Probabilistic Assessment of the Design of Steel Plate Girders

YUSUF A.
Department of Civil Engineering, Kampala International University, Uganda

Abstract- Steel plate girders under uniformly distributed load (UDL) and point loads designs were compared with test results obtained from literature. The results showed a reasonable prediction of the capacity of the plate girder, although there are cases of under and over estimation of the capacity. Probabilistic technique was therefore employed to examine the consistencies of the design provisions. The results obtained showed that with all the basic variables assuming normal distribution, the implicit reliability levels, β, decreases from 4.96 to 2.08 for span range of 11 to 14m in girders subjected to UDL in bending. For shear condition β values ranged from 4.89 to 2.00 for span range of 14 to 23m. With girder subjected to point load, in bending, β ranged from 4.90 to 2.15 for span varying from 12 to 19m while the shear condition showed a constant reliability level of about 4.69 for all the variables assuming normal distribution. The sensitivity analysis conducted to examine influence of the values of the basic design variables on the implied reliability levels showed that the most significant variables are the yield strength of the steel and the depth of the section of the plate girder. The implication of this is that these two design parameters need serious consideration.

Indexed Terms- Plate Girder, Probabilistic Techniques, Reliability Levels, Sensitivity Analysis.

I. INTRODUCTION

Plate girders are used to carry larger loads over longer spans than are possible with universal or compound beams. They are used in buildings for long span floor girders and bridges [1]. The designer uses his knowledge of structural mechanics, the codes of practice and practical experience to produce a safe solution to a given problem. From the analysis of structural frames, the shears, axial loads, moments on the structural elements are determined. The sizes of elements are so chosen so that the allowable stresses, deflections etc. given in the code are not exceeded and relevant requirements such as minimum thickness of material or maximum width/thickness are satisfied.

By and large, codes are compiled to meet up the following requirements [2]; the material selected must be suitable and adequate, proper load and service condition must be considered. Furthermore, computations and design must be made so that the structure and its detail possess the required strength and rigidity and the workmanship must be good. It follows then that code's specifications for structural design represent a compromise between theoretical considerations and practical requirements. Therefore, the specifications are not entirely rational and, for some structures or loading conditions, they may lead to more conservative results than for others. Also, the values of loads and allowable stresses are based on past experience and test data which have to be revised periodically to agree with latest findings.

BS 449 the code on which this investigation is focused is based on allowable stress design philosophy, wherein, the influence of the inherent uncertainties associated with engineering problems are supposedly taken care of by the introduction of factors of safety as an allowance to neutralize their undesirable effects. As the derivation of these safety factors are not probabilistic in nature, it was conjectured that the consistency of its provisions are questionable and therefore investigated. The risk or probability of failure is introduced as a measure in term of which the safety and reliability of various parts of a structural system can be defined, compared and uniform reliability or a safety of a complete system assured [3].Consequently, BS 449 possesses the following drawbacks [4]:

a. In the design procedures, internal stress resultants based on elastic theory is compared with specified allowable stress in the code without regards to uncertainties and consequence of failure.

b. The designer in his computation always compares his maximum stress; say for example, in a simply supported beam carrying a uniformly distributed load, occurring at the mid-span, with the allowable stress value in the code. This is a highly localized point. Most structures do not fail because of high local stress. The stresses may be redistributed by plastic flow to less highly stressed parts.
c. The allowable stress method does not give accurate picture of collapse conditions and cannot give reliable estimate of them. It only gives safe estimate and has a virtue of being simple, straight forward and easy to use.
d. Another disadvantage of this method is that it is not logically complete. It does not provide a framework for logical reasoning through which all the limiting conditions on a structure can be examined.
e. It is obvious that effects other than stresses have to be checked in a design, e.g. deflections, crack control etc. Clauses relating to these effects are ambiguous without unifying philosophy.
f. There is too much emphasis on elastic stress and too little emphasis on limiting conditions controlling the structure in use.

