Assessing Mathematical Understanding: Unveiling the True Essence of Feedback as a Process

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Abstract- ***This study unveiled the true essence of feedback as a process in assessing and enhancing learners' mathematical understanding within a socio-cultural learning environment. The study adopted the Socio-Cultural theory of learning as a theoretical framework to investigate the role of feedback in the learning process. The data for this qualitative study was derived from interviews conducted with two experienced mathematics teachers. Polkinghorne's narrative analysis was employed to analyse the interview data and identify key themes and patterns. The findings highlight the importance of feedback as a process in guiding learners towards self-assessment and self-correction, leading to deeper mathematical understanding. The study emphasizes the need for feedback to be meaningful, specific, and aligned with the goals and learning objectives of the task. It also highlights the importance of considering individual learner needs and learning styles in providing constructive feedback.***

Indexed Terms- ***Mathematical understanding; Feedback; Process; Product***

I. INTRODUCTION

In the study entitled "*Hlangwani et al. (2023). Exploring learners' conceptual obstacles of a quadratic function: A case of vertex concept. In Proceedings of the 28th National Congress of the Association for Mathematics Education of South Africa: AMESA (p.187)*", I analysed the comments received during a conference paper presentation. I observed that many researchers believed that a learner's ability to score well on mathematical problems indicated their understanding of the underlying concept. Conversely, those who struggled to score well were seen as lacking comprehension. However, upon further examination of the comments raised, I started to question the effectiveness of the feedback provided to learners. As such, I realized that the feedback often given to learners in mathematics tasks was often not constructive and merely treated as a final product, rather than an ongoing process. Motivated by this challenge to traditional feedback practices, I embarked on a quest to uncover the true essence of feedback as a process versus feedback as a product. In the realm of education, feedback as a product has long been the dominant approach to assessing learners' performance in various subjects such as mathematics (Ajjawi & Bound, 2017; Hattie & Gan, 2011; Van der Kleij et al., 2019). Thus, such approach typically involves assigning grades or scores based on learners' abilities to correctly answer questions or complete tasks. While feedback as a product has its merits in providing a quantifiable measure of achievement (Harks et al., 2014), there has been a noticeable shift in recent years towards acknowledging the significance of feedback as a process in fostering a deeper understanding of mathematics (Hattie & Timperley, 2007; Shute, 2008; Van der Kleij, 2019). Feedback as a process, in this context, refers to the information and guidance provided to learners regarding their mathematical tasks (Harks et al., 2014)). It goes beyond assigning a grade and focuses on providing constructive comments and suggestions for improvement (Harris et al., 2014; Hattie & Gan, 2011; Winstone et al., 2017). This shift is rooted in the recognition that mathematics is not just about getting the right answers but also about developing mathematical understanding. The concept of mathematical understanding is not a fixed notion but rather a fluid and multifaceted one (Hiebert & Lefevre, 1986; Hlangwani et al., 2023; Hlangwani, 2023; Kilpatrick et al., 2002; Rittle- Johnson, 2017). Scholars and researchers have explored various dimensions of mathematical understanding, examining factors such as learners ability to apply mathematical principles in different contexts (Kilpatrick et al., 2002), their capacity to reason and explain their mathematical

thinking (Groves, 2012), and their grasp of the underlying concepts and structures (Hlangwani et al., 2023). By prioritizing feedback as a process yields learners development of a deeper comprehension of mathematics concepts. Therefore, this notion is rooted in the fact that more studies have been on defining recommendations for how teachers should deliver good feedback (Hattie & Timperley, 2007; Shute, 2008). However, studies in primary (Dann, 2015), secondary (Havnes et al., 2012; Jónsson, Smith, & Geirsdóttir, 2018), and higher education (Carless, 2006) indicated the feedback given to learners is not constructive to learners. Therefore, the present study aims to shed light on scholars not to treat mathematics feedback as a one-time evaluation but as an ongoing and dynamic interaction between teachers and learners. Moreover, it sheds light on the importance of reevaluating our approach to feedback in the educational context. It highlights the need for feedback to be seen as an ongoing process that fosters growth and understanding, rather than a mere end result. By embracing this perspective, teachers and learners can work together to create a more constructive and impactful feedback culture.

RESEARCH PURPOSE AND QUESTIONS

The purpose of this study is to explore the role of feedback as a process in assessing and enhancing learners' mathematical understanding within a socio-cultural learning environment. By delving into the true essence of feedback, this research aims to provide insights into effective feedback practices that can support learners in developing deeper mathematical understanding. To achieve this purpose, the study answered the following research question:

- How does feedback as a process contribute to the assessment of students' mathematical understanding within a socio-cultural learning environment?
- What are the key characteristics and components of effective feedback that enhance students' mathematical understanding and problem-solving abilities?

