Modified Transient Energy Function for Power System Stability Studies

UGOCHUKWU E. ANIONOVO[1], EWAOCHE J. OKAMPO[2], OMOGBAI N. OYAKHILOMEN[3], IKARAOHA C. OBINNA[4], SYLVESTER E. ABONYI[5]

[1, 3, 4, 5] *Electrical Engineering Department, Nnamdi Azikiwe University, Awka Anambra State.*

[2] *Department of Electrical and Electronic Engineering Technology, University of Johannesburg, Johannesburg 2092, South Africa.*

Abstract- In this article, we examined the concept of the Transient Energy Function approach to power system transient stability analysis or direct method, originally suggested by Aleksandr Mikhailovich Lyapunov in his dissertation published in 1892. Leveraging the real-time possibility of the direct method, this research article presented a Modified Transient Energy Function (MTEF) approach based purely on real-time parameters of the power system network using simplified physics and mathematical modeling. The developed MTEF model or the alternative model proved to be clearer and easily adaptable to real-time control systems.

Indexed Terms- Direct Method, Transient Energy Function Method, Equal Area Criteria, Power System Stability Studies.

I. INTRODUCTION

The concept of power system stability in electrical engineering is as old as the advent of electricity, however it became a subject of concern to utility engineers as the network grew in complexity. Author [1] wrote; *"Power system stability is its ability to remain in equilibrium state under normal operating condition and regain acceptable equilibrium state after undergoing system disturbance"*. This involves large departure of alternator rotor angle from synchronous speed, leading to loss of synchronism and possible system breakdown in the event of system protection malfunction. Transient Stability Analysis is the basis of Power System Transient Stability Control which leads to stable operation in power system networks [2]. Over the years the conventional method based on time domain simulation approach were used to access power system stability, but due to its inability to provide straightforward screening tools, sound stability margins, sensitivity analysis tools and sound control suggestions, the non-conventional approaches began to receive appropriate attention [3]. Among the non-conventional methods which began to receive attention by the late sixties, include the Direct Method (DM) based on Transient Energy Function (TEF) and the automatic learning method. The authors in [4] and [5] made tremendous breakthrough in the development of this method for transient stability studies, capitalizing on the merits of direct method which involves reducing the Time domain simulations to the barest minimum during fault. It later became clear that the direct method suffers from certain difficulties which include over simplification of model to ensure construction of good Lyapunov function and poor computational efficiency and accuracy of the transient stability assessment of large systems [6]. Several interesting attempts which were made to outwit these difficulties include the "structure preserving modeling" [7] and the "pseudo-Lyapunov approaches" which involved the use of TDSM with detailed modeling together with pseudo-Lyapunov functions with simplified modeling [8]. More solutions proposed to tackle the second difficulty encountered by direct method includes; the method of [9], the acceleration approach [10], the exit point strategy of authors in [5], and other methods contributed by authors in [11] and [12].

Despite the inventive nature of these solutions, they were not able to completely overcome the limitations of direct method as the resulting solutions were overly conservative with unpredictably varying degrees of conservativeness and on the other hand computationally heavy, thereby removing the computer gains expected of direct method, making them much heavier than Time Domain Simulation Method even. However further research proposed the use of single or two machine equivalents to represent large power system and hybrid Direct Method / Time Domain Simulation Method to carry out the associated modeling for the purpose of stability domain estimation. This solution resulted on one hand to Bellman decomposition-aggregation approach and the Vector Lyapunov functions, where the multi-machine power system is reduced to 2-machine subsystems [6] and or the single machine equivalent approach [13], and [14]. On the other hand, the modeling aspect resulted to hybrid approach either of the multi-machine type [15] or of the single-machine equivalent type [16]. The merits of the resultant hybrid solution include full flexibility with respect to power system modeling, first-swing and multi-swing transient stability assessment, effective screening tool design, computation of stability margins, yielding sensitivity analysis and the identification of the relevant machines parameters which opens avenue for real-time emergency control [3]. Drawing from the real-time possibility of the direct method, this research article presented a Modified Transient Energy Function (MTEF) approach based purely on real-time parameters of the power system network using simplified physics and mathematical modeling.

