Optimization of Hybrid – Polypropylene - Steel Fibre Reinforced Concrete (HPSFRC)

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Abstract- Sometimes, in order to produce superior concrete, more than one type of fibre is added to concrete in various mixture design with promising results. Hybrid Fiber Reinforced Concrete (HFRC) is a concrete in which more than one type of fibre is added to concrete in order to improve its general properties, especially compressive strength. This research work is aimed at using Scheffe's Second Degree Model for six component mixtures, simply abbreviated as Scheffe's (6,2) to optimize the compressive strength of Hybrid – Polypropylene - Steel Fibre Reinforced Concrete (HPSFRC).Firstly, the Scheffe's second degree model for six component mixture was introduced and through the use of Scheffe's Simplex optimization method, the compressive strengths of HPSFRC were obtained for different mix proportions. The mix proportion of Polypropylene - Steel was in 50% - 50% ratio. Control experiments were also carried out, and the compressive strengths evaluated. By using the Student's t-test statistics, the adequacy of the model was confirmed .The optimum attainable compressive strength of HPSFRC was 57.65 MPa . This maximum value is higher than the minimum value specified by the American Concrete Institute (ACI), as 20 MPa for good concrete and also the minimum value specified by ASTM C 39 and ASTM C 469, as 30.75 for high performance concrete . Thus, the HPSFRC compressive strength value can sustain construction of light-weight and heavy-weight structures such as Bridges, Airports, maintenance hangars, access roads etc at the best possible economic and safety advantages.

Indexed Terms- HPSFRC, Scheffe's (6,2) Optimization,Model, Compressive Strength, Mixture Design

I. INTRODUCTION

In general , Hybrid Fibre Reinforced Concrete is the use of two or more fibres in a single concrete mixture matrix with the aim to improve its overall properties . In well-designed hybrid composites, there is always positive interaction between the fibres and hence, the resulting hybrid performance is expected to exceed the sum of individual fibres performances due to synergy between the fibre types. The process of combination of fibres is sometimes called hybridization but the best possible way of combining the fibres is actualized through optimization, which is less laborious. An optimization problem is one requiring the determination of the optimal (maximum or minimum) value of a given function, called the objective function, subject to a set of stated restrictions, or constraints placed on the variables concerned. Every optimization problem requires an objective which might be to maximize profit or benefit, to minimize cost or to minimize the use of material resources. Optimization of the concrete mixture design is a process of search for a mixture for which the sum of the costs of the ingredients is lowest, yet satisfying the required performance of concrete, such as workability, strength and durability. Furthermore, the design of concrete mix according to (Shetty, 2006) has not being a simple task on the account of the widely varying properties of the constituent materials, as well as the conditions that prevail at the site of work , the exposure condition, and the conditions that are demanded for a particular work for which the mix is designed. Again, concrete mix design according to Jackson and Dhir (1996) has been defined as the procedure which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. Thus, the cost of any concrete includes, in addition to that of the materials themselves, the cost of the mix design, of batching, mixing and placing the concrete and of the site supervision. In the context of the above guidelines, the methods and procedures proposed by Hughes (1971), ACI- 211(1994) and DOE (1988) are also more complex and time consuming as they involve a lot of trial mixes and deep statistical calculations before the desired strength of the concrete can be achieved. Thus, optimization of the concrete mixture design remains the fastest method, best option and the most efficient way of selecting concrete mix /proportion for better efficiency and performance of concrete when compared with usual empirical methods as listed above. A typical example of optimization model is Scheffe's Mathematical, which can be in form of Scheffe's Second Degree model or Scheffe's Third Degree model. Thus,in this present study, Scheffe's Second Degree Model for six components mixtures (namely cement, fine aggregate, coarse aggregate, water, polypropylene fibre and steel fibre) will be presented.

Concrete being the most widely used construction material has been undergoing changes both as a material and due to technological advancement. According to Oyenuga (2008), concrete is a composite inert material comprising of a binder course (cement), mineral filter or aggregates and water. Concrete, being a homogeneous mixture of cement, sand, gravel and water is very strong in carrying compressive forces and hence is gaining increasing importance as building materials throughout the world (Syal and Goel, 2007). Concrete, according to Neville (1990), plays an important part in all building structures owing to its numerous advantages which ranges from low built in fire resistance, high compressive strength to low maintenance. However, according to Shetty (2006) , plain concrete possesses a very low tensile strength, limited ductility and little resistance to cracking. This has resulted to continuous search for upgrading the properties of concrete. To remedy this situation, attempts have been made in the past to impact improvement in tensile properties of concrete members by way of using conventional reinforced steel bars and also by applying restraining techniques. Although both these methods provide tensile strength to the concrete members, they however, do not increase the inherent tensile strength of concrete itself.. Based on several further researches and recent developments, it has been observed that the addition of fibres (either as glass fibre, polypropylene fibre, nylon fibre, steel fibre , plastic fibre, asbestor (mineral fibre), or carbon fibres , etc.) to concrete would act as crack arrester and would substantially improve its static and dynamic properties. This type of concrete is known as Fibre reinforced concrete (FRC). FRC is a composite material consisting of mixtures of cement, mortar or concrete and discontinuous, discrete, uniformly dispersed. Hybrid – Polypropylene - Steel Fibre Reinforced Concrete (HPSFRC) is concrete mixture where the conventionally steel reinforcement in concrete production is replaced (wholly or partially) with polypropylene fibre and steel fibre. Already, works on optimization of compressive strength of PFRC and SFRC have been carryout out.

