

# Coefficient Inequality for Close to Starlike Functions Constructed Using Inverse Starlike Classes

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**Abstract-** Here we will discuss a newly constructed class of analytic functions and its subclasses by which we will be obtaining sharp upper bounds of the functional  $|a_3 - \mu a_2^2|$  for the analytic function  $f(z) = z + \sum_{n=2}^{\infty} a_n z^n, |z| < 1$  belonging to these classes and subclasses.

**Indexed Terms-** Univalent functions, Starlike functions, Inverse Starlike functions and bounded functions, Convex functions, Principle of Subordination, Coefficient inequality.

## I. INTRODUCTION

Let  $\mathcal{A}$  denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the unit disc  $\mathbb{E} = \{z: |z| < 1\}$ . Let  $\mathcal{S}$  be the class of functions of the form (1.1), which are analytic univalent in  $\mathbb{E}$ .

In 1916, Bieber Bach ([7], [8]) proved that  $|a_2| \leq 2$  for the functions  $f(z) \in \mathcal{S}$ . In 1923, Löwner [5] proved that  $|a_3| \leq 3$  for the functions  $f(z) \in \mathcal{S}$ .

With the known estimates  $|a_2| \leq 2$  and  $|a_3| \leq 3$ , it was natural to seek some relation between  $a_3$  and  $a_2^2$  for the class  $\mathcal{S}$ , Fekete and Szegő[9] used Löwner's method to prove the following well known result for the class  $\mathcal{S}$ .

Let  $f(z) \in \mathcal{S}$ , then

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3 - 4\mu, & \text{if } \mu \leq 0; \\ 1 + 2 \exp\left(\frac{-2\mu}{1-\mu}\right), & \text{if } 0 \leq \mu \leq 1; \\ 4\mu - 3, & \text{if } \mu \geq 1. \end{cases} \tag{1.2}$$

The inequality (1.2) plays a very important role in determining estimates of higher coefficients for some sub classes  $\mathcal{S}$  (See Chhichra [1], Babalola [6]).

Let us define some subclasses of  $\mathcal{S}$ .

We denote by  $S^*$ , the class of univalent starlike functions

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \mathcal{A} \text{ and satisfying the condition}$$

$$Re \left( \frac{zg(z)}{g(z)} \right) > 0, z \in \mathbb{E}. \tag{1.3}$$

We denote by  $\mathcal{K}$ , the class of univalent convex functions

$$h(z) = z + \sum_{n=2}^{\infty} c_n z^n, z \in \mathcal{A}$$

and satisfying the condition

$$Re \left( \frac{zh'(z)}{h'(z)} \right) > 0, z \in \mathbb{E}. \tag{1.4}$$

A function  $f(z) \in \mathcal{A}$  is said to be close to convex if there exists  $g(z) \in S^*$  such that

$$Re \left( \frac{zf'(z)}{g(z)} \right) > 0, z \in \mathbb{E}. \tag{1.5}$$

The class of close to convex functions is denoted by  $C$  and was introduced by Kaplan [3] and it was shown by him that all close to convex functions are univalent.

$$S^*(A, B) = \left\{ f(z) \in \mathcal{A}; \frac{zf'(z)}{f(z)} < \frac{1+Az}{1+Bz}, -1 \leq B < A \leq 1, z \in \mathbb{E} \right\} \tag{1.6}$$

$$\mathcal{K}(A, B) = \left\{ f(z) \in \mathcal{A}; \frac{(zf'(z))'}{f'(z)} < \frac{1+Az}{1+Bz}, -1 \leq B < A \leq 1, z \in \mathbb{E} \right\} \quad (1.7)$$

It is obvious that  $S^*(A, B)$  is a subclass of  $S^*$  and  $\mathcal{K}(A, B)$  is a subclass of  $\mathcal{K}$ .

We introduce a new subclass as

$$\left\{ f(z) \in \mathcal{A}; (1-\alpha) \frac{zf(z)}{2 \int_0^z f(z) dz} + \alpha \frac{zf'(z)}{f'(z)} < \left( \frac{1+z}{1-z} \right)^\delta; z \in \mathbb{E} \right\}$$

and we will denote this class as  $KS^{*-1}(\alpha, \delta)$ .