II. TECHNICAL CONSIDERATION

The Direct method or Transient Energy Function method cannot be discussed without emphasis on equal area criteria, thus in the next section we will dissect the onus of equal area criteria and establish its relation with the TEF method.

2.1 Equal Area Criteria

In stability analysis of a single-machine infinite bus system, the information regarding the maximum angle excursion (δ_m) and stability limit (δ_L) can readily be obtained graphically from the power angle curve of figure 2.1 (a) without formally solving the swing equation.

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = P_{mec} - P_e \tag{2.1}$$

From equation (2.1), the relationship between the rotor angle and the accelerating power can be written as follows:

$$\frac{d^2\delta}{dt^2} = \frac{\omega_0}{2H}P_a \tag{2.2}$$

Where $P_a = P_{mec} - P_e$. At the initial condition, the alternator operates at synchronous speed with rotor angle δ_0 and mechanical power P_{mec0}. At this point $P_{mec} = P_e$. When fault occurs, difference in power which must be accounted for by rate of stored kinetic energy change in rotor masses occurs [17]. This is as a result of increase in speed caused by constant accelerating power P_a. For any time less than the clearing time t_c, the acceleration is constant as seen from equation (2.2).

While the fault exists, the velocity increase during fault can be obtained by integrating equation (2.2) as follows with respect to time:

$$\frac{d^2\delta}{dt^2} = \int_0^t \frac{\omega_0}{2H}P_a dt => \frac{d\delta}{dt} = \frac{\omega_0}{2H}P_a t \tag{2.3}$$

A further integration of equation (2.3) with respect to time yields the rotor angle position as follows:

$$\frac{d\delta}{dt} = \int_t^{t_c} \frac{\omega_0}{2H}P_a(t)dt => \delta = \frac{\omega_0 P_a}{4H}t^2 + \delta_1 \tag{2.4}$$

Equations (2.3) and (2.4) shows that the velocity of the rotor, relative to the synchronous speed increases linearly with time, while the rotor angle moves from $\delta_0 or\ \delta_1 to\ \delta_2$, the clearing angle as in the case of figure 2.1.

At the time of fault clearing, equation (2.4) can be written as equation (2.5), from where the critical clearing time can be obtained.

$$t_c = \sqrt{\frac{4H(\delta_2-\delta_1)}{\omega_0 P_a}} \tag{2.5}$$

We can define the angular velocity of rotor ω_r relative to synchronous speed ω_0 from the swing equation for a single machine connected to an infinite bus, or two machine system of equation (2.1) as follows:

$$\omega_r = \frac{d\delta}{dt} = \omega - \omega_0 \tag{2.6}$$

Differentiating equation (2.6) with respect to time t and substituting in the swing equation of equation (2.1) we obtain;

$$\frac{2H}{\omega_0}\frac{d\omega_r}{dt} = P_{mec} - P_e \qquad (2.7)$$

Hence multiplying both side of equation (2.7) with $\omega_r = \frac{d\delta}{dt}$, equation (2.8) is obtained.

$$\omega_r \frac{2H}{\omega_0}\frac{d\omega_r}{dt} = (P_{mec} - P_e)\frac{d\delta}{dt} \qquad (2.8)$$

Rearranging equation (2.8), multiplying both side by dt and integrating, we obtain equation (2.9).

$$\frac{H}{\omega_0}\frac{d(\omega^2{}_r)}{dt}.dt = \int_{\delta_1}^{\delta_m}(P_{mec} - P_e)\frac{d\delta}{dt}.dt \qquad (2.9)$$

The change in rotor angular acceleration $d(\omega^2{}_r)$ corresponds to the rotor angle excursion limit $\delta_1 and\ \delta_m$. Since ω_r is the difference between the rotor angular speed and the synchronous speed, we can conclude that if the rotor angular acceleration is synchronous at the two-machine angle limits considered, $d(\omega^2{}_r)\ or\ \omega^2{}_{r2} - \omega^2{}_{r1}$ equals zero. Hence equation (2.9) reduces to equation (2.10) which applies to any two points $\delta_1$and $\delta_m$or boundary condition on the power angle curve provided the rotor speed is at synchronous speed.

$$\int_{\delta_1}^{\delta_m}(P_{mec} - P_e)d\delta = 0 \qquad (2.10)$$

Integrating equation (2.10) in parts or steps produces equations (2.11) and (2.12) when the accelerating part is equated to the decelerating part.