A single span steel girder with a symmetrical I-section, supporting a uniformly distributed load and a point load were respectively investigated. The bending and shear conditions with strict compliance to provision of BS 449 were assessed such that the failure probabilities (as a measure of safety indices) implicit in the design code for varying design parameters were computed. Sensitivity of the design equations to some relevant design parameter to ascertain their influence and relative importance was also investigated. Digital computations became necessary to arrive at the objective of the study. Consequently, a well coded algorithm based on probability technique (i. e FORM 5) was adopted.

II. THEORETICAL FORMULATION OF RELIABILITY METHOD

Problems of reliability of engineering systems can be essentially reduced to problem of supply to meet up a demand. In case of a structure, we are concerned with ensuring that the strength of the structure (supply) is enough to withstand the life time maximum applied load (demand). In reality of course, the determinations of the available supply as well as the maximum demand are not simple problems. Estimation and predictions are invariably necessary for these purposes, implying that uncertainties are unavoidable for the simple reason that engineering information is invariably incomplete. In the light of such uncertainties the available supply and actual demand cannot be determined precisely. They may be described as belonging to the respective ranges (or population) of possible supply and demand. In order to explicitly represent or reflect the significance of uncertainty, the available supply and required demand may be modeled as random variables [5].

A structural system with resistance capability R (i.e., the supply) acted upon by the load effect S (i.e., the demand), the objective of reliability analysis is to ensure the event that (R > S) throughout the useful life system. Conversely, the probability of the complimentary event (R < S) is the corresponding measure of unreliability or failure. Assume also that the necessary probability distributions of R and S are available (though rarely possible), that is, the cumulative probability function (CDF) or probability density (PDF), $F_R(r)$ or $f_R(r)$ and $F_S(s)$ or $f_S(s)$ respectively.

The required probability is given by [6],

$$P_f = \int_0^\infty F_R(s) f_S(s) ds \qquad 2.1$$

This is diagrammatically represented in fig (1)

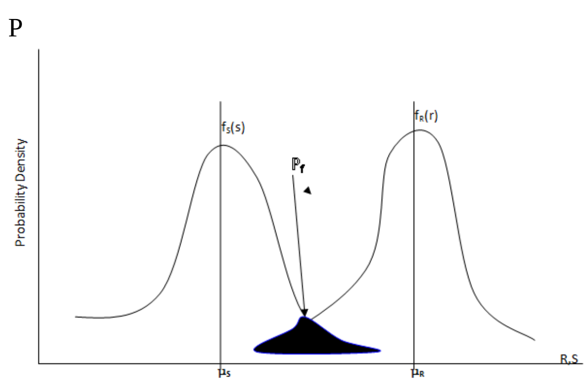

Fig. 1: Basic Resistance/Load Effect problem

III. ALLOWABLE STRESS DESIGN CONSIDERATION OF GIRDERS

For the determination of the resistance to pure bending (flexural and no axial) in most flexural members a single calculation will suffice in the form:

$$Z = \frac{M_{max}}{\sigma_{all}}$$

Where

Z = the required section modulus (mm^3)

M_{max}= the maximum bending moment (Nmm)

σ_{all} = the allowable bending stress (N/mm^2)

The principal assumptions for justification of this procedure include the following condition [7]:

(a) The material is homogenous.
(b) Section plane before bending remains plane.
(c) The cross-section has an axis of symmetry.
(d) Stress is proportional to strain.
(e) The moment is applied in the plane of symmetry.
(f) The member is straight in the plane of symmetry.
(g) The member is stable laterally to the plane of bending.

Usually the maximum moment, M_{max} resisted by the member is obtained by the simple method of structural mechanics and section modulus Z for the trial section is computed. The ratio M_{max}/Z is such that

$M_{max}/Z \leq \sigma_{all}$ (prescribed in the code)

The elastic formula for analysis of shear stress on sectionis too complex for routine use with variety of shapes available or possible for steel members. For a member that possesses an axis of symmetry in the plane of loading, two simplifying assumptions that result in a negligible loss of accuracy are permitted [7].

a. The contribution of flanges to shear capacity is neglected.
b. The use of an average value of shear stress on the gross area of the web.

