Stochastic Assessment of the Design of Moment Resisting Timber Joint

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Abstract - Effective and accurate algorithms were developed to evaluate the reliability of the design of moment resisting joints in timber frames. Generally, the results obtained showed that the reliability levels, β, showed that bending consideration, rather than the shear controls the design of timber joints. For the bending consideration, failure mode III, (due to the development of plastic hinge in bolt, in main or side members) gave β = 1.10 at mean values of the design parameters, thereby making it the most critical. Failure Mode IV, (due to the development of plastic hinge in bolt, in both main and side members) follows with β = 1.88, then failure Mode I, (due to bearing of the main and side members) gave β = 2.66, failure Mode II, (due to bolt rotation without bending) has the least effect with β = 3.84. However, it was found that, under certain design situations, this order changes. This brings to the fore, the inconsistencies in code provisions.The sensitivity analysis conducted to examine the influence of the basic design variables on the implicit reliability levels showed that the most significant variables are the diameter of fasteners and density of timber. The implication of this is that during the design of joints in timber frames, these variables are considered seriously.

Indexed Terms- Failure Modes, Probabilistic Techniques, Reliability Levels, Sensitivity Analysis

I. INTRODUCTION

The dowel connections are the main fastening technique used in timber frames. Facing the actual architectural trends, timber-to-timber connections are mainly replaced by connection using slotted-in steel. In this case, better aesthetic appearance and greater fire safety is obtained. According to the actual world-wide design rules [1,2], the calculation of mechanical timber joint is based on Johansen's yield model [3]. Considering elasto-plastic behavior of the components, the main assumption of this approach is that no brittle failure occurs. However, recent studies [4-5] show that brittle failure mainly governs the load carrying capacity of the connections when using slotted-in steel. In regard to these results, the design criteria of timber joint need to be re-examined to take account of all the potential failure modes.

The application of risk and reliability in analysis, design and planning in engineering systems has received worldwide acceptance. As a result of extensive efforts by different engineering disciplines during the last five decades, design guidelines and codes are being modified or already been modified to incorporate the concept of risk-based analysis and design. Most of the commonly used risk or reliability-based methods require that a functional relationship among the load-resistance related variables, commonly known as the limit state or performance function be available in implicit form. However, it can be difficult to explicitly define the relationship of structural response as a function of the basic variables such as the geometries, material properties, the associated constitutive relations, the loads acting on the structure, various sources of nonlinearity expected in the structural behavior just before failure, etc. For large structural systems, it may be impossible. Thus, simple, commonly used risk-based analysis and design problem cannot be applied when the performance function is implicit. However, accurate reliability evaluation of such structures has been demanded by the profession.

II. FORMULATION OF RELIABILITY PROBLEMS

2.1. THE CONCEPT OF RELIABILITY

To make sense of the term "reliability" we must understand the implied meaning of a failure event. The reliability is defined as the probability that failure will not occur. In Structural reliability analysis, failure does not necessarily imply collapse or other spectacular events. By failure we simply mean exceeding prescribed thresholds. Hence, the key problem in reliability analysis is to compute the failure probability, denoted P_f, for a predefined failure event. The reliability is then defined as the compliment of the probability of failure; Reliability = $(1 – P_f)$.

Before proceeding to the formal definitions of a reliability problem, the meaning of a calculated reliability and how it can be used are briefly discussed. One may ask; what does the calculated P_f mean? Can it be related to observed rates of failure for the real problem? And most importantly, can the knowledge of P_f be used to achieve better and safer infrastructure? Using the case of structural reliability as an example, it is clear that structural failure (which is a rare thing) often occur due to human error, neglected loading, unforeseen events, etc. That is, failure may occur due to uncertain factors that are not even included in the analysis model. Therefore one may argue that P_f cannot be interpreted by the "frequentist" definition of probability. However, subjective engineering approach embraced by Bayesian statisticians brings the importance of calculated P_f to the fore.