THEORETICAL FRAMEWORK

The present study adopted the Socio-Cultural Theory of Learning, proposed by Vygotsky (1978), as a theoretical lens to investigate the assessment of mathematical understanding and the role of feedback in the learning process. The Socio-Cultural Theory posits that learning is a social and cultural activity that occurs through interaction with more knowledgeable individuals and within a specific cultural context (Goos, 2014). The theory suggests that feedback is not merely a product or an end result, but rather a dynamic process that occurs within social interactions (Goos, 2004). It emphasizes the importance of scaffolding and guided participation, where more knowledgeable individuals provide support and guidance to learners to enhance their understanding. Through the lens of the Socio-Cultural Theory, this study seeks to examine how feedback as a process influences learners' mathematical understanding. By analysing the teachers' perspectives on interrogating feedback, the study aimed to uncover the underlying mechanisms and strategies used by teachers to facilitate meaningful learning experiences. In summary, the adoption of the Socio-Cultural Theory of Learning in this study provided a lens for exploring the assessment of mathematical understanding and the role of feedback as a process. By investigating the teachers' perspectives on interrogating feedback, the study aimed to contribute to the existing body of knowledge on effective feedback practices in mathematics education.

LITERATURE REVIEW

Assessment feedback plays a crucial role in enhancing the mathematical understanding of concepts (Harks et al., 2014). It goes beyond being a mere performance-related grading system employed by teachers towards learners. According to Bound and Molloy (2013), feedback should not be viewed as a one-way process where teachers provide grades to learners based on their assessment performance. Instead, it should be seen as a dynamic and interactive process that fosters growth and improvement in mathematical skills and comprehension (Mason & Bruning, 2001). In the traditional approach to assessing mathematics tasks in classrooms, the focus has often been on treating feedback as a product rather than as an ongoing process (Cizek et al., 1996; Lipnevich & Smith, 2009). This approach tends to overlook the

importance of supporting the learner's agency and ability to improve their work. According to Boud and Molloy (2013), this type of delivery of feedback does little to empower learners and provide them with the necessary process to enhance their mathematical understanding of concepts. On the other hand, there is a growing recognition of the need to shift towards a more process-oriented approach to feedback (Harks et al., 2014; Van der Kleij, 2919). This approach emphasizes the importance of ongoing feedback and guidance to help learners develop their mathematical understanding. Carless and Boud (2018) argue that this mode of delivering feedback encourages teachers to move beyond simply assigning grades and instead focus on providing meaningful feedback that guides learners towards improvement. By adopting a process-oriented approach to assessment in mathematics classrooms, teachers can create an environment that supports the agency of the learner and promotes continuous growth and development in mathematics. This approach recognizes that feedback is not just about assigning grades, but about providing effective feedback that helps learners understand their strengths and areas for improvement (Harks et al., 2014). It empowers learners to take ownership of their learning journey and actively engage in the process of improving their mathematical skills. When learners receive feedback on their assessments, it offers them valuable insights into their strengths and areas for improvement. This feedback serves as a guidepost for learners to reflect on their learning, identify misconceptions, and make necessary adjustments in their understanding of mathematical concepts. By providing specific and constructive feedback, teachers can help learners identify their strengths, build upon them, and address any misunderstandings or gaps in their knowledge. Moreover, feedback encourages active engagement and participation from learners in the learning process (Nicol 2010). When learners receive feedback that is meaningful and actionable, it empowers them to take ownership of their learning and strive for continuous improvement (Price et al. 2010). They become more motivated to explore and experiment with different approaches, seek clarification, and ask questions to deepen their understanding of mathematical concepts. This two-way communication between teachers and learners through feedback creates a supportive and collaborative learning environment. With this understanding, it becomes evident that assessment feedback is not merely about grading learners' performance, but rather an opportunity to facilitate their growth and development in mathematics. By embracing feedback as a valuable tool, teachers can empower learners to become active learners who are eager to engage with mathematical concepts and enhance their understanding.

METHODOLOGY

This study used an ethnographic approach to explore the essence of feedback in assessing mathematical understanding. The primary data was derived from a previously presented paper and audience comments at an AMESA 2023 conference proceedings. Thereafter, two experienced mathematics teachers from different schools in Limpopo province were conveniently selected for semi-structured interviews, providing valuable insights into their perspectives on feedback. The interviews were audio-recorded, transcribed, and analysed using Polkinghorne's (1995) narrative analysis. The ethnographic approach allowed me to focus on the personal narratives and subjective experiences of the two sampled mathematics teachers (Morris et al., 1999), aiming to uncover the true essence of feedback as a process.