This assumption has been justified by an approximate reduction of the allowable shear stress. With these assumption, derivation and application of accepted shear capacity formula becomes simple. Neglecting the flanges, all symmetrical rolled shapes, box shapes and all built-up sections reduce to an equivalent rectangular section with dimensions $\Sigma t_w D$ and shear stress becomes

$$\mathbf{f_v} = \frac{V}{\Sigma t_w D}$$

where,

f_v = the shear stress at any point in the depth D

V = the vertical load at that point

t_w = web thickness

D = Overall depth

BS 449 prescribes the average shear stress on the gross section for plate girder as being carried by the depth of the web multiplied by the thickness. Thus the shear requirement is given by

$$\frac{V_{max}}{(D-2t_f)t_w} \geq \tau_{all}$$

Where,

V_{max} = maximum shear force

t_f = thickness of flange

τ_{all} = allowable shear stress (prescribed in the code)

D = overall depth

For allowable stress (elastic) design, ‘safety factor’ is usually expressed and accepted as the ratio of the specified minimum yield stress to the allowable stress. It ignores the overload capacity of the structure between the first yielding to final overall instability. It also disregards the entire approach of probabilities of overload, the ratio of live load to dead load, the actual strength versus minimum specified yield strength and variation of the actual section strength, etc. Although adequate data relative to strength of a given material may not be available, it is a well-known fact that the strength varies from specimen to specimen and the nature of that variation has been investigated by several investigators. Previous studies [e.g., 8, 9] on plate girder showed noticeable variation between the ultimate stresses obtained by theoretical analysis based on elastic method as in allowable stress design concept with values obtained experimentally. Nevertheless, the conventional approach has shortcomings that are well recognised; principally the approach is limited to the extent that all variables and factors must be treated as though they are deterministic. In spite of the steps taken to include statistical aspects of engineering information in the formulation of safety factors, the procedure is essentially remain deterministic in the sense that any risk associated with randomness is not determined.

Therefore in the presence of statistical variability, such as evidenced in loads and material strength, proper evaluation of structural design requires explicit consideration of probability. Clearly, the conventional design approach is not adequate from reliability stand point. Hence another design methodology that considers the probabilistic nature of design is needed so that component reliability becomes intrinsic part of design. However, the direction this study is to show the safety levels implicit in the BS 449 requirements for design when the design variables are explicitly considered random.

IV. COMPUTATION OF SAFETY INDICES

The probability distribution of the basic variables (design parameters) can be obtained either by direct inference from observed data or by subjective assessment or combination of these techniques. In practice, direct inference is rarely always possible since there is seldom sufficient data available to identify unambiguously only one distribution as appropriate. This further implies that past observations and experience for similar structures can be validly used for the structure under assessment. A physical reasoning may be used to suggest an appropriate distribution. Thus where the basic variables consist of many variables (which are not explicitly considered), the central limit theorem may be invoked to suppose that normal distribution is appropriate [10]. Central limit theorem stated loosely says that the sum of large number of individual random components none of which is dominant, tends to a normal distribution as the number of component (regardless of their initial distribution) increase without limit [11].Therefore, if a physical process is the result of totality of a large number of individual effects, then according to the central limit theorem, the process would tend to be normally distributed.

Consequently, a preferable approach is to make use of physical reasoning about the nature of each particular random variable to guide in the choice of its distribution. Imposing central limit theorem, the basic variables were assumed normally distributed. In addition yield strength was modeled by other appropriate distributions. Logarithmic normal (or lognormal) distribution was used because it has a theoretical advantage of precluding negative values. Weibul distribution is also used because it characterises strength of component whose strength is governed by the size of its largest defect. Gumbel distribution was also tried for its similarity with weibul as an extreme-value distribution. This subjective judgement in the choice of the distribution should not be seen as a limitation, since the aim is not to produce a perfect image of the reality but to develop a mathematical model of the phenomenon which embodies its salient features and which can be used to make optimal design decisions using the available data [10].

As explained previously in second moment reliability method, the parameters of interest are first and second moments (mean and variance) for the computation of safety indices [12]. It is also important to note that it is the coefficient of variation (ratio of standard deviation to mean values) that are published in literature [e. g 13, 14]. The values of these relevant parameters for the basic variables used in this study are shown in Table 1.