$$\int_{\delta_1}^{\delta_2}(P_{mec} - P_e)\,d\delta + \int_{\delta_2}^{\delta_m}(P_e - P_{mec})\,d\delta = 0 \quad (2.11)$$

$$\int_{\delta_1}^{\delta_2}(P_{mec} - P_e)\,d\delta = \int_{\delta_2}^{\delta_m}(P_e - P_{mec})\,d\delta \qquad (2.12)$$

This entails that the area under the function $(P_{mec} - P_e)$ plotted against δ equals zero for the system to be stable and is achievable when area A_1 equals area A_2. Splitting the integral function of equation (2.11) into accelerating and decelerating kinetic energy (KE) periods, we obtain equations (2.13) and (2.14).

$$KE_1 = \int_{\delta_1}^{\delta_2}(P_{mec} - P_e)\,d\delta = area\ A_1$$

…accelerating (2.13)

$$KE_2 = \int_{\delta_2}^{\delta_m}(P_e - P_{mec})\,d\delta = area\ A_2$$

…decelerating (2.14)

Since energy losses had not been considered, the kinetic energy gained is equal to that lost; thus area A_1 is equal to area A_2. This is the basis of equal area criterion which enables the maximum swing of δ and hence the stability of the system to be determined without explicitly computing the time response through formal solution of swing equation [18].

This criterion can readily be used to determine the maximum permissible increase in P_{mec} for the system under consideration. The value of A_1depends on the fault clearing time. Delay in fault clearing increases clearing angle δ_c or δ_2 as shown in figure 2.1, consequently increasing area A_1. Stability criterion requires that area A_2 increase at the expense of larger rotor maximum excursion angle δ_m. This system is considered stable only if area A_2 equal to A_1 is located above P_{mec}. For values of A_1 greater than A_2, A_2 located any other place apart from above P_{mec} along the curve or vice versa, the system becomes unstable. At this point $\delta_m > \delta_L or\ \delta_{max}$, the maximum permissible rotor angle deviation the system can withstand. Stability is lost owing to the fact that the net torque at this time is generative instead of degenerative.

Hence in order to satisfy the requirements of the equal-area criterion for stability, there is a critical angle for clearing the fault called the *critical clearing angle* δ_{cr}. The corresponding critical time for removing the fault is called the *critical clearing time* t_{cr}. δ_{cr} and t_{cr} can be calculated. Thus, from equation (2.13)

$$A_1 = \int_{\delta_1}^{\delta_{cr}} P_{mec}\,d\delta = P_{mec}(\delta_{cr} - \delta_1) \qquad (2.15)$$

Where δ_1 is the initial rotor angle. At this instance $P_e = 0$ and from equation (2.14)

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}}(P_e - P_{mec})\,d\delta => \int_{\delta_{cr}}^{\delta_{max}}(P_{max}\sin\delta - P_{mec})\,d\delta \qquad (2.16)$$

$$=> A_2 = P_{max}(\cos\delta_{cr} - \cos\delta_{max}) - P_{mec}(\delta_{max} - \delta_{cr}) \qquad (2.17)$$

Equating the new expression for A_1 and A_2, that is equations (2.15) and (2.17), we can obtain the critical clearing angle as follows:

$$P_{mec}\delta_{cr} - P_{mec}\delta_1 = P_{max}\cos\delta_{cr} - P_{max}\cos\delta_{max} - P_{mec}\delta_{max} - P_{mec}\delta_{cr} \qquad (2.18)$$

$\cos\delta_{cr} = -\frac{P_{mec}}{P_{max}}\delta_1 + \cos\delta_{max} + \frac{P_{mec}}{P_{max}}\delta_{max}$ (2.19)

$\cos\delta_{cr} = \frac{P_{mec}}{P_{max}}(\delta_{max} - \delta_1) + \cos\delta_{max}$ (2.20)

But $\delta_{max} = \pi - \delta_1 (elec\ rad.)$ from figure 2.1 and $P_{mec} = P_{max}\sin\delta$, hence substituting these into equation (2.20) produces equation (2.21).