According to Ettu (2001), the major aim of engineering design is to ensure that the structure being designed will not reach a Serviceability Limit State (SLS), which is connected with deflection, cracking, vibration etc, and Ultimate Limit State (ULS), which is generally connected with collapse. In all of the above, the concrete's compressive strength is one of the most important properties of concrete that require close investigation because of its important role. Compressive strength of concrete is the Strength of hardened concrete measured by the compression test. It is a measure of the concrete's ability to resist loads which tend to compress it. It is measured by crushing cylindrical concrete specimens in a universal testing machine. Again, the compressive strength of the concrete cube test also provides an idea about all the characteristics of concrete under examination.

The present study therefore examines the use of Scheffe's Second Degree Model in optimizing the compressive strength of HPSFRC. As a matter of fact many researchers have done related works on polypropylene fibre as well as steel fibre. For instance, on PFRC , Bayasi and Zeng (1993) and Patel and others (2012) have investigated the properties of PFRC. On SFRC and related works, Baros and others (2005) investigated the post – cracking behaviour of SFRC. Jean-Louis and Sana (2005) investigated the corrosion of SFRC from the crack. Lima and Oh (1999) carried out an experimental and theoretical investigation on the shear of SFRC beams. Similarly, Lau and Anson (2006) carried out research on the effect of high temperatures on high performance SFRC. The work of Lie and Kodar (1996) was on the study of thermal and mechanical properties of SFRC at elevated temperatures. Blaszczynski and Przybylska-Falek (2015) investigated the use of SFRC as a structural material. Huang and Zhao (1995) investigated the properties of SFRC containing larger coarse aggregate. Arube and others (2021) investigated the Effects of Steel Fibres in Concrete Paving Blocks. Again, Khaloo and others (2005) examined the flexural behaviour of small SFRC slabs. And Ghaffer and others (2014) investigated the use of steel fibres in structural concrete to enhance the mechanical properties of concrete. On HPSFRC and related works, M-K.Yew and others (2011) have investigated the strength properties of Hybrid Nylon-Steel fibre-reinforced concrete in comparison to that of polypropylene-steel fibre-reinforced concrete. In their contribution, Varma and Raji (2019) have presented an experimental investigation to quantity the improved mechanical properties of Hybrid - Polypropylene-Steel Fibre-Reinforced Concrete. Nuaklong and others (2020) investigated the effect of hybrid polypropylene- steel- fibres on strength characteristics of UHPFRC. Qian and Stroeven (2000) investigated the optimization of fibre size , fibre content, and fly ash content in hybrid polypropylene-steel fibre concrete based on general mechanical properties. Recent works on optimization show that many researchers have used Scheffe's method to carry out one form of optimization work or the other. For example, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere (2006) were also based on the use of Scheffe' mathematical model in the optimization of compressive strength of Perwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's (4,2) and Scheffe's (4,3). Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also, Nwachukwu and others (2022a) developed and used Scheffe's Third Degree Polynomial model, Scheffe's (5,3) to optimize the compressive strength of GFRC where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). Nwachukwu and others (2022b) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). Furthermore, Nwachukwu and others (2022e) used Scheffe's Third Degree Regression model, Scheffe's (5,3) to optimize the compressive strength of PFRC. Nwachukwu and others (2022f) applied Modified Scheffe's Third Degree Polynomial model to optimize the compressive strength of NFRC. And lastly, Nwachukwu and others (2022g) applied Scheffe's Third Degree Model to optimize the compressive strength of SFRC. From the forgoing, it appears that the subject matter has not been wholly addressed as it can be envisaged that no work has been done on the use of Scheffe's Second Degree Model to optimize the compressive strength of HPSFRC. Henceforth, the need for this present research work.