Table 1 Statistical values of design parameters

S/No	Basic Variables	Mean	Coeff. of Variation
1	Yield strength in bending	$1.05f_y$	0.10
2	Yield strength in shear	$0.64f_y$	0.10
3	Live load	Q, q	0.2 – 0.35
4	Dead load	G, g	0.05 -0.10
5	Width of flange	B	0.002
6	Overall depth of section	D	0.002
7	Thickness of flange	t_f	0.05
8	Thickness of web	t_w	0.05

V. DISCUSSION OF RESULT

Appropriate computer programs, FORM5 [15] were employed in this study to compute the safety levels, β, implicit in BS 449 provision for design of girders subjected to UDL and point load for bending and shear considerations. The β values in the selected plots were studied with the following deductions.

With respect to varying span for UDL in bending condition (Fig. 2), for 12m the following reliability levels in descending (with respect to choice of distribution for f_y) were obtained: weibull, β = 5.22; normal, β = 4.96; lognormal, β = 3.35; and gumbel, β = 3.04. Shear condition (Fig. 3) also gave similar order with respect to choice of distributions except that higher reliability levels were obtained (i.e weibull, β > 6; normal, β = 5.85; lognormal, = 4.80; and gumbel, β = 4.20). With basic variables prescribed normal, computed β ranged from 4.96 to 2.08 for a corresponding span of 11-14m I bending condition and β ranged from 5.85 to 2.00 for span range of 12m to 23m in shear condition. This implies that the rate of change of β is higher for bending condition and also under UDL consideration shear requirement gave higher reliability levels. In girders subjected to point load (Fig. 4), 12m span gave the following reliability levels in descending order with respect to the prescribed distributions: weibull, β = 5.01; normal, β = 4.90; lognormal, β = 4.90; and gumbel, β = 3.63. The trend is similar to what was obtained for UDL consideration but variation in span does not influence β value for shear condition in girders subjected to point load.

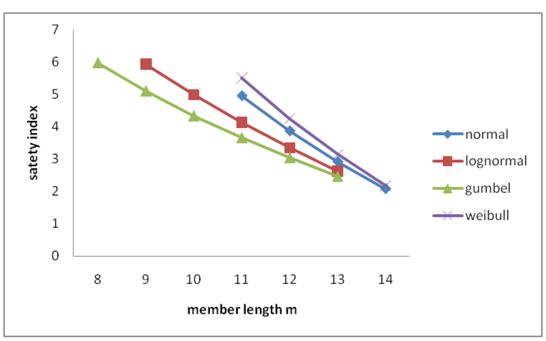

Fig. 2: Variation of β with the span, L (UDL in bending)

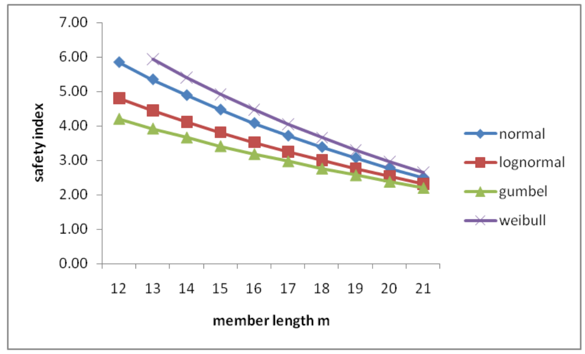

Fig. 3: Variation of β with the span, L (UDL in shear)

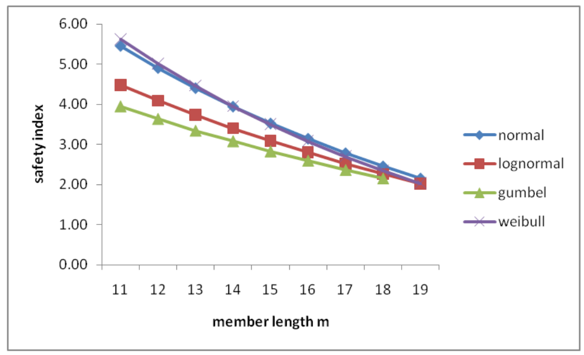

Fig. 4: Variation of β with span, L (Point load in bending)

