Significance of Elzaki Transform

DR. AFTAB ALAM[1], DR. DINESH VERMA[2]
[1]*Associate Professor, Department of Mathematics, Swami Vivekanand Subharti University Meerut U.P*
[2] *Professor, Department of Mathematics, NIILM University, Kaithal, Haryana (India)*

Abstract- A mathematical technique known as the Elzaki transformation is used to solve differential equations by changing one from in to another from. It frequently works well to solve partial or ordinary linear differential equations. By transforming the time domain function into a frequency domain function, the Elzaki transformation is employed to solve the time domain problem. The Laplace transform method is typically used to solve cooling problems according to Newton's Law. The Elzaki transform approach is used in the study to investigate Newton's Law of Cooling. The goal of the paper is to demonstrate how Elzaki transform can be used to analyse Newton's Law of Cooling.

Indexed Terms- Elzaki Transform, Inverse ElzakiTransform, Newton's Law of Cooling, Temperature of environment, Temperature of body.

I. INTRODUCTION

Boundary value issues have been solved using the Elzaki Transform in the majority of science and engineering professions [1]. It also proves to be a very efficient method for solving conventional, non-linear differential equations, as well as partial differential equations, and is widely used in physics. A linear differential equation is converted using the Elzaki transform into an algebraic equation that can be solved using algebraic principles. Several branches of science, engineering, and technology have successfully employed the Elzaki Transform [1], [2], [7], [9], [24], [25]. The cooling of a heated body located in a cold environment is predicted by Newton's Law of Cooling, which is referred to as an ordinary differential equation [3, [4, [5], [6]]. Newton's law of cooling states that the rate of heat transfers from

$T' = -k(T - T_e)$ ………………………….(I)

with initial condition as $\mathrm{T}(t_0) = T_0$ ………(II)

Where,T is the temperature of the object

T_e is the constant temperature of the environment,

k is the constant of proportionality,

T_0 is the initial temperature of the object at time t_0

The negative sign of RHS in (1), indicate temperature of the body is decreasing with time and so the derivative $\frac{dT}{dt}$ must be negative.

II. BASIC DEFINITION

Elzaki Transform

If the function $\hbar(\mathrm{y})$, $\mathrm{y} \geq 0$ is having an exponential order and is a piecewise continuous function on any interval, then the Elzaki transform of $\hbar(\mathrm{y})$ is given by

$$\mathrm{E}\{\hbar(\mathrm{y})\} = \bar{\hbar}(p) = \mathrm{p}\int_0^\infty e^{-\frac{y}{p}}\hbar(\mathrm{y})dy.$$

The Elzaki Transform [1, 2, 3] of some of the functions are given by

- $E\{y^n\} = n!\,p^{n+2}\ , where n = 0,1,2,..$
- $E\{e^{ay}\} = \frac{p^2}{1-ap}$,
- $E\{sinay\} = \frac{ap^3}{1+a^2p^2}$,
- $E\{cosay\} = \frac{ap^2}{1+a^2p^2}$,
- $E\{sinhay\} = \frac{ap^3}{1-a^2p^2}$,
- $E\{coshay\} = \frac{ap^2}{1-a^2p^2}$.

2.2 Inverse Elzaki Transform

The Inverse Elzaki Transform of some of the functions are given by

- $\mathrm{E}^{-1}\{p^n\} = \frac{y^{n-2}}{(n-2)!}\ , n = 2, 3, 4 \dots$
- $\mathrm{E}^{-1}\{\frac{p^2}{1-ap}\} = e^{ay}$
- $\mathrm{E}^{-1}\{\frac{p^3}{1+a^2p^2}\} = \frac{1}{a}\sin ay$
- $\mathrm{E}^{-1}\{\frac{p^2}{1+a^2p^2}\} = \frac{1}{a}\cos ay$
- $\mathrm{E}^{-1}\{\frac{p^3}{1-a^2p^2}\} = \frac{1}{a}\sin hay$
- $\mathrm{E}^{-1}\{\frac{p^2}{1-a^2p^2}\} = \frac{1}{a}\cos hay$

2.3 Elzaki Transform of Derivatives

The Elzaki Transform [1, 2, 3] of some of the Derivatives of h(y) are given by

- $E\{h'(y)\} = \frac{1}{p}E\{h(y)\} - \text{p}\, h(0)$

  $or E\{\hbar'(y)\} = \frac{1}{p}\bar{\hbar}(p) - \text{p}\, \hbar(0),$
- $E\{\hbar''(y)\} = \frac{1}{p^2}\bar{\hbar}(p) - \hbar(0) - \text{p}\hbar'(0),$

  $and so on$

III. METHODOLOGY

From (I),

$$T' = -k(T - T_e)$$

Taking ElzakiTransform on both sides,

$$E\{T'\} = -E\{k(T - T_e)\}$$

$$E\{T'\} = -kE\{T(t)\} + kE\{T_e\}$$

$$\frac{1}{p}E\{T(t)\} - pT(0) = -kE\{T(t)\} + kT_e E\{1\}$$

From (II), As T $(t_0) = T_0$

Now,

$$\frac{1}{p}E\{T(t)\} - pT(0) = -kE\{T(t)\} + kT_e p^2$$

$$\left(\frac{1}{p} + k\right)E\{T(t)\} = pT(0) + kT_e p^2$$

$$E\{T(t)\} = \frac{pT_0}{\left(\frac{1}{p} + k\right)} + kT_e \frac{p^2}{\left(\frac{1}{p} + k\right)}$$

Taking inverse Elzaki,

$$\{T(t)\} = T_0 e^{-kt} + kT_e - kT_e e^{-kt}$$

$$T(t) = (T_0 - kT_e)e^{-kt} + kT_e$$

$$T(t) = Ce^{-kt} + kT_e \text{ ... ... (III)}$$

$$where, C = (T_0 - kT_e)$$

While this function decreases exponentially, it approaches T_e as t⟶ ∞ instead of zero.