$\cos\delta_{cr} = \sin\delta_1(\pi - 2\delta_1) + \cos(\pi - \delta_1)$ (2.21)

Recalling the trigonometry identity $\cos(\pi - x) = -\cos x$, equation (2.21) becomes

$\delta_{cr} = \cos^{-1}[(\pi - 2\delta_1)\sin\delta_1 - \cos\delta_1]$ (2.22)

Substituting the value of δ_{cr} into equation (2.5) and setting $\delta_2 = \delta_{cr}$, t_{cr} becomes

$$t_{cr} = \sqrt{\frac{4H(\delta_{cr}-\delta_1)}{\omega_0 P_a}} \quad (2.23)$$

We can solve for δ_0 from the pre-fault swing equation, (with 0 acceleration) according to equation (2.1) from where δ_{max} can be obtained.

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = P_{mec} - P_{max}sin\delta$$

$$=> 0 = P_{mec} - P_{max}sin\delta_0 \quad (2.26)$$

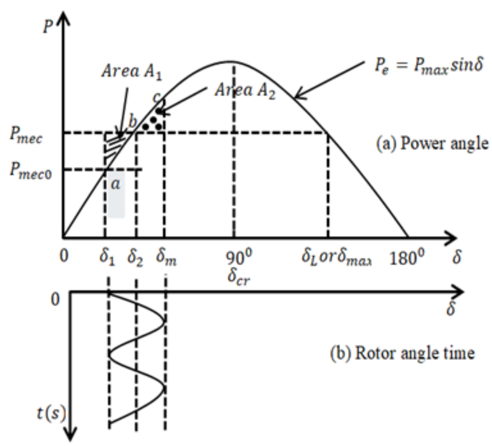

Figure 2.1: Equal area criteria illustration
(Source: Adapted from [1]).

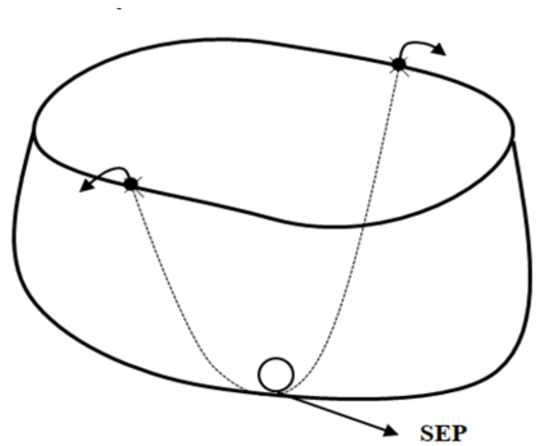

Figure 2.2: A ball rolling on the inner surface of a bowel
(Source: adapted from [1])

2.2 Direct Method (DM)

This method evaluates system stability without explicitly solving the system set of differential equations [1]. It uses transient energy for assessment of system transient stability. A more general Lyapunov's second method called the direct method uses the energy-based method. The transient energy method makes use of rolling ball analogy [8], for its implementation. As shown in figure 2.2, the area inside the bowel represents the region of stability and the area outside is the region of instability [1]. The bowel has irregular rim surface representing different heights.

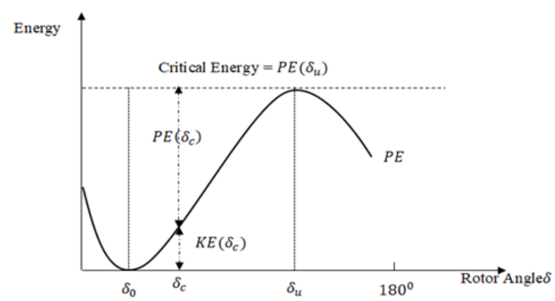

Figure 2.3: Energy angle relationship (source [1])

Under equilibrium state, the ball is stationed at the bottom of the bowel; a condition referred to as Stable Equilibrium Point (SEP). It will remain in that state, until kinetic energy of a given magnitude and direction is injected into the system causing the ball to move up the bowel in the direction determined by the kinetic energy injected. Depending on the magnitude of the kinetic energy injected, the ball could rise and fall back to equilibrium point or escape the bowel through the rim to state of instability. The surface inside the bowel represents the Potential Energy Surface (PES), and the rim of the bowel represents the Potential Energy Boundary Surface (PEBS) [19]. With reference to power system, the application of Transient Energy Function (TEF) method to power system transient analysis is synonymous to that of a ball rolling in a bowel, briefly described above [1]. Prior to the presence of fault in the power system, it is operating at stable equilibrium point. When fault occur, the system gains kinetic energy which causes the synchronous machine to accelerate. While the fault persists, the power system gains kinetic energy and potential causing it to move away from SEP.