At a yield stress, f_y = 257 N/mm^2 under a UDL in bending condition (Fig. 5), obtained β valuesfor the prescribed distributions are: β = 3.15 for weibull, β = 2.92 for normal, β = 2.63 for lognormal and β = 2.46 for gumbel. Similarly for shear condition (Fig, 5), β = 5.93 for weibull, β = 5.35 for normal, β = 4.44 for lognormal and β = 5.92 for gumbel. The lower reliability in bending condition shows that it controls the design. Under varying live load values the computed reliability levels are dependent on the types of distributions assumed by the material yield strength. This is clearly shown in Fig. 6. At various levels of q different order for the prescribed distributions were obtained.

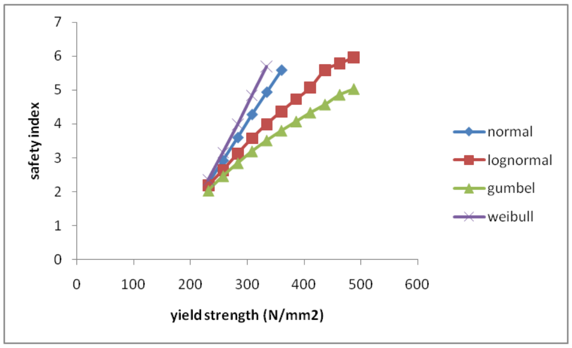

Fig. 5: Variation of β with yield strength, f_y (UDL in bending)

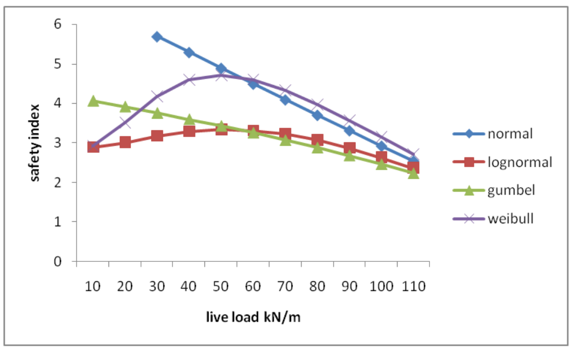

Fig. 6: Variation of β with live load, q (UDL in bending}

The variations of load ratios are shown in Fig. 7 and Fig. 8; considerations are given to bending and shear under UDL respectively. The values of β are higher for shear than for bending. Similarly Fig. 9 and Fig.10depict the variation of load ratios for bending and shear requirements under point load. In all cases the probability of failure levels are higher for bending criterion than shear. Varying aspect ratio B_f/t_f with all distributions prescribed normal showed that, under UDL in bending condition (Fig. 11), reliability levels increased from 2.27 to 5.87 for a ratio of 8 to 17 while for point load in bending (Fig. 12) give β range of 2.34 to 5.78 for a ratio 12 to 24. Higher reliability was obtained for UDL than point load.

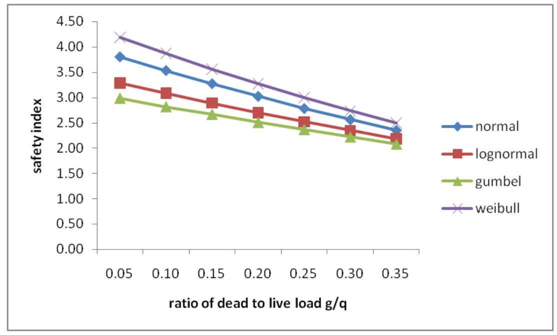

Fig. 7: Variation of β with ratio of dead load to live load, g/q (UDL bending)

