2}\right) = \frac{cosh}{a} \]
\[ D^{-1}\left(p^5\right) = \delta(t) \]

DINESH VERMA TRANSFORM (DVT) OF DERIVATIVES

\[ D\{f'(t)\} = p^5 \frac{d}{dp} (p^5 f(0)) \]
\[ D\{f''(t)\} = p^5 \frac{d^2}{dp^2} (p^5 f(0) - p^5 f'(0)) \]
\[ D\{f'''(t)\} = p^5 \frac{d^3}{dp^3} (p^5 f(0) - p^5 f'(0) - p^5 f''(0)) \]

And so on.

METHODOLOGY

When current is flowing in a coil of moving-coil galvanometer, it is turned by the deflecting couple performing on it, and if \( \phi \) is the deflection of the coil from the equilibrium position at any instant \( t \), then the motion of the coil is opposed by the following couples [1-5]:

(a) If \( r \) is damping constant and the negative sign shows that the motion of the coil is opposed by the damping couple. Then Damping couple will be \( (\tau_d) \) i.e. \( \tau_d = -r \phi(t) \).

(b) If \( C \) is torsional rigidity of suspension fibre, and the minus sign indicates that the motion is also opposed by the restoring couple. Then Restoring couple \( (\tau_r) \) i.e. \( \tau_r = -C \phi(t) \).

(c) A couple \( \tau_e = \frac{d}{dt} \phi(t) \) arises due to electromagnetic damping i.e. due to induced eddy currents in the coil and depends directly upon the angular velocity of the coil and inversely upon its resistance \( R \) and also depends upon the magnetic field strength. All these factors are included in the constant \( K \).

The application of Newton’s second law of motion gives the equation of motion of the coil as follows

\[ I \ddot{\phi}(t) = -r \phi(t) - C \phi(t) - \frac{K}{R} \phi(t) \], where \( I \) is the moment of inertia of the coil about its axis of rotation and \( \dot{\phi}(t) \) is its angular acceleration.

This equation can be rewritten as

\[ \phi(t) = - \frac{r}{I} \phi(t) - \frac{C}{I} \phi(t) - \frac{K}{IR} \phi(t) \]