When the fault is cleared, the kinetic energy is converted into potential energy like in the case of rolling ball system. For stability, the criterion is that the system must be capable of absorbing the kinetic energy at a time when the forces on the generators tends to bring them toward a new equilibrium point [1]. This depends on the potential energy-absorbing capacity of the post-disturbance system. For a given post disturbance system, there is a maximum or critical amount of transient energy that the system can absorb. Thus, transient stability assessment requires two considerations.

(a) Functions that adequately describe the transient energy required for one or more machines to fall out of synchronism.
(b) A good estimation of the critical energy required for the machines to lose synchronism.

For a two-machine system, the critical energy is uniquely defined [1], and the TEF analysis is equivalent to the equal area criterion described in section 2.1. See figure (2.3) which shows a plot of transient energy to rotor angle (δ)[8]. Figure (2.3) can be used to specify the critical clearing angle in terms of potential and kinetic energy.

To determine stability, the sum of the kinetic energy and the potential energy gained by the system during fault at a given rotor angle, is compared to the critical potential energy $PE(\delta_u)$, equation (2.27).

$$PE(\delta_c) + KE(\delta_c) = PE(\delta_u) \qquad (2.27)$$

For a given disturbance, there is a stable equilibrium point for the post fault system. Figure (2.4) shows region of attraction for a given post fault condition / SEP. Any state of the system at fault clearing (x_{cl}) inside the region of attraction will eventually converge to SEP, thus the system is said to be stable. But if the state of the system at (x_{cl}) lies outside the region of attraction, the system is said to be unstable. The state of the system at fault clearing (x_{cl}) can be described by the value of the energy function evaluated at x_{cl} i e. $V(x_{cl})$. Hence the direct method solves the stability problem by comparing $V(x_{cl})$ to the critical energy V_{cr}. The system is stable if $V(x_{cl})$ is less than V_{cr} and the quantity $V_{cr} - V(x_{cl})$ is a good measure of the systems relative stability defined as the transient energy margin. The quantity $V(x_{cl})$ measures the amount of transient energy injected into the system by the fault while the critical energy measures the strength of the post fault system.

With reference to figure (2.4), if the rotor oscillates within the range δ_{u1} and δ_{u2}, the system will remain transiently stable. If it swings out of this region, instability sets in. Hence, the two points δ_{u1} and δ_{u2} on the potential energy curve form a boundary to all stable rotor angle trajectories. This boundary is called the PEBS and the points on the boundary are local potential energy peaks.

The boundary of the stability region is usually approximated locally by a constant energy surface $\{K = V(x)|_x\}$ as shown in figure (2.4), where K represents the critical energy V_{cr} of the post-fault system.

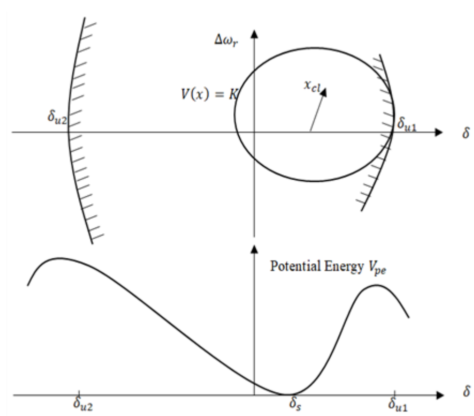

Figure 2.4: Region of stability and its local approximation (Source: [1]).