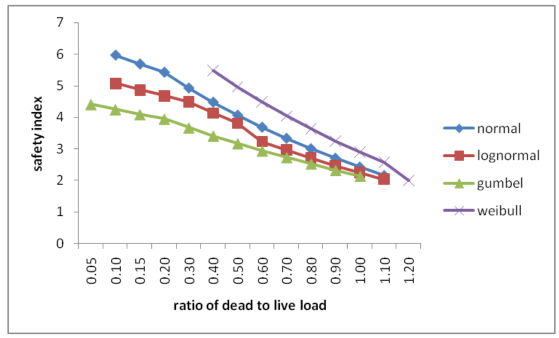

Fig. 8: Variation of β with ratio of dead load to live load, g/q (UDL shear)

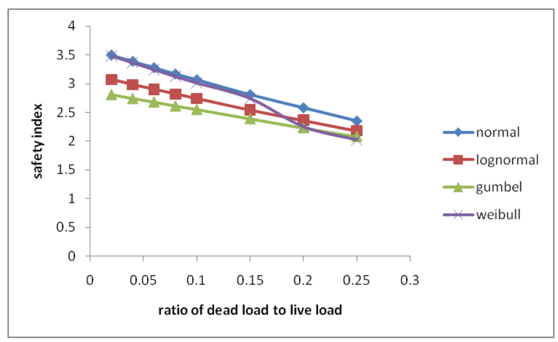

Fig. 9: Variation of β with ratio of dead load to live load, G/Q (point load in bending)

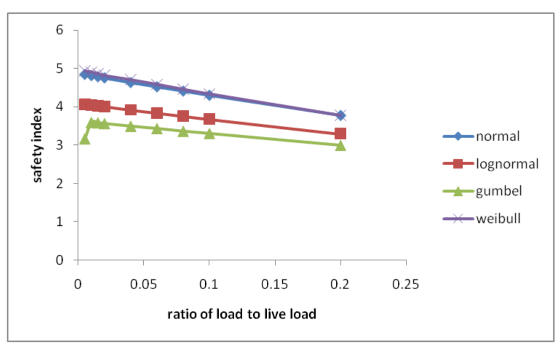

Fig. 10: Variation of β with ratio of dead load to live load, G/Q (point load in shear}

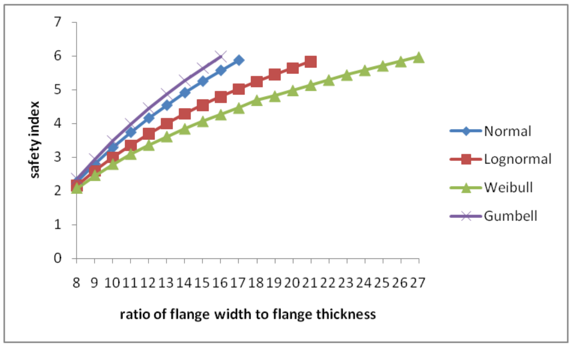

Fig. 11: Variation of β with the ratio of flange width to thickness, B_f/t_f (UDL in bending)

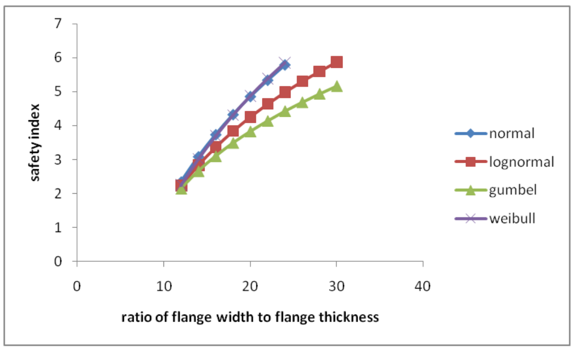

Fig. 12: Variation of β with the ratio of flange width to thickness, B_f/t_f (Point load in bending)

To evaluate the significance of design parameters, the specified yield strength (f_y), flange width (B_f), overall depth (D), flange thickness (t_f) and thickness of web (t_w) were considered. Graphical plots for the various sensitivity studies are shown in Figs. 12 to 15. For each of these parameters considered, a central reliability level is taken as β_0 and the corresponding value of the parameter is noted (reference point). For an increase or decrease above or below the reference point of the design parameter the reliability ratio β/β_0 is computed for the varying β values. Comparisons made in the plots for the different design parameters are as follows.