It is to be noted that

- $KS^{*-1}(\alpha, 1) = KS^{*-1}$
- $KS^{*-1}(0, \delta) = S^{*-1}(\delta)$
- $KS^{*-1}(1, \delta) = S^*(\delta)$
- $KS^{*-1}(0, 1) = S^{*-1}$
- $KS^{*-1}(1, 1) = S^*$

Symbol  $<$  stands for subordination, which we define as follows:

- Principle of Subordination: Let  $f(z)$  and  $F(z)$  be two functions analytic in  $\mathbb{E}$ . Then  $f(z)$  is called subordinate to  $F(z)$  in  $\mathbb{E}$  if there exists a function  $w(z)$  analytic in  $\mathbb{E}$  satisfying the conditions  $w(0) = 0$  and  $|w(z)| < 1$  such that  $f(z) = F(w(z)); z \in \mathbb{E}$  and we write  $f(z) < F(z)$ .

By  $\mathcal{U}$ , we denote the class of analytic bounded functions of the form

$$w(z) = \sum_{n=1}^{\infty} d_n z^n, w(0) = 0, |w(z)| < 1. \quad (1.8)$$

$$\text{It is known that } |d_1| \leq 1, |d_2| \leq 1 - |d_1|^2 \quad (1.9)$$

## II. PRELIMINARY LEMMAS

For  $0 < c < 1$ , we write

$$w(z) = \left( \frac{c+z}{1+cZ} \right)$$

so that

$$\frac{1+Aw(z)}{1+Bw(z)} = 1 + (A-B)c_1z + (A-B)(c_2 - Bc_1^2)z^2 + \dots \quad (2.1)$$

## III. MAIN RESULTS

THEOREM 3.1: Let  $f(z) \in KS^{*-1}(\alpha, \delta)$ , then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{4\delta^2(4\alpha^2+18\alpha+5)}{(3\alpha+1)(2\alpha+1)^2} - \frac{36\mu\delta^2}{(2\alpha+1)^2} \text{ if } \mu \leq \frac{\alpha^2(4\delta-4)+\alpha(18\delta-4)+(5\delta-1)}{9\delta(3\alpha+1)} \\ \frac{4\delta}{3\alpha+1} \text{ if } \frac{\alpha^2(4\delta-4)+\alpha(18\delta-4)+(5\delta-1)}{9\delta(3\alpha+1)} < \mu < \frac{\alpha^2(4\delta+4)+\alpha(18\delta+4)+(5\delta+1)}{9\delta(3\alpha+1)} \\ \frac{36\mu\delta^2}{(2\alpha+1)^2} - \frac{4\delta^2(4\alpha^2+18\alpha+5)}{(3\alpha+1)(2\alpha+1)^2} \text{ if } \mu \geq \frac{\alpha^2(4\delta+4)+\alpha(18\delta+4)+(5\delta+1)}{9\delta(3\alpha+1)} \end{cases}$$

The results are sharp.

Proof: By definition of  $f(z) \in KS^{*-1}(\alpha, \delta)$ , we have

$$(1-\alpha) \frac{zf(z)}{2 \int_0^z f(z) dz} + \alpha \frac{zf'(z)}{f'(z)} = \left( \frac{1+w(z)}{1-w(z)} \right)^\delta; w(z) \in \mathcal{U}. \quad (3.4)$$

Expanding the series (3.4), we get

$$\alpha \left( z^3 + z^4 \left( \frac{2}{3} a_2 + 2a_2 \right) + z^5 \left( \frac{1}{2} a_3 + \frac{4}{3} a_2^2 + 3a_3 \right) + \dots \right) + (1-\alpha)z(z^2 + 2a_2z^3 + a_2^2z^4 + 2a_3z^4 + \dots) = \left( z^3 + z^4 \left( \frac{2}{3} a_2 + a_2 \right) + z^5 \left( \frac{1}{2} a_3 + \frac{2}{3} a_2^2 + a_3 \right) + \dots \right) \left( 1 + 2\delta c_1z + 2\delta(c_2 + c_1^2)z^2 + \frac{\delta(\delta-1)}{2} 4(c_1^2)z^2 + \dots \right) \quad (3.5)$$