If it is accepted that in some cases, reliability models are not yet refined enough to properly take into account all sources of uncertainty; reliability analysis is still useful for several reasons [7]:

- An integral part of a reliability analysis is ranking of the model parameters according to their relative importance. This represents invaluable information that may provide physical insight, lead to improved design by focusing more effort on the most important components, etc.
- An alternative to the classical statisticians' view of P_f is that probability expresses a "degree of belief" about the occurrence of an event. Hence, instead of viewing $\mathbf{P_f}$ as a direct indicator of frequency of failure, it can be used as a nominal value.
- One use of P_f as a nominal value is for comparison purposes. By computing P_f for example, in alternative design solutions we can compare relative reliability and use that to make a decision.
- The concept of nominal failure probability is used in code calibration applications. If we imagine that the current code regulations are the result of many years of trial and error to arrive at a level of safety that is acceptable to the society, then new designs can be calibrated towards that safety level. Based on the above notion of nominal probability, reliability methods can be used in such calibrations.

Depending on the problem at hand, rapid developments in modeling analysis capabilities may even enable an actually interpretation of P_f as the real probability of failure. Even then it is important to keep in mind the following word of wisdom [8]:
"Reliability analysis is not a substitute for the individual responsibility of an engineer to promote or think about safety, nor is it necessarily any better; properly used, however, it has the potential to clarify and expose issues of importance".

2.2 FORMULATION OF RELIABILITY ANALYSIS

Modern structural reliability theory has its origin in a landmark paper by A. M. Freudenthal which appeared in 1947 Transaction of the American Society of Civil Engineers [9]. Later he and E. J. Gumbel collaborated in the development of methods of fatigue reliability focusing on statistical modeling of S-N data [10-12]. The concept of risk-based design was then summarized by Freudenthal, Garrelts and Shinozuka in1966. The idea was to address the lapses observed in the conventional factor safety methods. Nominal factor of safety fails to convey the actual margin of safety in design since the intended conservatism introduced by the safety factor depends largely on uncertainty involved in the load and resistance variables, and the experience of a structural engineer. Therefore, a more rational approach will be to compute the margin of safety by accounting for the uncertainty in design variables explicitly. This is the concept reliability analysis.

The load effect S and resistance Rare the random variables, with probability density function (PDF) $f_S(s)$ and $f_R(r)$respectively. The reliability can be evaluated by the relationship betweenthese the two variables called the performance function g(R,S).

Mathematically, expression of the relationship or performance function for this case can be described as

$$Z \equiv G(R,S) \equiv R - S \tag{1}$$

Where g(R, S) < 0 indicates the failure state. Mathematically, the probability of failure of the event (P (R<S)) for this case can be written as,

$$p_f = P(R<S) = P[g(R,S)<0] = \iint_{\Omega} f_{R,S}(r,s)drds \tag{2}$$

Where $f_{R,S}$ (r,s) is the joint probability function of the two variables. The integration of Equation (2) is performed over the failure regionΩ.

In general, the load effect S and resistance R are function of other variables such as individual loads, geometric and material properties. Therefore, Equation (1) assumes a more general form as

$$g(X) = g(X_1, ..., X_n) \tag{3}$$

where X is the relevant load and resistance parameters, called the basic random variables. The failure surface or limit state can be defined as g(X)=0 which is the boundary between the safe and unsafe region. The failure probability is calculated as [14-17],

$$p_f = \iint ...... \iint_{g(x)<0} f_x(x_1, x_2, ..., x_n)dx_1.........dx_n \tag{4}$$

Where $f_X(x_1, \ldots, x_n)$is the joint probability density function of the multivariable X.

The practical approximate approaches are often restricted to the use of the mean value and coefficient of variation (COV) because the information or data may only be sufficient to evaluate mean and COV of a random variable [15,18]. This leads to the development of the Mean Value First Order Second Moment (MVFOSM) method. However, the MVFOSM fails to incorporate the distribution information of the random variables, even if it is available. This led to the development of First Order Reliability Method (FORM) and Second Order Reliability Method (SORM).