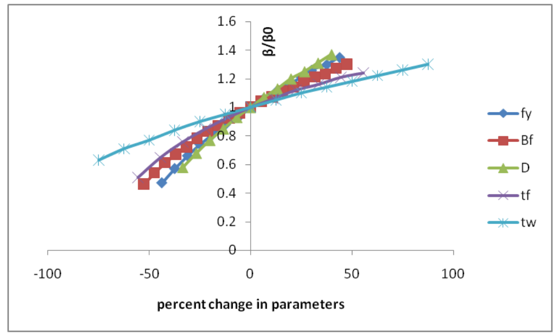

Fig. 13: Sensitivity plots for bending consideration under uniformly distributed load

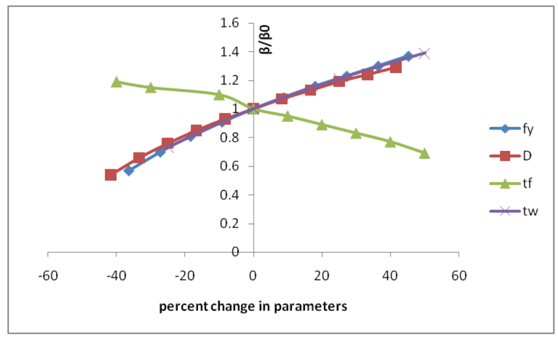

Fig. 14: Sensitivity plots for shear consideration under uniformly distributed load

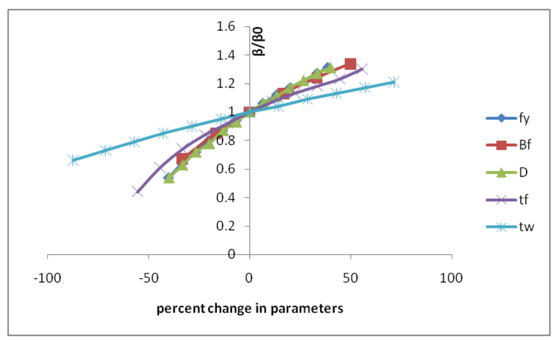

Fig. 15: Sensitivity plots for bending consideration under point load.

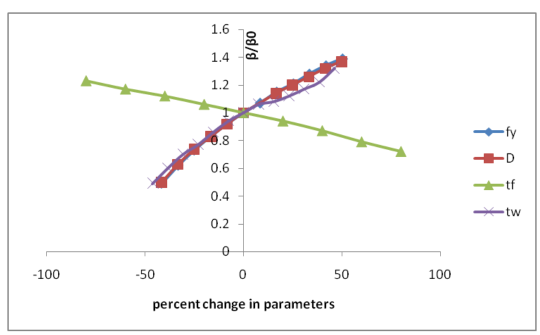

Fig. 16: Sensitivity plots for shear consideration under point load.

Considering UDL in bending (Fig. 12) a 10% increase in t_w showed about 4% change in reliability, an equal increase showed about 6% increase for t_f and 7.8% increase was observed in B_f, f_y and D have about 8.2% increase in reliability level. A 10% decrease give about 3.8% decrease for t_w, 5% decrease for t_f, 8.8% decrease for B_f and 10% decrease for f_y and D. These indicate that f_y and D influence reliability most and the effect is more with the decrease in value of these parameters. In shear condition also for UDL (Fig. 13) a 10% increase in the value of the parameters showed about 7.9% increase in the reliability level for D, 8% increase for t_w and 9% increase for f_y, t_f decreased reliability by 5%. A 10% decrease in the value of the parameters showed 8% decrease for D, 10% decrease for f_y and t_w and a 5% increase for t_f. This implies that t_f influence reliability inversely in shear condition.