Identifying terms in (3.5), we get

$$a_2 = \frac{6\delta c_1}{2\alpha+1}, \alpha \neq \frac{-1}{2} \quad (3.6)$$

$$a_3 = \frac{4}{3\alpha+1} \left( \frac{\delta^2 c_1^2 (4\alpha^2+18\alpha+5)}{(2\alpha+1)^2} + \delta c_2 \right) \quad (3.7)$$

From (3.6) and (3.7), we obtain

$$a_3 - \mu a_2^2 = c_1^2 \left( \frac{\delta^2(16\alpha^2+72\alpha+20)-36\mu\delta^2(3\alpha+1)}{(3\alpha+1)(2\alpha+1)^2} \right) + \frac{4\delta c_2}{(3\alpha+1)} \quad (3.8)$$

Taking absolute value, (3.8) can be rewritten as

$$|a_3 - \mu a_2^2| \leq |c_1|^2 \left( \left| \frac{\delta^2(16\alpha^2+72\alpha+20)-36\mu\delta^2(3\alpha+1)}{(3\alpha+1)(2\alpha+1)^2} \right| - \frac{4\delta}{(3\alpha+1)} \right) + \frac{4\delta}{(3\alpha+1)} \quad (3.9)$$

Case I:  $\mu \geq \frac{(4\alpha^2+18\alpha+5)}{9(3\alpha+1)}$ . (3.9) can be rewritten as

$$|a_3 - \mu a_2^2| \leq |c_1|^2 \left( \frac{-4\delta^2(4\alpha^2+18\alpha+5)}{(3\alpha+1)(2\alpha+1)^2} - \frac{4\delta}{3\alpha+1} + \frac{36\mu\delta^2}{(2\alpha+1)^2} \right) + \frac{4\delta}{(3\alpha+1)} \quad (3.10)$$

Subcase I (a):  $\mu \leq \frac{\alpha^2(4\alpha+4)+\alpha(18\delta+4)+5\delta+1}{9\delta(3\alpha+1)}$

Using (1.8), (3.10) becomes

$$|a_3 - \mu a_2^2| \leq \frac{36\mu\delta^2}{(2\alpha+1)^2} - \frac{4\delta^2(4\alpha^2+18\alpha+5)}{(3\alpha+1)(2\alpha+1)^2} \quad (3.11)$$

Subcase I (b):  $\mu \leq \frac{\alpha^2(4\alpha+4)+\alpha(18\delta+4)+5\delta+1}{9\delta(3\alpha+1)}$  We obtain from (3.10)

$$|a_3 - \mu a_2^2| \leq \frac{4\delta}{(3\alpha+1)} \quad (3.12)$$

Case II:  $\mu < \frac{(4\alpha^2+18\alpha+5)}{9(3\alpha+1)}$

Proceeding as in case I, we get

$$|a_3 - \mu a_2^2| \leq |c_1|^2 \left( \frac{4\delta^2(4\alpha^2+18\alpha+5)}{(3\alpha+1)(2\alpha+1)^2} - \frac{4\delta}{3\alpha+1} - \frac{36\mu\delta^2}{(2\alpha+1)^2} \right) + \frac{4\delta}{(3\alpha+1)} \quad (3.13)$$

Subcase II (a):  $\mu \leq \frac{\alpha^2(4\alpha-4)+\alpha(18\delta-4)+5\delta+1}{9\delta(3\alpha+1)}$

(3.13) takes the form

$$|a_3 - \mu a_2^2| \leq \frac{4\delta^2(4\alpha^2+18\alpha+5)}{(3\alpha+1)(2\alpha+1)^2} - \frac{36\mu\delta^2}{(2\alpha+1)^2}$$