II. SCHEFFE'S (6, 2) REGRESSION EQUATION

A simplex lattice is a structural representation of lines joining the atoms of a mixture, whereas these atoms are constituent components of the mixture. For HPSFRC mixture, the constituent elements are these six components, water, cement, fine aggregate, coarse aggregate, polypropylene fibre and steel fibre. Thus, a simplex of six-component mixture is a five - dimensional solid. According to Obam (2009), mixture components are subject to the constraint that the sum of all the components must be equal to 1. That is:

$$X_1 + X_2 + X_3 + \dots + X_q = 1 \; ; \quad \Rightarrow \sum_{i=1}^{q} X_i = 1 \qquad (1)$$

where $X_i \geq 0$ and $i = 1, 2, 3\dots q$, and q = the number of mixtures.

2.1. INTRODUCTION OF SIX COMPONENT MIXTURES IN SCHEFFE'S SIMPLEX LATTICE DESIGN

The (q, m) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen regressionl equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains $^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \dots, 1; \quad i = 1, 2, \dots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degee, which in this present study is 2.

For example a (3, 2) lattice consists of $^{3+2-1}C_2$ i.e. $^4C_2$ = 6 points. Each X_i can take m+1 = 3 possible values; that is $x = 0, \frac{1}{2}, 1$ with which the possible design points are: $(1,0,0), (0,1,0), (0,0,1), \left(\frac{1}{2},\frac{1}{2},0\right), \left(0,\frac{1}{2},\frac{1}{2}\right), \left(\frac{1}{2},0,\frac{1}{2}\right)$. The general formula for evaluating the number of coefficients/terms/points required for a given lattice is given by:

$$k = \frac{(q+m-1)!}{(q-1)!\,.\,m!} \quad \text{Or} \quad {}^{q+m-1}C_m \qquad 2(a\text{-}b)$$

Where k = number of coefficients/ terms / points

q = number of components = 6 in this study

m = number of deqree of polynomial = 2 in this present work

Using either of Eqn. (2), $k_{(6,2)} = 21$

Thus, the possible design points for Scheffe's (6,2) lattice can be as follows:

A_1 (1,0,0,0,0,0); A_2 (0,1,0,0,0,0); A_3 (0,0,1,0,0,0); A_4 (0,0,0,1,0,0), A_5 (0,0,0,0,1,0); A_6 (0, 0,0,0, 0, 1); A_{12} (0.67,0.33, 0, 0, 0, 0); A_{13} (0.67, 0, 0.33,0,0,0); A_{14} (0.67, 0, 0, 0.33,0,0); A_{15} (0.67, 0, 0, 0,0.33, 0); A_{16} (0.67, 0, 0, 0, 0, 0.33); A_{23} (0,0.50,0.50, 0,0,0); A_{24} (0, 0.50, 0, 0.50, 0,0); A_{25}, (0, 0.50, 0, 0,0.50, 0); A_{26} (0, 0.50,0,0, 0.50); A_{34} (0.50, 0.50, 0, 0,0,0); A_{35} (O.50, 0,0.50, 0,0,0); A_{36} (0.50,0, 0,0.50, 0, 0); A_{45} (0.50, 0, 0, 0,0.50, 0); A_{46}(0.50,0,0,0,0,0.50);A_{56}(0,0,0.50,0.50,0,0); (3)

According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable X_1, X_2, X_3, X_4 … X_q is given in the form of Eqn.(4)

$$N = b_0 + \sum bi\ xi + \sum bij x j + \sum bi\, jxjxk + + \sum bi^{\,j}{}_2 + \ldots i_n x i_2 x i_n \qquad (4)$$

where ($1 \le i \le q$, $1 \le i \le j \le k \le q$, $1 \le i_1 \le i_2 \le \ldots \le i_n \le q$ respectively) , b = constant coefficients and N is the response which represents the property under investigation, which ,in this case is the compressive strength.

As this research work is based on the Scheffe's (6, 2) simplex, the actual form of Eqn. (4) for six component mixture , degree two (6, 2) will be developed subsequently.

2.2. PSEUDO AND ACTUAL COMPONENTS.

In Scheffe's mix design, there exist a relationship between the pseudo components and the actual components which has been established as Eqn.(5):

$$Z = A * X \qquad (5)$$

where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging Eqn. (5) yields:

$$X = A^{-1} * Z \qquad (6)$$

2.3. FORMULATION OF HPSFRC REGRESSION EQUATION FOR SCHEFFE'S (6, 2) LATTICE

The Regression equation by Scheffe (1958), otherwise known as response is given in Eqn.(4) .Hence, for Scheffe's (6,2) simplex lattice, the regression equation for six component mixtures is formulated based on Eqn.(4) and the work of Nwachukwu and others (2017) as under:

$$\begin{aligned} N = {} & ß_1X_1 + ß_2X_2 + ß_3X_3 + ß_4X_4 + ß_5X_5 + ß_6X_6 + ß_{12}X_1X_2 + ß_{13}X_1X_3 + ß_{14}X_1X_4 + ß_{15}X_1X_5 + ß_{16}X_1X_6 + \\ & ß_{23}X_2X_3 + ß_{24}X_2X_4 + ß_{25}X_2X_5 + ß_{26}X_2X_6 + ß_{34}X_3X_4 + ß_{35}X_3X_5 + ß_{36}X_3X_6 + ß_{45}X_4X_5 + ß_{46}X_4X_6 \\ & + ß_{56}X_5X_6 \end{aligned} \qquad (7)$$