Sensitivity study of girders subjected to point load in bending is shown in Fig. 14. A 10% increase in the parameters showed 4% increase in reliability for t_w, 6% increase in reliability of t_f, 8% increase for B_f and 8.2% increase for f_y and D. A corresponding 10% decrease showed 3.8% decrease for t_w, 6.8% decrease for t_f, 8.2%decrease for B_f and 10% decrease for f_y and D. It therefore indicates that decrease in parameter is more critical to reliability level and f_y and D have the highest effect. With girders subjected to point in shear (Fig. 15), a 10% increase in the parameters gave 8% increase in reliability for t_w, and about 8.2% increase for D and f_y. At 20% increase in reliability increased by 15% for t_w, 16% for D and !7% for f_y. t_f like in the case for UDL influence reliability level inversely by the same margin. It shows here also that a decrease in the value of parameters is more critical.

CONCLUSION

The reliability levels associated with the bending and shear limitations were studied. With the basic variables assuming gaussian distribution, varying reliability levels (β) were computed for different values of the basic variables. It was found that β values decrease with increasing values of some of these parameters and decreases with others. Consequently it is observed that the minimum information required for assessing the implied safety levels in design are the expected values (e.g mean) of the design variable plus a measure of the uncertainty (e.g coefficient of variation).This information may be obtained from observed data coupled generally with professional judgment.

In probability method of design, unforeseen changes could be simulated and their influences on safety and performance of a structure predicted. For instance, the usage of a structure is altered, with a corresponding drastic change in loading condition. Gross errors occur in geometric dimensions of sections as induced by steel rolling procedures, fabrication and erection processes on site. This method gives the associated measure of risk and will also will also allow for introduction of precautionary signs or measure as to the limitations in the usage of a structure.

The appreciable influence of specified yield strength f_y of steel and the overall depth of the I-section girder considered calls for high quality control measures. Laboratory tests to assess the steel grade and good quality workmanship should be assured.

REFERENCES

[1] MacGinley, T. J. (1973); Structural Steelwork Calculation and Detailing, Butterworth and coy. Ltd, UK.

[2] Brestler, B., Lin, T. Y. and Scalzi J. B. (1968): Design of Steel Stuctures, 2nd Edition, John Wiley and Sons, Inc. New York.

[3] Freudenthal, A. M., Garrelts, J. M. and Shinozuka, M. (1966): The Analysis of Structural Safety, Journal of Structural Division, ASCE, Vol 92, No ST1 pp 267-235.

[4] Blockley, D. I. (1980): The nature of Structural Design and Safety, Ellis Horwood, Ltd, England.

[5] Ang, A. H-S. and Tang, W. H. (1984): Probability Concept in Engineering and Design, Vol. II; Decision, Risk and Reliability, John Willey and Sons, New York.

[6] Ang, A. H-S. (1973): Structural Risk Analysis and Reliability Based Design, Journal of the Structural Division, ASCE, Vol. 99, No ST9, pp 1891-1910.

[7] Rice, P. F. and Hoffman, E. S. (1976): Structural Design Guide to AISC Specification for Buildings, Van Norstrand Reinhold Coy. New York.

[8] Basler, K. and Thulinann, B. (1961): Strength of Plate Girders in Bending, Journal of the Structural Divisions, ASCE, Vol 87. No. ST6 pp 153-181.

[9] Basler, K. (1961): Strength of Plate Girder in Shear: Journal of the Structural Division, ASCE, Vol 87, No ST7 pp 151-180.

[10] Thoft-christensen, p. and Baker, M. J. (1982): Structural Reliability Theory and its Applications, Springer Verlag, Berlin.

[11] Ang, H-S. and Tang, W. H. (1975): Probability Concept in Engineering Planning and Design. Vol I: Basic Concept, John Willey and Sons, New York.

[12] Ayyub, B. M. and Halder, A. (1984): PracticalStructural Reliability Techniques, Journal of Structural Division, ASCE, Vol 110, No. ST8, pp 1707-1724.

[13] Galanbos, T. V. and Ravindra, M. K. (1976): Properties of Steel for Use in LRFD, Journal of Structural Division, ASCE, Vol. 104, No ST9. Pp 14598-14608.

[14] Melchers, R. E. (1987): Structural Reliability Analysis and Prediction, Ellis Horwood Ltd. England.

[15] FORM (1988): User Manual: RCP-GmbH, Munich, West Germany.