Applications of Mohand Transform

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Abstract- The Mohand transformation is a mathematical tool which is used in the solving of differential equations by converting it from one from in to another from. Regularly it is effective in solving linear differential equations either ordinary or partial. The Mohand transformation is used in solving the time domain function by converting it into frequency domain function. The Newton's Law of cooling are generally solved by adopting Laplace transform method. The paper inquires the Newton's Law of cooling by Mohand transform technique. The purpose of paper is to prove the applicability of Mohand transform to analyze Newton's Law of cooling.

Indexed Terms- Mohand Transform, Inverse Mohand Transform, Newton's Law of Cooling, Temperature of Environment, Temperature of Body.

I. INTRODUCTION

Mohand Transform has been applied in solving boundary value problems in most of the science and engineering disciplines [1]. It also comes out to be very effective tool to solve linear and non linear ordinary and partial differential equations and is used extensively in physics. The Mohand transform reduces a linear differential equation to an algebraic equation, which can by rules of algebra.. The Mohand Transform has been effectively used to in different areas of science, engineering and technology [1], [2], [7], [9], [24], [25]. Newton's Law of Cooling is called an ordinary differential equation that expects the cooling of a warm body sited in a cold environment [3], [4], [5], [6]. According to Newton's law of cooling , the rate of loss of heat from a body is directly proportional to the difference in the temperature of the body and its surroundings. [17], [18], [19], [20], [22], [23], [24].In this paper, we present Mohand transform technique to Newton's Law of cooling.

$T' = -k(T - T_e)$ ....(I)
with initial condition as $T(t_0) = T_0$ ....(II)
Where, T is the temperature of the object
T_e is the constant temperature of the environment,
k is the constant of proportionality,
T_0 is the initial temperature of the object at time t_0

The negative sign of RHS in (1), indicate temperature of the body is decreasing with time and so the derivative $\frac{dT}{dt}$ must be negative.

Basic Definition: Mohand Transform

If the function $ℏ(y)$, $y \geq 0$ is having an exponential order and is a piecewise continuous function on any interval, then the Mohand transform of $ℏ(y)$ is given by

$$M\{ℏ(y)\} = \bar{ℏ}(p) = p^2 \int_0^\infty e^{-py}\, ℏ(y)dy.$$

The Mohand Transform [1, 2] of some of the functions are given by

- $M\{y^n\} = n!/p^{n-1}, where\ n = 0,1,2, ..$
- $M\{e^{ay}\} = \frac{p^2}{p-a},$
- $M\{sinay\} = \frac{ap^2}{p^2+a^2},$
- $M\{cosay\} = \frac{p^3}{p^2+a^2},$
- $M\{sinhay\} = \frac{ap^2}{p^2-a^2},$
- $M\{coshay\} = \frac{p^3}{p^2+a^2},$
- $M\{\delta(t)\} = p^2$

Inverse Mohand Transform
The Inverse Mohand Transform [1, 2] of some of the functions are given by

- $M^{-1}\{\frac{1}{p^n}\} = \frac{y^{n+1}}{(n+1)!}$, $n = 0, 1, 2, 3, 4 ...$
- $M^{-1}\{\frac{p^2}{p-a}\} = e^{ay}$
- $M^{-1}\{\frac{p^2}{p^2+a^2}\} = \frac{1}{a}\sin ay$

- $M^{-1}\{\frac{p^3}{p^2+a^2}\} = \cos ay$
- $M^{-1}\{\frac{p^2}{p^2-a^2}\} = \frac{1}{a}\sin hay$
- $M^{-1}\{\frac{p^3}{p^2+a^2}\} = \cos hay$

Mohand Transform of Derivatives

The Mohand Transform [1, 2] of some of the Derivatives of h(y) are given by

- $M\{h'(y)\} = pM\{h(y)\} - p^2\, h(0)$
  $or\ M\{\hbar'(y)\} = p\bar{\hbar}(p) - p^2\, \hbar(0),$
- $M\{\hbar''(y)\} = p^2\bar{\hbar}(p) - p^3\hbar(0) - p^2\hbar'(0), \quad and\ so\ on$