2.4 . DETERMINATION OF COEFFICIENTS OF THE SCHEFFE'S (6, 2) POLYNOMIAL

From the work of Nwachukwu and others (2017), the coefficients of the Scheffe's (6, 2) polynomial can be determined as under. :

$\beta_1 = N_1$; $\beta_2 = N_2$; $\beta_3 = N_3$; $\beta_4 = N_4$; $\beta_5 = N_5$ and $\beta_6 = N_6$ 8(a-f)

$\beta_{12} = 4N_{12} - 2N_1 - 2N_2$; $\beta_{13} = 4N_{13} - 2N_1 - 2N_3$; $\beta_{14} = 4N_{14} - 2N_1 - 2N_4$; 9(a-c)

$\beta_{15} = 4N_{15} - 2N_1 - 2N_5$; $\beta_{16} = 4N_{16} - 2N_1 - 2N_6$; $\beta_{23} = 4N_{23} - 2N_2 - 2N_3$; $\beta_{24} = 4N_{24} - 2N_2 - 2N_4$; 10(a-d)

$\beta_{25} = 4N_{25} - 2N_2 - 2N_5$; $\beta_{26} = 4N_{26} - 2N_2 - 2N_6$, $\beta_{34} = 4N_{34} - 2N_3 - 2N_4$; $\beta_{35} = 4N_{35} - 2N_3 - 2N_5$; 11(a-d)

$\beta_{36} = 4N_{36} - 2N_{3} - 2N_{6}$; $\beta_{45} = 4N_{46} - 2N_{4} - 2N_{6}$, $\beta_{46} = 4N_{46} - 2N_{4} - 2N_{6}$; $\beta_{56} = 4N_{56} - 2N_{35} - 2N_{6}$; 12(a-d)

Where N_i = Response Function (Compressive Strength) for the pure component, i

2.5. SCHEFFE'S (6, 2) MIXTURE DESIGN MODEL

Substituting Eqns. (8)-(12) into Eqn. (7), yields the mixture design model for the HPSFRC Scheffe's (6,2) lattice.

2.6. PSEUDO AND ACTUAL MIX PROPORTIONS OF SCHEFFE'S (6, 2) DESIGN LATTICE

Since the requirement of simplex lattice design based on Eqn. (1) criteria makes it impossible to use the conventional mix ratios such as 1:2:4 etc., at a given water/cement ratio for the actual mix ratio., there is need for the transformation of the actual components proportions to meet the above criterion. Based on experience and previous knowledge from literature, the following arbitrary prescribed mix ratios are always chosen for the six vertices of Scheffe's (6,2) lattice. They are :

A_1 (0.67:1:1.7:2:0.5:0.5); A_2 (0.56:1:1.6:1.8:0.8:0.8); A_3 (0.5:1:1.2:1.7:1:1); A_4 (0.7:1:1:1.8:1.2:1.2);

A_5 (0.75:1:1.3:1.2:1.5:1.5), and A_6 (0.80:1:1.3:1.2:0.9:0.9)
(13)

which represent water/cement ratio, cement, fine aggregate, coarse aggregate, polypropylene fibre and steel fibre respectively.

For the pseudo mix ratio, the following corresponding mix ratios at the vertices for six component mixtures are always chosen:

A_1(1:0:0:0:0:0), A_2(0:1:0:0: 0:0), A_3(0:0:1:0:0:0), A_4(0:0:0:1:0:0), A_5(0:0:0:0:1:0) and A_6(0:0:0:0:0:1)
(14)

For the transformation of the actual component, Z to pseudo component, X, and vice versa, Eqns. (5) and (6) are used. Substituting the mix ratios from point A_1 into Eqn. (5) yields:

$$\begin{Bmatrix} 0.67 \\ 1.00 \\ 1.70 \\ 2.00 \\ 0.50 \\ 0.50 \end{Bmatrix} = \begin{Bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{Bmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (15)$$

Transforming the R.H.S matrix and solving, we obtain as follows:

$A_{11}(1) + A_{21}(0) + A_{31}(0) + A_{41}(0) + A_{51}(0) + A_{61}(0) = 0.67$.