Or

\[ \phi(t) + \left( \frac{r}{I} + \frac{K}{IR} \right) \phi(t) + \frac{C}{I} \phi(t) = 0 \ldots (I) \]
For convenience let us put \((\frac{\tau}{C} + \frac{K}{IR}) = 2\varepsilon\) and \(\frac{\varepsilon}{C} = \mu^2\), then equation (I) can be rewritten as
\[\dot{\Theta}(t) + 2\varepsilon \Theta(t) + \mu^2 \Theta(t) = 0 \ldots \ldots \text{(II)}\]
The equation (8) is known as a differential equation of the PMMC instrument.
To solve equation (8), the initial boundary conditions are as follows [1-5]:
(a) If the maximum deflection of the coil from the equilibrium position is assumed to be \(\phi_0\) and we measure the time from the instant when the coil is at the position of its maximum deflection, then at \(t = 0\), \(\phi(0) = \phi_0\).
(b) At the instant \(t = 0\), the angular velocity \(\dot{\phi}(0) = 0\) as the coil is at rest at the instant \(t = 0\).
The Dinesh Verma transform [8, 9] of equation (II) provides
\[s^2 \ddot{\phi}(s) - s^6 \phi(0) - s^5 \dot{\phi}(0) + 2\varepsilon \{s \dot{\phi}(s) - s^5 \phi(0)\} + \mu^2 \ddot{\phi}(s) = 0 \ldots \ldots \text{(III)}\]
Here \(\ddot{\phi}(r)\) denotes the Dinesh Verma Transform of \(\phi(t)\).
Applying initial conditions [5] \(\phi(0) = \phi_0\) and \(\ddot{\phi}(0) = 0\), equation (III) becomes,
\[s^6 \ddot{\phi}(s) - s^5 \phi_0 + 2\varepsilon \{s \ddot{\phi}(s) - s^5 \phi_0\} + \mu^2 \ddot{\phi}(s) = 0\]
Or
\[s^2 + 2\delta s + \mu^2 \] \(\ddot{\phi}(s) = [s^6 + 2\varepsilon s^5] \phi_0 \]
Or
\[\ddot{\phi}(s) = \frac{[s^6 + 2\varepsilon s^5] \phi_0}{s^2 + 2\delta s + \mu^2} \]
Or
\[\ddot{\phi}(s) = \frac{[s^6 + 2\varepsilon s^5] \phi_0}{(s + \varepsilon)^2 - \sqrt{\varepsilon^2 - \mu^2}} \]
For convenience let us substitute \(\delta + \sqrt{\varepsilon^2 - \mu^2} = \alpha_1\) and \(\delta - \sqrt{\varepsilon^2 - \mu^2} = \alpha_2\) such that \(\alpha_1 - \alpha_2 = 2\sqrt{\varepsilon^2 - \mu^2}\), then equation (IV) can be rewritten as
\[\ddot{\phi}(s) = \frac{[s^6 + 2\varepsilon s^5] \phi_0}{(s + \alpha_1)(s + \alpha_2)} \]
Or
\[\ddot{\phi}(s) = \frac{-[(a_1 + 2\varepsilon) \phi_0]}{(s + \alpha_1)(s + \alpha_2)} + \frac{[-a_2 + 2\varepsilon] s^5 \phi_0}{(s + \alpha_1)(s + \alpha_2)} \]
Or
\[\ddot{\phi}(s) = \frac{-[\varepsilon - \sqrt{\varepsilon^2 - \mu^2} \varepsilon] s^5 \phi_0}{(-a_1 + a_2)(s + \alpha_1)(s + \alpha_2)} + \frac{[\varepsilon + \sqrt{\varepsilon^2 - \mu^2} \varepsilon] s^5 \phi_0}{(-a_1 + a_2)(s + \alpha_1)(s + \alpha_2)} \]
Or
\[\ddot{\phi}(s) = \frac{[\varepsilon - \sqrt{\varepsilon^2 - \mu^2} \varepsilon] s^5 \phi_0}{2\sqrt{\varepsilon^2 - \mu^2}(s + \alpha_1)(s + \alpha_2)} + \frac{[\varepsilon + \sqrt{\varepsilon^2 - \mu^2} \varepsilon] s^5 \phi_0}{2\sqrt{\varepsilon^2 - \mu^2}(s + \alpha_1)(s + \alpha_2)} \ldots \ldots (V)\]
The application of inverse Dinesh Verma Transform [6, 12, 13,] provides
\[\phi(t) = \frac{[\varepsilon - \sqrt{\varepsilon^2 - \mu^2} \varepsilon] \phi_0 e^{-\alpha_1 t}}{2\sqrt{\varepsilon^2 - \mu^2}} + \frac{[\varepsilon + \sqrt{\varepsilon^2 - \mu^2} \varepsilon] \phi_0 e^{-\alpha_2 t}}{2\sqrt{\varepsilon^2 - \mu^2}} \]
Or
\[\phi(t) = \frac{\phi_0 e^{-\alpha_1 t}}{2\sqrt{\varepsilon^2 - \mu^2}}((1 + \frac{\varepsilon}{\sqrt{\varepsilon^2 - \mu^2}}) e^{\sqrt{\varepsilon^2 - \mu^2} t} + (1 - \frac{\varepsilon}{\sqrt{\varepsilon^2 - \mu^2}}) e^{-\sqrt{\varepsilon^2 - \mu^2} t}) \ldots \ldots (VI)\]
This equation (12) provides the deflection of the coil of the When current is flow flowing coil of moving coil galvanometer, it is turned by the deflecting couple performing on it, and if \(\phi\) is the deflection of the coil from the equilibrium position at any instant \(t\), then the motion of the coil is opposed by the following couples [1-5]:
If \(r\) is damping constant and the negative sign shows that the motion of the coil is opposed by the damping couple. Then Damping couple will be \((\tau_d)\) i.e. \(\tau_d = -r \phi(t)\).
If \(C\) is torsional rigidity of suspension fibre, and the minus sign indicates that the motion is also opposed by the restoring couple. Then Restoring couple \((\tau_r)\) i.e. \(\tau_r = -C \phi(t)\).
A couple \(\tau_e = -\frac{K}{r} \phi(t)\) arises due to electromagnetic damping i.e. due to induced eddy current in the coil and depends directly upon the angular velocity of the coil and inversely upon its resistance R and also depends upon the magnetic field strength. All these factors are included in the constant \(K\).
The application of Newton’s second law of motion gives the equation of motion of the coil as follows
and reveals that the nature of its deflection depends on the nature of the quantity $\sqrt{\varepsilon^2 - \mu^2}$ which may be real, zero or imaginary depending upon the values of $\varepsilon$ and $\mu$. We have the following three cases:

Case I: When $\varepsilon > \mu$, then the quantity $\sqrt{\varepsilon^2 - \mu^2}$ is real and therefore, the equation (VI) can be rewritten as

$$
\varphi (t) = \varphi_0 e^{-\epsilon t} \left[ \frac{\delta}{\sqrt{\varepsilon^2 - \mu^2}} \sin \sqrt{\varepsilon^2 - \mu^2} \right] t + 
$$

$$
\cos \sqrt{\varepsilon^2 - \mu^2} t \quad \cdots \quad \cdots \quad (VII)
$$

It is clear from the equation (VII) that the motion of the coil of the moving-coil galvanometer is non-oscillatory and the coil approaches equilibrium quite slowly without any oscillation when a steady current is passed through it. The galvanometer in such a case is said to be over-damped or dead beat [5, 15].

Case II: When $\varepsilon = \mu$, then the quantity $\sqrt{\varepsilon^2 - \mu^2}$ is zero. In this case, equation (VI) reveals that the motion of the coil of the moving-coil galvanometer indeterminate, which is not possible. If the quantity $\sqrt{\varepsilon^2 - \mu^2}$ is so small that it approaches zero, then on expanding the exponential terms containing the quantity $\sqrt{\varepsilon^2 - \mu^2}$ and neglecting higher order terms, we can rewrite equation (VI) as

$$
\varphi (t) = \varphi_0 e^{-\epsilon t} \left\{ \frac{\epsilon}{\sqrt{\varepsilon^2 - \mu^2}} \left[ 1 + \frac{(\sqrt{\varepsilon^2 - \mu^2}) t - [1 - (\sqrt{\varepsilon^2 - \mu^2}) t]}{2} \right] + \frac{1 + (\sqrt{\varepsilon^2 - \mu^2}) t - [1 - (\sqrt{\varepsilon^2 - \mu^2}) t]}{2} \right\}
$$

Or $\varphi (t) = \varphi_0 (1 + \epsilon t) e^{-\epsilon t} \quad \cdots \quad (VIII)$

It is clear from the equation (VIII) that the motion of the coil of the moving-coil galvanometer is non-oscillatory and the coil approaches equilibrium as fast as possible without any oscillation when a steady current is passed through it. The galvanometer in such a case is said to be critically damped [5, 16]. This type of damping is very desirable feature in the PMMC instrument.

Case III: In the case of light damping [5], $\varepsilon < \mu$. In such a case, the quantity $\sqrt{\varepsilon^2 - \mu^2}$ is imaginary. We can rewrite the quantity $\sqrt{\varepsilon^2 - \mu^2}$ as $\sqrt{\varepsilon^2 - \mu^2} = i \sqrt{\varepsilon^2 - \mu^2}$  

$$
\varphi (t) = \varphi_0 e^{-\epsilon t} \left[ \frac{\epsilon}{\sqrt{\varepsilon^2 - \mu^2}} \sin \sqrt{\varepsilon^2 - \mu^2} t + \right. 
$$

$$
\cos \sqrt{\varepsilon^2 - \mu^2} t \quad \cdots \quad \cdots \quad (X)
$$

Let us substitute $\frac{\varphi_0}{\sqrt{\varepsilon^2 - \mu^2}} = A \cos \Theta$ and $\varphi_0 = A \sin \Theta$ such that $A = \frac{\varphi_0}{\sqrt{\varepsilon^2 - \mu^2}}$ and $\Theta = \tan^{-1} \frac{\sqrt{\varepsilon^2 - \mu^2}}{\epsilon}$, then equation (X) becomes

$$
\varphi (t) = A e^{-\epsilon t} \sin \left[ \left( \sqrt{\varepsilon^2 - \mu^2} t + \Theta \right) \right] \quad \cdots \quad (XI)
$$

It is clear from the equation (XI) that the motion of the coil of the moving-coil galvanometer is oscillatory with amplitude $A e^{-\epsilon t}$ which is decreasing exponentially with time over many oscillations, and the oscillating angular frequency is $\sqrt{\varepsilon^2 - \mu^2}$. The galvanometer in such a case is said to be under-damped [5, 17, 18, 19] or ballistic galvanometer.

CONCLUSION

In this paper, an effort made to exemplify the Dinesh Verma transform for talk about the theory of a moving-coil galvanometer. This paper brought up the Dinesh Verma transform as a powerful mathematical tool for determining the response of a moving-coil galvanometer. The response obtained is the same as obtained with other the methods or approaches [1-5, 14].

REFERENCES


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