RESULTS AND DISCUSSION

In this section, the qualitative data are simultaneously analysed and discussed. The study began by delving into the comments raised during the presentation of the study entitled "*Hlangwani et al. (2023). Exploring learners' conceptual obstacles of a quadratic function: A case of vertex concept. In Proceedings of the 28th National Congress of the Association for Mathematics Education of South Africa: AMESA (p.187).*" and then this is followed by the analysis of the responses of two experienced mathematics teachers who were interviewed regarding their perceptions of feedback as a process and how they interrogate in order to make sense of the feedback provided to learners. Through Polkinghorne's (1995) narrative analysis, I identified key themes and patterns within the teachers' responses, shedding light on the true essence of

feedback in enhancing learners' mathematical understanding.

The paper entitled "*Hlangwani et al. (2023). Exploring learners' conceptual obstacles of a quadratic function: A case of vertex concept*" was presented during the 28th AMESA proceedings 2023. Several comments emerged from teachers whom held the notion that feedback as a product can denote mathematical understanding if a learner for example scores full marks on a mathematics question posits understanding of the concept. Hence, their comments are captured in Table 1 below.

Table 1: Comments raised during the presentation of the article entitled "Hlangwani et al. (2023).

Commenter-1: Learner ALPHA scored everything correctly, demonstrating a strong understanding of the vertex concept. It can be noted that the learners was able to determine the vertex by substituting the point (1;0) into the function equation and finding the value of*b*. I do not agree with your analysis when you state that the has not attained the full schema of vertex concept since the learner scored total.
Comment-2: Learner BETA struggled with the concept of the vertex and made errors in determining the value of p and q. The learner's lack of understanding of the vertex concept is evident as he could not find the intercept.
Comment 3: It is clear that Learner ALPHA has a solid grasp of the vertex concept. The learner was able to solve the problem correctly and obtained full marks.
Comment 4: Learner BETA's response indicates a lack of understanding of the vertex concept. While the learner attempted to simplify the function algebraically. The learner arrived at an incorrect value of p and q.

From the comments raised during the presentation of the article, it was evident that most of the commenters were accustomed to feedback as a product. For example, Commenter-1 and Commenter-2, only focused on the marks obtained by the learner. Such, of denoting understanding to marks scoring was noted by Harks et al. (2014). This meant that these commenters viewed feedback as a quantifiable measure of understanding. However, mathematical understanding is a covert behaviour and cannot be quantified (Hiebert & Lefevre, 1986). In addition, Commenter-2, disregarded the fact that the learner correctly determined the axis of symmetry but concluded that the learner do not posit an understanding of the vertex. similarly, this was a clear indication that Commenter-2, also shared the notion as held by Commenter-1 that feedback should be oriented to a product rather than a process (Ajjawi and Bound, 2017). Lastly, Commenter-3, even failed to note that the learner determined the axis of symmetry correctly. The commenter only focused on the intercept written as the final answer and this is the culture in our mathematics classroom we judge learners based on the final answers. Ajjawi and Bound (2017) concur to this assertion, as they noted that feedback as a product was a dominant approach when assessing learners. Such feedback is not constructive since it does not interrogate the learner's mathematical understanding of concept thoroughly (Carless, 2006). Therefore, from this analysis it can be noted that these commenters are accustomed to feedback as a product and not as a process.

Hence, looking into this aspect of feedback as a process I opined to unveil the true essence of feedback by interviewing two experienced mathematics teachers. Their responses to the interview questions are captured in Table 1, Table 2, and Table 3 below. Firstly, I looked into how teachers perceive feedback as a process drawing from their experiences as mathematics teachers. Their responses are captured in Table 1 below.

Table 2: Transcribed responses of teachers' perception of the feedback as a process

Researcher: How do you perceive feedback as a process in the context of assessing mathematical understanding? *Teacher-1:* In my experience, feedback is an essential component of the learning process. When it comes to assessing mathematical understanding, feedback serves as a guide for learners to identify their strengths and areas for improvement. *Teacher-2:* Feedback as a process is crucial for assessing mathematical understanding because it promotes metacognition and self-regulation. When learners receive feedback that is specific, timely, and actionable, they can reflect on their own thinking and make connections between concepts.