Thus , $A_{11} = 0.67$

Similarly, $A_{21}= 1$; $A_{31}= 1.7$; $A_{41}= 2$; $A_{51}= 0.5$; $A_{61}= 0.5$

The same approach is used to obtain the remaining values as shown in Eqn. (16)

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{Bmatrix} = \begin{Bmatrix} 0.67 & 0.56 & 0.50 & 0.50 & 0.75 & 0.75 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 1.70 & 1.60 & 1.20 & 1.00 & 1.30 & 1.30 \\ 2.00 & 1.80 & 1.70 & 1.80 & 1.20 & 1.20 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \end{Bmatrix} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{pmatrix} \quad (16)$$

Considering mix ratios at the mid points from Eqn.(3) and substituting these pseudo mix ratios in turn into Eqn.(16) will yield the corresponding actual mix ratios.

For instance, considering point A_{12} we have: A_{12} (0.67,0.33, 0, 0, 0, 0). This implies:

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.50 & 0.50 & 0.75 & 0.75 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 1.70 & 1.60 & 1.20 & 1.00 & 1.30 & 1.30 \\ 2.00 & 1.80 & 1.70 & 1.80 & 1.20 & 1.20 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \end{pmatrix} \begin{pmatrix} 0.67 \\ 0.33 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.63 \\ 1 \\ 1.67 \\ 1.90 \\ 1.60 \\ 1.60 \end{pmatrix} \quad (17)$$

Solving, $Z_1 = 0.63$; $Z_2 = 1.00$; $Z_3 = 1.67$' $Z_4 = 1.90$; $Z_5 = 1.60$ and $Z_6 = 1.60$

The same approach goes for the remaining mid-point mix ratios.

Hence, to generate the polynomial coefficients, twenty-one (21) experimental tests will be carried out and the corresponding mix ratios are depicted in Table 1.

Table 1: Pseudo (X) and Actual (Z) Mix Ratio for HPSFRC based on Scheffe's (6,2) Lattice

S/N	POINTS	PSEUDO COMPONENT						RESPONSE SYMBOL	ACTUAL COMPONENT					
		X_1	X_2	X_3	X_4	X_5	X_6		Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
1	1	1	0	0	0	0	0	N_1	0.67	1.00	1.70	2.0	0.5	0.5
2	2	0	1	0	0	0	0	N_2	0.56	1.00	1.60	1.8	0.8	0.8
3	3	0	0	1	0	0	0	N_3	0.50	1.00	1.20	1.7	1.0	1.0
4	4	0	0	0	1	0	0	N_4	0.70	1.00	1.00	1.8	1.2	1.2
5	5	0	0	0	0	1	0	N_5	0.75	1.00	1.30	1.2	1.5	1.5
6	6	0	0	0	0	0	1	N_6	0.63	1.00	1.67	1.9	1.6	1.6
7	12	0.67	033	0	0	0	0	N_{12}	0.60	1.00	1.63	1.8	0.7	0.7
8	13	0.67	0	0.33	0	0	0	N_{13}	0.61	1.00	1.54	1.9	0.6	0.6
9	14	0.67	0	0	0.33	0	0	N_{14}	0.56	1.00	1.37	1.8	0.8	0.8
10	15	0.67	0	0	0	0.33	0	N_{15}	0.68	1.00	1.47	1.9	0.7	0.7
11	16	0.67	0	0	0	0	0.33	N_{16}	0.69	1.00	1.23	1.8	0.9	0.9
12	23	0	0.50	0.50	0	0	0	N_{23}	0.70	1.00	1.57	1.7	0.8	0.8
13	24	0	0.50	0	0.50	0	0	N_{24}	0.72	1.00	1.43	1.4	1.1	1.1
14	25	0	0.50	0	0	0.50	0	N_{25}	0.55	1.00	1.40	1.7	0.8	0.8
15	26	0	0.50	0	0	0	0.50	N_{26}	0.52	1.00	1.20	1.7	0.9	0.9
16	34	0.50	0.50	0	0	0	0	N_{34}	0.61	1.00	1.67	1.8	0.9	0.9
17	35	0.50	0	0.50	0	0	0	N_{35}	0.66	1.00	1.73	1.8	1.0	1.0
18	36	0.50	0	0	0.50	0	0	N_{36}	0.63	1.00	1.50	1.6	0.7	0.7
19	45	0.50	0	0	0	0.50	0	N_{45}	0.69	1.00	1.40	1.4	0.6	0.6
20	46	0.50	0	0	0	0	0.50	N_{46}	0.57	1.00	1.13	1.7	1.0	1.0
21	56	0	0	0.50	0.50	0	0	N_{56}	0.64	1.00	1.07	1.7	1.1	1.1

2.7. THE CONTROL POINTS

Twenty- one (21) different controls were predicted which according to Scheffe's (1958) ,their summation should not be greater than one. The same approach for component transformation adopted for the initial experimental points are also adopted for the control points and the results are shown in Table 2.