III. METHODOLOGY

In direct method, the system stability is accessed using the method of transient energy. Here, to determine stability, the sum of the kinetic energy and the potential energy gained by the system during fault, at any given rotor angle is compared to the critical potential energy $PE(\delta_u)$, see equation (2.27) restated here for clarity. That is

$$PE(\delta_c) + KE(\delta_c) = PE(\delta_u) \quad (3.1)$$

3.1 Modified Transient Energy Function Model Formation

In simple terms, electrical energy is defined as the work done in an electric circuit. This is synonymous to the work done in moving one coulomb of charge within the circuit. Charged particles hold potential energy which is released when attractive or repulsive force is applied in the form of heat energy. Since movement of charge in electric circuit constitutes current, we can say that energy required to generate electric current is called electrical energy. Now for three phase star connected circuit, if (V) is the potential difference or pressure or attractive or repulsive force causing the current of (I) to flow when a charge (Q) is moved, work done becomes;

$$W = Q\sqrt{3}V \quad (3.2)$$

But $Q = it \; or \; I = Q/t$, therefore

$$W = \sqrt{3}VI\cos\theta \,.\, t = PE = \sqrt{3}VI\cos\theta \,.\, t(W - S \; or \; Joule) \quad (3.3)$$

Where $\cos\theta$ is power factor. The potential energy of the system at any point in time must equalize the injected kinetic energy due to fault for the system to maintain stability, thus critical clearing energies;

$$PE(\delta_{cr}) = KE(\delta_{cr}) \quad (3.4)$$

Or

$$KE(\delta_{cr}) = \sqrt{3}VI\cos\theta \,.\, t_{cr}(\delta_{cr}) \quad (3.5)$$

In terms of power, work done per unit time or 1 joule of energy expended per unit time is defined as power. From equation (3.3), it follows that;

$$\frac{W}{t} = \frac{E}{t} = \sqrt{3}VI\cos\theta = P \quad (3.6)$$

Or

$$E = Pt \quad (3.7)$$

Where E is the energy expended and the P is the power consumed in t time.

With stator resistance neglected, air-gap power is same as the terminal power. That is $P_{mec} = P_e = \bar{I}\bar{V}\sin\bar{\delta}$. Hence, expressing the terminal power in terms of energy potential of the system, we have;

$$E = P_e t = \bar{I}\bar{V}\sin\bar{\delta} \,.\, t \quad (3.8)$$

From where the transient time can be computed as shown in equation (3.9)

$$t = \frac{\bar{I}\bar{V}\sin\bar{\delta}}{P_e} \quad (3.9)$$

The rotor speed change $\Delta\omega_r = \frac{\partial\delta}{\partial t}$ can be obtained from equation (3.10)

$$\Delta\omega_r = \frac{\partial\delta}{\partial(IV\sin\delta)/P_e}, => \omega_r = \int\left(\frac{(P_{mec}-P_e)\omega_0}{IV\sin\delta}\right).d\delta \quad (3.10)$$

This is synonymous to equation (2.1), after simplification, restated here for clarity; a variant of the swing equation.

$$d(\delta^2{}_r) = \int_{\delta_1}^{\delta_m}\left(\frac{(P_{mec}-P_e).\omega_0}{2H}\right)d\delta \quad (3.11)$$

$\frac{\omega_0}{IV\sin\delta} = \frac{\omega_0}{2H} = costant,$ a measure of the inertia constant of the alternator given in megawatts-seconds per megavolt ampere.

We can solve for δ_0 from the pre-fault swing equation, (with 0 acceleration) according to equation (2.1) from where δ_{max} can be obtained.

$$\frac{2H}{\omega_0}(\delta^2{}_r) = \int_{\delta_1}^{\delta_m}(P_{mec} - P_{max}\sin\delta)d\delta$$

$$=> 0 = P_{mec} - P_{max}\sin\delta_0 \quad (3.12)$$

IV. DISCUSSION

The MTEF model formation represents a new approach to power system transient stability analysis mathematical modeling based on the direct method. The method also eliminated the vague introduced by the term "*construction of good Lyapunov's energy function*" as obtainable in the previous energy function method as suggested by A. M. Lyapunov in his dissertation published in 1892. In order words, it simplified the process of energy function determination and extended it to compute the transient time which readily produces the rotor angle-time response characteristics. Since the parameters utilized for this model formation can be obtained from Phasor Measurement Unit, the model is adaptable to real-time control systems.

CONCLUSION

We studied the direct method of power system transient stability analysis which is based on transient energy function suggested initially by A. M Lyapunov, and came up with a modified version of it called MTEF. The modified version model was spelt out with simple physics and mathematical equations obtainable in the power system engineering. Owing to the model's empirical nature, a clear insight on how the MTEF method and the previous TEF method can be applied to power system transient analysis was made apparent.