Application:

An apple pie with an initial temperature of 170^0 C is removed from the oven and left to cool with an air temperature 20^0C. Given that the temperature of the pie initially decreases at a rate of 3.0^0 C/min. How long will it take for the pie to cool to a temperature of 30^0 C? [22]

Suppose the pie is in compliance with newton's cooling law; we have the following information

$$T' = -k(T - 20), T(0) = 170\ , T'(0) = -3.0$$

Where, T is the temperature of the pie in degree Celsius, T' is the time in minutes and k is an unknown constant.

Now, we will find the value of k by putting the given information we know about t = 0 directly into the differential equation:

$$-3 = -k(170 - 20)$$

$$k = 0.02$$

So, the differential equation can be written as

$$T' = -\frac{1}{50}(T - 20)$$

Taking Elzakion both sides,

$$E\{T'\} = -\frac{1}{50}E\{(T(t) - 20)\}$$

$$E\{T'\} = -\frac{1}{50}E\{T(t)\} + \frac{2}{5}E\{1\}$$

$$\frac{1}{p}E\{T(t)\} - pT(0) = -\frac{1}{50}E\{T(t)\} + \frac{2}{5}p^2$$

$$pE\{T(t)\} - pT(0) = -\frac{1}{50}E\{T(t)\} + \frac{2}{5}p^2$$

$$\left(p + \frac{1}{50}\right)E\{T(t)\} = pT(0) + \frac{2}{5}p^2$$

$$\left(p + \frac{1}{50}\right)E\{T(t)\} = 170p + \frac{2}{5}p^2$$

$$E\{T(t)\} = \frac{170p}{\left(p + \frac{1}{50}\right)} + \frac{2p^2}{5\left(p + \frac{1}{50}\right)}$$

$$E\{T(t)\} = \frac{170p}{\left(p + \frac{1}{50}\right)} - \frac{20p^2}{\left(p + \frac{1}{50}\right)} + 20p^2$$

$$E\{T(t)\} = \frac{150p^2}{\left(p + \frac{1}{50}\right)} + 20p^2$$

Taking inverse Elzaki on both sides, we get,

$$T(t) = 150e^{-\frac{1}{50}t} + 20 \text{ ... ...} (IV)$$

Putting T=30 in (IV),

$$30 = 150e^{-\frac{1}{50}t} + 20$$

$$e^{-\frac{1}{50}t} = \frac{1}{15}$$

$$e^{\frac{1}{50}t} = 15$$

$$\frac{1}{50}t = \ln 15$$

$$t = 50 \ln 15$$

$$t = 50 * 2.7080502011$$

$$t = 135.4\ minute$$

Hence, this will require 135.4 minutes for the pie to cool to a temperature of 30^0 C.

CONCLUSION

The Elzaki Transform has been effectively established in this research to address issues with Newton's Law of Cooling. The examples provided show how well the Elzaki Transform works for Newton's Law of Cooling issues. The proposed approach has broad use in many areas of physics, electrical engineering, control engineering, economics, mathematics, signal processing, and electronics engineering.

REFERENCES

[1] Dinesh Verma ,Elzaki Transform Approach to Differential Equatons with Leguerre Polynomial, International Research Journal of Modernization in Engineering Technology and Science (IRJMETS)" Volume-2, Issue-3, March 2020.

[2] Dinesh Verma, Aftab Alam, Analysis of Simultaneous Differential Equations By Elzaki Transform Approach, Science, Technology And Development Volume Ix Issue I January 2020.

[3] Sunil Shrivastava, Introduction of Laplace Transform and Elzaki Transform with Application (Electrical Circuits),International Research Journal of Engineering and Technology (IRJET) ,volume 05 Issue 02 , Feb-2018.

[4] Tarig M. Elzaki, Salih M. Elzaki and ElsayedElnour, On the new integral transform Elzaki transform fundamental properties investigations and applications, global journal of mathematical sciences: Theory and Practical, volume 4, number 1(2012).

[5] Dinesh Verma and Rahul Gupta ,Delta Potential Response of Electric Network Circuit, Iconic Research and Engineering Journal (IRE) Volume-3, Issue-8, February 2020.

[6] Mohand M. Abdelrahim Mahgoub, The New Integral Transform "Mohand Transform: Advances in Theoretical and Applied Mathematics, ISSN 0973-4554 Volume 12, Number 2 (2017), pp. 113-120.

[7] Mohand M. Abdelrahim Mahgoub, The New Integral Transform "Mohand Transform, Advances in Theoretical and Applied Mathematics ISSN 0973-4554 Volume 12, Number 2 (2017), pp. 113-120

[8] Mohamed E. Attaweel and Haneen Almassry, On the Mohand Transform and Ordinary Differential Equations with Variable Coefficients, Al-Mukhtar Journal of Sciences 35 (1): 1-6, 2020

[9] Dinesh Verma, Putting Forward a Novel Integral Transform: Dinesh Verma Transform (DVT) and its Applications, International Journal of Scientific Research in Mathematical and Statistical Sciences, Volume -7, Issue-2, April-2020, pp: 139-145.

[10] Govind Raj Naunyal , Updesh Kumar and Dinesh Verma, Applications of dinesh verma transform to an electromagnetic device, Iconic Research and Engineering Journals *(IRE Journals)*, Volume-5, Issue-12, June 2022, ISSN: 2456-8880; PP: 235-240.

[11] Ahsan Z., Differential Equation and Their Applications, PHI, 2006.