Table 2 : Actual and Pseudo Component of HPSFRC Based on Scheffe (6,2) Lattice for Control Points

S/N	POINTS	PSEUDO COMPONENT						CONTROL POINTS	ACTUAL COMPONENT					
		X_1	X_2	X_3	X_4	X_5	X_6		Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
1	1	0.25	0.25	0.25	0.25	0	0	C_1	0.61	1	1.38	1.83	0.5	0.50
2	2	0.25	0.25	0.25	0	0.25	0	C_2	0.62	1	1.45	1.68	0.8	0.8
3	3	0.25	0.25	0	0.25	0.25	0	C_3	0.67	1	1.40	1.70	1	1
4	4	0.25	0	0.25	0.25	0.25	0	C_4	0.66	1	1.30	1.68	1.2	1.2
5	5	0	0.25	0.25	0.25	0.25	0	C_5	0.63	1	1.28	1.63	1.5	1.5
6	6	0.20	0.20	0.20	0.20	0.20	0	C_6	0.64	1	1.36	1.70	0.65	0.65
7	12	0.30	0.30	0.30	0.10	0	0	C_{12}	0.59	1	1.45	1.83	0.75	0.75
8	13	0.30	0.30	0.30	0	0.10	0	C_{13}	0.59	1	1.48	1.77	0.85	0.85

9	14	0.30	0.30	0	0.30	0.10	0	C_{14}	0.65	1	1.42	1.80	1	1
10	15	0.30	0	0.30	0.30	0.10	0	C_{15}	0.64	1	1.30	1.77	0.9	0.9
11	16	0	0.30	0.30	0.30	0.10	0	C_{16}	0.60	1	1.27	1.71	1	1
12	23	0.10	0.30	0.30	0.30	0	0	C_{23}	0.60	1	1.31	1.79	1.55	1.55
13	24	0.30	0.10	0.30	0.30	0	0	C_{24}	0.62	1	1.33	1.83	1.1	1.1
14	25	0.30	0.10	0.30	0.30	0	0	C_{25}	0.63	1	1.41	1.85	1.25	1.25
15	26	0.10	0.20	0.30	0.40	0	0	C_{26}	0.61	1	1.25	1.79	1.35	1.35
16	34	0.30	0.20	0.10	0.40	0	0	C_{34}	0.64	1	1.35	1.85	0.89	0.89
17	35	0.20	0.20	0.10	0.10	0.40	0	C_{35}	1.40	1	1.04	1.59	1.08	1.08
18	36	0.30	0.10	0.30	0.20	0.10	0	C_{36}	0.62	1	1.36	1.77	0.92	0.92
19	45	0.25	0.25	0.15	0.15	0.20	0	C_{45}	0.61	1	1.51	3.16	0.91	0.91
20	46	0.30	0.30	0.20	0.10	0.10	0	C_{46}	0.68	1	1.56	1.96	0.98	0.98
21	56	0.10	0.30	0.30	0.30	0	0	C_{56}	1.30	1	1.31	1.79	0.95	0.95

The actual component as transformed from Eqn. (17) , Table (1) and (2) were used to measure out the quantities of water/cement ratio (Z_1), cement (Z_2), fine aggregate (Z_3), coarse aggregate (Z_4), polypropylene fibre (Z_5) and steel fibre (Z_6) in their respective ratios for the concrete cube strength test.

III. MATERIALS AND METHODS

3.1 MATERIALS

Cement, water, fine and coarse aggregates, polypropylene fibre and steel fibre are the materials under investigation in this research work. The cement is Dangote cement, a brand of Ordinary Portland Cement, conforming to British Standard Institution BS 12 (1978). The fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite of 20mm size was obtained from a local stone market and was downgraded to 4.75mm. The same size and nature of polypropylene fibre and steel fibre used previously by Nwachukwu and others (2022c) and Nwachukwu and others (2022b) respectively, are the same as the one being used in this present work. Also, potable water from the clean water source was used in this experimental investigation.

3.2. METHOD

3.2.1. SPECIMEN PREPARATION / BATCHING/ CURING

The specimens for the compressive strength were concrete cubes. They were cast in steel mould measuring 15cm*15cm*15cm. The mould and its base were damped together during concrete casting to prevent leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 42 mix ratios were to be used to produce 84 prototype concrete cubes. Twenty- one (21) out of the 42 mix ratios were as control mix ratios to produce 42 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (12). Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28days of curing the specimens were taken out of the curing tank.