II. METHODOLOGY

From (I),

$$T' = -k(T - T_e)$$

Taking Mohand Transform on both sides,

$$M\{T'\} = -M\{k(T - T_e)\}$$
$$M\{T'\} = -kM\{T(t)\} + kM\{T_e\}$$
$$pM\{T(t)\} - p^2T(0) = -kM\{T(t)\} + kT_eM\{1\}$$

From (II), As T $(t_0) = T_0$

Now, $pM\{T(t)\} - p^2T_0 = -kM\{T(t)\} + kT_ep$

$$(p+k)M\{T(t)\} = p^2T_0 + kT_ep$$
$$M\{T(t)\} = \frac{p^2T_0}{(p+k)} + kT_e\frac{p}{(p+k)}$$

Taking inverse Mohand, $\{T(t)\} = T_0e^{-kt} + kT_e - kT_ee^{-kt}$

$$T(t) = (T_0 - kT_e)e^{-kt} + kT_e$$
$$T(t) = Ce^{-kt} + kT_e \ldots\ldots\ldots\ldots\ldots \text{(III)}$$
$$where, C = (T_0 - kT_e)$$

While this function decreases exponentially, it approaches T_e as t→ ∞ instead of zero.

Application:

An apple pie with an initial temperature of 170^0 C is removed from the oven and left to cool with an air temperature 20^0C. Given that the temperature of the pie initially decreases at a rate of 3.0^0 C/min. How long will it take for the pie to cool to a temperature of 30^0 C? . [22]

Suppose the pie is in compliance with newton's cooling law; we have the following information

$$T' = -k(T-20), T(0) = 170\ , T'(0) = -3.0$$

Where, T is the temperature of the pie in degree Celsius, T' is the time in minutes and k is an unknown constant.

Now, we will find the value of k by putting the given information we know about t = 0 directly into the differential equation:

$$-3 = -k(170 - 20)$$
$$k = 0.02$$

So, the differential equation can be written as

$$T' = -\frac{1}{50}(T - 20)$$

Taking Mohand on both sides,

$$M\{T'\} = -\frac{1}{50}M\{(T(t) - 20)\}$$
$$M\{T'\} = -\frac{1}{50}M\{T(t)\} + \frac{2}{5}M\{1\}$$
$$pM\{T(t)\} - p^2T(0) = -\frac{1}{50}M\{T(t)\} + \frac{2}{5}p$$
$$pM\{T(t)\} - p^2T(0) = -\frac{1}{50}M\{T(t)\} + \frac{2}{5}p$$
$$\left(p + \frac{1}{50}\right)M\{T(t)\} = p^2T(0) + \frac{2}{5}p$$
$$\left(p + \frac{1}{50}\right)M\{T(t)\} = 170p^2 + \frac{2}{5}p$$
$$M\{T(t)\} = \frac{170p^2}{\left(p+\frac{1}{50}\right)} + \frac{2p}{5\left(p+\frac{1}{50}\right)}$$
$$M\{T(t)\} = \frac{170p^2}{\left(p+\frac{1}{50}\right)} - \frac{20p^2}{\left(p+\frac{1}{50}\right)} + 20p$$
$$M\{T(t)\} = \frac{150p^2}{\left(p+\frac{1}{50}\right)} + 20p$$

Taking inverse Mohand on both sides, we get,

$$T(t) = 150e^{-\frac{1}{50}t} + 20 \ \ldots\ldots\ldots\ldots\ldots\ldots (IV)$$

Putting T=30 in (IV),

$$30 = 150e^{-\frac{1}{50}t} + 20$$
$$e^{-\frac{1}{50}t} = \frac{1}{15}$$
$$e^{\frac{1}{50}t} = 15$$
$$\frac{1}{50}t = \ln 15$$
$$t = 50\ln 15$$
$$t = 50 * 2.7080502011$$
$$t = 135.4\ minute$$

Hence, this will require 135.4 minutes for the pie to cool to a temperature of 30^0 C.

CONCLUSION

In this paper, we have well developed the Mohand Transform to solve problems related to Newton's Law of Cooling. The applications presented demonstrate effectiveness of Mohand Transform in the problems of Newton's Law of Cooling. The proposed scheme is widely in various field of Physics, Electrical engineering, Control engineering, Economics, Mathematics, Signal processing and Electronics engineering.

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