Drawing from their responses it can be noted that both teachers highlighted the importance of feedback as a process in assessing mathematical understanding. As noted that Bound and Molly (2013) that feedback should not be a one way process. Therefore, these teachers affirms that feedback should be an interactive learning process. In addition, Teacher-2, emphasizes that feedback should be specific, timely, and actionable to promote metacognition and self-regulation. Therefore, such notion of feedback opposes what Carless (2006) noted. Carless, found that feedback given to learners was not constructive. However, the notion held by Teacher-2 supports the constructive nature of feedback. In the next interview question, I looked at an example of how feedback can facilitate learning and support learners' mathematical understanding within a socio-cultural context. The teachers' responses to the interview question are captured in Table 2 below.

Table 3: Transcribed teachers' responses on how feedback support mathematical understanding

Researcher: Can you provide examples of how feedback can facilitate learning and support learners' mathematical understanding within a socio-cultural context? *Teacher-1:* For example, if a learner is struggling with solving quadratic functions, I might provide feedback that highlights their correct steps and the specific errors they made. This allows them to reflect on their thought process and make necessary adjustments. By providing feedback as a process, learners can actively engage in self-assessment and self-correction, leading to deeper mathematical understanding. *Teacher-2:* For instance, if a learner is struggling with understanding the relationship between the graph of a quadratic function and its equation, I would provide feedback that encourages them to analyze the features of the graph and make connections to the equation. By guiding them through this process, I aim to enhance their mathematical understanding and problem-solving abilities.

It can be noticed from the teachers' responses that feedback was considered as a guide for learners to identify their strengths and areas for improvement rather than a product. As opposed to Harks et al. (2014) that more teachers often viewed feedback as a quantifiable measure of understanding. These teachers held that the goal of feedback to encourage learners to reflect on their thinking and make connections between concepts. Lastly, these teachers were asked how they interrogate and make sense of the feedback provided to learners in order to enhance their mathematical understanding. Their answers to this interview question posed are captured in Table 3 below.

Table 4: Transcribed teachers responses on the interrogation and sense making of feedback

Researcher: How do you interrogate and make sense of the feedback provided to learners in order to enhance their mathematical understanding? *Teacher-1:* When examining the feedback provided to learners, I pay close attention to the specific goals and learning objectives of the task. I analyze whether the feedback aligns with these goals and provides the necessary support for learners to improve their mathematical understanding. For example, if a learner receives feedback on a quadratic function task, I will assess whether the feedback addresses their misconceptions, provides alternative strategies, and encourages them to make connections to prior knowledge. By interrogating the feedback in this way, I can ensure that it is meaningful and targeted to enhance learners' understanding. *Teacher-2:* To make sense of the feedback provided to learners, I consider the individual needs and learning styles of my learners. I analyze how they have interpreted and applied the feedback in their subsequent work. By closely observing their progress, I can identify patterns and trends in their understanding. For instance, if a learner consistently struggles with understanding the concept of vertex form in quadratic functions, I will tailor my feedback to address their specific challenges and provide additional resources or examples to support their learning. By adapting the feedback based on their needs, I aim to enhance their mathematical understanding and foster a growth mindset.

From their responses, it can be noted that both teachers first, stated the importance of interrogating and making sense of the feedback provided to learners before handing it out. Contrarily, to the comments received from the presentation as the commenters only paid attention to scores without interrogating the essence of the feedback. Second, the teachers asserted that feedback should align with the goals and learning objectives of the task given to learners. Thirdly, the teachers noted that the feedback should address misconceptions, provide alternative strategies, and encourage connections to prior knowledge. Similar to the findings of Mason and Burning (2001), feedback should be interactive. These teachers' responses indicated the acknowledgement of such notion. Lastly, these teachers stressed that feedback should address individual learner needs. In the next section, I synthesise the principal findings of the study in order to draw a meaningful conclusion.

SYNTHESIS AND CONCLUSIONS

Based on the Polkinghorne's (1995) narrative analysis, it is evident that feedback plays a crucial role in assessing mathematical understanding. It serves as a process that guides learners towards self-assessment and self-correction, leading to deeper mathematical understanding. The feedback should be meaningful, specific, and targeted to address individual learner needs and learning styles. By aligning the feedback with the goals and learning objectives of the task, teachers can support learners in making connections between concepts and enhancing their problem-solving abilities. These findings highlight the importance of providing timely and actionable feedback within a socio-cultural context. By scaffolding and guiding learners' mathematical understanding through feedback, teachers can create a supportive learning environment that fosters growth mindset and metacognitive skills. The findings also emphasize the need for teachers to continuously reflect on their own feedback practices and adapt them to meet the diverse needs of their learners.

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