3.2.2. COMPRESSIVE STRENGTH TEST

Compressive strength testing was done in accordance with BS 1881 – part 116 (1983) - Method of determination of compressive strength of concrete cube and ACI (1989) guideline .Two samples were crushed for each mix ratio and in each case, the compressive strength was then calculated using Eqn.(18)

$$\text{Compressive Strength} = \frac{\text{Average failure Load,P (N)}}{\text{Cross– sectional Area,A (mm2)}} \quad (18)$$

IV. RESULTS AND DISCUSSION

4.1. COMPRESSIVE STRENGTH RESULTS OF HPSFRC FOR THE INITIAL EXPERIMENTAL TESTS.

The results of the compressive strength ($R_{esponse,}$ N_i) of HPSFRC based on a 28-days strength is presented in Table 3. These are calculated from Eqn..(18)

Table 3: 28th Day Compressive Strength Test Results for HPSFRC Based on Scheffe's (6, 2) Model for the Initial Experimental Tests.

S/N	POINTS	EXPERIMENTAL NUMBER	RESPONSE N_i, MPa	RESPONSE SYMBOL	$\sum N_i$	AVERAGE RESPONSE N, MPa
1	1	1A 1B	50.43 51.64	N_1	102.07	51.04
2	2	2A 2B	53.89 54.09	N_2	107.98	53.99
3	3	3A 3B	49.76 50.53	N_3	100.29	50.15
4	4	4A 4B	54.32 54.33	N_4	108.65	54.33
5	5	5A 5B	57.66 57.64	N_5	115.30	57.65
6	6	6A 6B	49.98 49.87	N_6	99.85	49.93
7	12	7A 7B	34.76 35.76	N_{12}	70.52	35.26
8	13	8A 8B	43.97 43.87	N_{13}	87.84	43.92
9	14	9A 9B	49.94 48.54	N_{14}	98.48	49.24
10	15	10A 10B	52.08 52.54	N_{15}	104.64	52.31
11	16	11A 11B	44.32 43.65	N_{16}	87.97	43.99
12	23	12A 12B	46.87 47.28	N_{23}	94.15	47.08
13	24	13A 13B	39.84 40.05	N_{24}	79.89	39.95
14	25	14A 14B	33.64 33.68	N_{25}	67.32	33.66
15	26	15A 15B	50.08 49.86	N_{26}	99.94	49.97
16	34	16A 16B	45.98 45.88	N_{34}	91.86	45.93
17	35	17A 17B	47.98 47.86	N_{35}	95.84	47.92
18	36	18A 18B	39.82 40.21	N_{36}	80.03	40.02
19	45	19A 19B	48.66 47.98	N_{45}	96.64	48.32

20	46	20A 20B	51.44 50.76	N_{46}	102.20	51.10
21	56	21A 21B	48.76 48.84	N_{56}	97.60	48.80

4.2 COMPRESSIVE STRENGTH RESULTS OF HPSFRC FOR THE EXPERIMENTAL (CONTROL) TEST.

Table 4 shows the 28th day Compressive strength results for the Experimental (Control) Test

Table 4: 28TH Day Compressive Strength Results for HPSFRC Based on Scheffe's (6,2) Model for the Experimental (Control) Tests.

S/N	CONTROL POINTS	EXPERIMENTAL NUMBER	RESPONSE, MPa	AVERAGE RESPONSE, MPa
1	C_1	1A 1B	49.98 50.65	50.32
2	C_2	2A 2B	54.75 54.22	54.49
3	C_3	3A 3B	46.76 47.08	46.92
4	C_4	4A 4B	52.65 53.08	52.87
5	C_5	5A 5B	51.86 52.04	51.95
6	C_6	6A 6B	50.76 49.44	50.10
7	C_{12}	7A 7B	37.86 36.98	37.42
8	C_{13}	8A 8B	42.98 41.77	42.38
9	C_{14}	9A 9B	45.66 46.22	45.94
10	C_{15}	10A 10B	51.06 51.32	51.19
11	C_{16}	11A 11B	44.43 44.64	44.54
12	C_{23}	12A 12B	47.87 47.12	47.50
13	C_{24}	13A 13B	40.22 40.33	40.28
14	C_{25}	14A 14B	31.82 32.00	31.91
15	C_{26}	15A 15B	45.87 46.22	46.05
16	C_{34}	16A 16B	45.55 45.32	45.44
17	C_{35}	17A	49.86	49.65

		17B	49.43	
18	C_{36}	18A 18B	40.54 40.32	40.43
19	C_{45}	19A 19B	48.34 48.43	48.39
20	C_{46}	20A 20B	50.32 50.23	50.28
21	C_{56}	21A 21B	49.44 49.22	49.33