Subcase II (b):  $\frac{\alpha^2(4\alpha-4)+\alpha(18\delta-4)+5\delta+1}{9\delta(3\alpha+1)} < \mu < \frac{(4\alpha^2+18\alpha+5)}{9(3\alpha+1)}$

Proceeding as in subcase I (a), we get

$$|a_3 - \mu a_2^2| \leq \frac{4\delta}{3\alpha+1} \quad (3.14)$$

Thus, the theorem is proved. Extremal function for (3.1) and (3.3) is defined by

$$f_1(z) = z \left[ 1 + \frac{4\delta^3(4\alpha^2+18\alpha+5)^2 - 3(3\alpha+1)^2(2\alpha+1)^3}{\delta(3\alpha+1)(2\alpha+1)^2(4\alpha^2+18\alpha+5)} z \right]^{\frac{4\delta^3(4\alpha^2+18\alpha+5)^2}{4\delta^3(4\alpha^2+18\alpha+5)^2 - 3(3\alpha+1)^2(2\alpha+1)^3}}$$

Extremal function for (3.2) is defined by  $f_2(z) = z(1 + \delta z)^{\frac{4}{3\alpha+1}}$

Corollary 3.2: Putting  $\delta = 1$  in the theorem, we get

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{4(4\alpha^2+18\alpha+5)}{(3\alpha+1)(2\alpha+1)^2} - \frac{36\mu}{(2\alpha+1)^2} & \text{if } \mu \leq \frac{14\alpha+4}{9(3\alpha+1)} \\ \frac{4}{3\alpha+1} & \text{if } \frac{14\alpha+4}{9(3\alpha+1)} < \mu < \frac{8\alpha^2+22\alpha+6}{9(3\alpha+1)} \\ \frac{36\mu}{(2\alpha+1)^2} - \frac{4(4\alpha^2+18\alpha+5)}{(3\alpha+1)(2\alpha+1)^2} & \text{if } \mu \geq \frac{8\alpha^2+22\alpha+6}{9(3\alpha+1)} \end{cases}$$

These are the results of the class  $KS^{*-1}$ .

Corollary 3.3: Putting  $\alpha = 0$  in the theorem, we get

$$|a_3 - \mu a_2^2| \leq \begin{cases} (20 - 36\mu)\delta^2 & \text{if } \mu \leq \frac{5\delta-1}{9\delta} \\ 4\delta & \frac{5\delta-1}{9\delta} < \mu < \frac{5\delta+1}{9\delta} \\ (36\mu - 20)\delta^2 & \text{if } \mu \geq \frac{5\delta+1}{9\delta} \end{cases}$$

These are the results of the class  $S^{*-1}(\delta)$ .

Corollary 3.4: Putting  $\alpha = 1$  in the theorem, we get

$$|a_3 - \mu a_2^2| \leq \begin{cases} (3 - 4\mu)\delta^2 & \text{if } \mu \leq \frac{3\delta-1}{4\delta} \\ \delta & \frac{3\delta-1}{4\delta} < \mu < \frac{3\delta+1}{4\delta} \\ (4\mu - 3)\delta^2 & \text{if } \mu \geq \frac{3\delta+1}{4\delta} \end{cases}$$

These are the results of the class  $S^*(\delta)$ .

Corollary 3.5: Putting  $\alpha = 0, \delta = 1$  in the theorem, we get

$$|a_3 - \mu a_2^2| \leq \begin{cases} 20 - 36\mu & \text{if } \mu \leq \frac{4}{9} \\ 4 & \frac{4}{9} < \mu < \frac{2}{3} \\ 36\mu - 20 & \text{if } \mu \geq \frac{2}{3} \end{cases}$$

These are the results of the class  $S^{*-1}$ .

Corollary 3.6: Putting  $\alpha = 1, \delta = 1$  in the theorem, we get

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3 - 4\mu & \text{if } \mu \leq \frac{1}{2} \\ 1 & \frac{1}{2} < \mu < 1 \\ 4\mu - 3 & \text{if } \mu \geq 1 \end{cases}$$

These estimates were derived by Keogh and Merkes [8] and are results for the class of univalent starlike functions.

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