REFERENCES

[1] Kundur Prabhashankar (1994), *"Power System Stability and Control",* Electric Power Research Institute, Tata MacGraw Hill Publishing Company Ltd, 7 Patel Nager, New Delhi, India.

[2] Lu Qiang, and Mei Shengwei (2000) *"Important basic researches on powersystems facing 21 century,"* Natural Science Development, vol. 10, no.10, pp. 870-876.

[3] Mania Pavella (1997), *Power System Stability and Control Comparative Analysis*, In IFAC Control of Industrial Systems, Belfort France.

[4] Pai M.A. (1981), *Power System Stability Analysis by the Direct Method of Lyapunov*. North-Holland Publishing Company, Amsterdam.

[5] Chiang H. D., .F .F. Wu and P. PVaraiya (1994), *A BCU method for Direct Analysis of Power System Transient Stability*. In IEEE Transactions, PWRS-9, pp. 1194-2000.

[6] Grujic Lj.T, A.A. Martynyuk and M. Ribben-Pavella (1987). *Large Scale Systems Stability Under Structural and Singular Perturbations.* Lecture notes in Control and Information Sciences Series, Springer Verlag, Berlin Germany.

[7] Bergen A. R. and D.J. Hill (1981), *A Structure Preserving Model for Power System Stability Analysis.* In IEEE Transactions on PAS, PAS-100, pp.25-33.

[8] Athay T., R.R Sherkat, R. Podmore, S. Virmani and C. Puech (1979), *Transient Energy Stability Analysis.* In System Engineering for Power: Emergency Operating State Control- Section IV, US Dept. of Energy Publication, No CONF-790904-P1.

[9] Kakimoto N., Y. Ohsawa and M. Hayashi (1980), *Transient Stability Analysis of Multimachine Power System with Field Flux Decays via Lyapunov's Direct Method.* In IEEE Transactions, PAS-99, pp. 1819-1827.

[10] Ribbens-Pavella M., P.G. Murthy and J.L. Horward (1981), *An Acceleration Approach to Practical Stability Domain Estimation in Power System.* In Electrical and Power System Modelling and Simulation (j. Robert and w. Midvidy, Eds), J.C BaltwerAgentina, Scientific Publishing Co., pp. 3-10.

[11] Pai. M.A (1989*), Energy Function Analysis for Power System Stability*. Kluwer Academic Publishers, Boston USA.

[12] Fonseca L.G.S., and I.C Decker (1985), *Iterative Algorithm for Critical Energy Determination in Transient Stability of Power System.* In IFAC Symposium on Planning and Operation Electric Energy Systems, Rio de Janeiro, Brazil, South America.

[13] Rahimi F.A., and G. Schaffer (1987*), Power System Transient Stability Indexes for Online Analysis of 'Worst Case' Dynamic Contingencies*. In IEEE Transactions, PWRS-2, pp. 660-668.

[14] Xue Y., T.H Van Cutsem and M. Ribbens-Pavella (1988), *A Simple Direct Method for Fast Transient Stability Assessment of Large Power Systems*. In IEEE Transactions, PWRS-3, pp, 400-412.

[15] Maria G.A., C. Tang and J. Kim (1990), *Hybrid Transient Stability Analysis*. In IEEE transactions, PWRS-5, pp. 348-394.

[16] Zhang, Y., L. Wehenkel, P. Rousseaux and M. Pavella (1997). SIME: *a Hybrid Approach to Fast Transient Stability Assessment and Contingency Selection*. In International Journal of Electrical Power and Energy Systems, Vol. 19, No. 3, March, pp. 195-208.

[17] Ferraro, Robert Matthew (2021), *"Method of Modeling the Swing Equation Using Time Synchronized Measurements"*. Dissertations and

Theses. Paper 5729. https://doi.org/10.15760/etd.7600

[18] Kimbark E. N (1948), *Power System Stability, Vol. I, Elements of Stability Calculations*, New York John Wiley and Sons Inc.

[19] Chi-Keung Tang (1984), *Evaluation of Direct Method for Power System Transient Stability Analysis*, M. Eng, degree thesis, University of Toronto, Canada.