4.3 SCHEFFE'S (6,2) REGRESSION MODEL FOR COMPRESSIVE STRENGTH OF HPSFRC

Substituting the values of the compressive strengths (responses) from Table 3 into Eqns.(8) through (12), we obtain the coefficients (in MPa) of the Scheffe's second degree polynomial as follows:

$\beta_1 = 51.04$; $\beta_2 = 20.53.99$; $\beta_3 = 50.15$; $\beta_4 = 54.33$; $\beta_5 = 57.65$; $\beta_6 = 49.93$; $\beta_{12} = 69.02$; $\beta_{13} = 26.70$; $\beta_{14} = -13.78$; $\beta_{15} = -8.14$; $\beta_{16} = -25.98$; $\beta_{23} = -19.96$; $\beta_{24} = -56.84.$; $\beta_{25} = -88.69$; $\beta_{26} = -7.96$; $\beta_{34} = -25.24$; $\beta_{35} = -23.92$; $\beta_{36} = -40.14$; $\beta_{45} = -30.68$; $\beta_{46} = -4.12$ $\beta_{56} = -19.96$; (19)

Substituting the values of these coefficients in Eqn.(19) into Eqn. (9), we obtain the polynomial model for the optimization of the compressive strength of HPSFRC based on Scheffe's (6,2) lattice as given in Eqn.(20)

$$N = 51.04X1 + 20.53X2 + 50.15X3 + 54.33X4 + 57.65X5 + 49.93X6 + 69.02X1X2 + 26.70X1X3 - 13.78X1X4 - 8.14X1X5 - 25.18X1X6 - 19.96X2X3 - 56.84X2X4 - 88.69X2X5 - 7.96X2X6 - 25.24X3X4 - 23.92X3X5 - 40.14X3X6 - 30.68X4X5 - 4.12X4X6 - 19.96X5X6 \quad (20)$$

4.4. SCHEFFE'S (6,2) MODEL RESPONSES FOR HPSFRC AT CONTROL POINTS

By substituting the pseudo mix ratio of points c_1, c_2, c_3, c_4, c_5, … ….. c_{56} of Table 2 into Eqn.(20) , we obtain the second degree model responses for the control points of HPSFRC.

4.5 VALIDATION AND TEST OF ADEQUACY OF THE SCHEFFE'S (6,2) MODEL FOR HPSFRC

The reason for performing the test of adequacy is to check if there is any significant difference between the compressive strength results (lab responses) given in Table 4 and model responses from the control points based on Eqn.(20). Here, the Student's – T - test is adopted as the means of validating the Sheffe's model. Note that the procedures for using the Student's – T - test have been explained by Nwachukwu and others (2022 c). The result of the test shows that there is no significant difference between the experimental results and model responses. Therefore, the model is very adequate for predicting the compressive strength of HPSFRC based on Scheffe's (6,2) lattice.

4.6. DISCUSSION OF RESULTS

The highest compressive strength of HPSFRC based on Scheffe's (6,2) lattice is 57.65MPa . This corresponds to mix ratio of 0.75:1.00:1.30:1.20:1.50:1.50 for water/cement ratio, cement, fine aggregate, coarse aggregate, polypropylene fibre and steel fibre respectively. Similarly, the lowest compressive strength was found to be 33.66 MPa values from the model were found to be greater than the minimum value specified by the American Concrete Institute for the compressive strength of good concrete and also minimum standard (of 4500psi or 30.75MPa) specified by the American Society of Testing and Machine, ASTM C39 and ASTM C 469. Thus, using the model, compressive strength of HPSFRC of all points (1 - 56) in the simplex can be evaluated based on Scheffe's second degree model.

V. CONCLUSION

Scheffe's Second Degree Optimization Model, for six component mixtures, Scheffe's (6,2) has been presented so far and used to predict the mix proportions as well as a model for predicting the compressive strength of HPSFRC cubes. By using Scheffe's (6,2) simplex model, the values of the compressive strength were obtained for HPSFRC at all

21 points. The result of the student's t-test confirmed that there is a good correlation between the strengths predicted by the models and the corresponding experimentally observed results. The optimum attainable compressive strength of HPSFRC predicted by the Scheffe's (6,2) model based on Scheffe's (6,2) model. However, both values meet the minimum standard requirement (of 20 MPa and 30.75MPa) stipulated by American Concrete Institute (ACI) and American Society of Testing and Machine, ASTM C39 and ASTM C 469 respectively, for the compressive strength of good concrete. Thus, with the Scheffe's (6,2) model, any desired strength of HPSFRC given any mix proportions can be easily predicted and evaluated and vice versa. Finally, by the utilization of this Scheffe's optimization model, the problem of having to go through vigorous, time-consuming and laborious mixture design procedures to obtain a desiring strength of HPSFRC has been reduced to the barest minimum.

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