The FORM uses the first-order (linear) approximation for the performance function, being capable of performing the reliability analysis for a linear performance function of uncorrelated variables or linear approximation of the nonlinear performance function. The SORM performs the reliability analysis by approximating the nonlinear performance function, including a linear performance function with uncorrelated non-normal variables, to a second-order representation.

2.3. FIRST-ORDER RELIABILITY METHOD (FORM)

The development of FORM can be traced historically to second-moment methods. It uses the information on first and second moments of the random variables. These are first-order second-moment (FOSM) and advanced first-order second-moment (AFOSM) methods. In FOSM methods, the information on the distribution of the random variable is ignored; however, in AFOSM methods, the distribution information is appropriately used.

2.4. FIRST-ORDER SECOND-MOMENT METHOD OR MVFOSM METHOD

The FOSM method also referred to as MVFOSM derives its name from the fact that it is based on a first-order Taylor series approximation of the performance function linearized at the mean values of the random variables, and because it uses only second-moment statistics (mean and covariance) of the random variables. Considering Equation (1) for the simple two variables, R and S performance function. Assuming that these are statistically independent normally distributed random variables, the variable Z is also normally distributed. The probability of failure depends on the ratio of the mean value of Z to its standard deviation. The ratio is commonly known as the safety index or reliability index and is denoted as β:

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 - \sigma_S^2}} \tag{5}$$

The probability of failure in term of safety index is thus

$$p_f = \Phi(-\beta) = 1 - \Phi(\beta) \tag{6}$$

An alternative formulation was proposed, by assuming that the variables R and S are statistically independent lognormal random variables [19]. For physical reasons, these variables are restricted to positive values; hence it is more reasonable to assume that they are log-normally distributed. In this case, another random variable Y is introduced as

$$Y = \frac{R}{S} \quad (7)$$

or

$$\ln Y = Z = \ln R - \ln S \quad (8)$$

The failure event can be defined as Y< 1.0 or Z < 0.0. Since R and S are lognormal, lnR and lnS are normal.Using the relationship between the mean (μ), variance (s^2), coefficient of variation (δ) and the parameters of lognormal distribution, the probability of failure can be written as

$$p_f = 1 - \Phi\left[\frac{\ln\left(\mu_R / \mu_S\right)\sqrt{(1+\delta_S^2)/(1+\delta_R^2)}}{\sqrt{\ln(1+\delta_S^2)/(1+\delta_R^2)}}\right] \quad (9)$$

If δ_R and δ_S are not large, say ≤ 0.30, Equation (9) can be simplified as

$$p_f \cong 1 - \Phi\left[\frac{\ln(\mu_R / \mu_S)}{\sqrt{\delta_S^2 + \delta_R^2}}\right] \quad (10)$$

The reliability index in this case is given as

$$\beta = \Phi^{-1}(1 - P_f) \cong \frac{\ln(\mu_R / \mu_S)}{\sqrt{\delta_S^2 + \delta_R^2}} \quad (11)$$

These formulations may also be generalized for many random variables, denoted by a vector X. A tailor series expansion of the performance function, that is, Equation (3) gives

$$Z = g(\mu_x) + \Sigma\sum_{i=1}^{n}\frac{\partial g}{\partial X_i}(X_i - \mu_{X_i}) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2 g}{\partial X_i \partial X_j}(X_i - \mu_{X_i})(X_j - \mu_{X_j}) + ... \quad (12)$$

where the derivatives are evaluated at the mean values of the random variables (X_1, X_2,…………X_n) and μ_{X_i} is the mean value of X_i. Truncating the series at the linear terms, we obtain the first order approximate mean and variance of Z as,

$$\mu_Z \approx g(\mu_{X_1}, \mu_{X_2}, ..., \mu_{X_n}) \quad (13)$$

and

$$\sigma_Z^2 \approx \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial g}{\partial X_i}\frac{\partial g}{\partial X_j}Cov(X_i, X_j) \quad (14)$$

Where Cov(X_i, X_j) is the covariance of X_i and X_j. However, if the variables are uncorrelated, then variance is simply,

$$\sigma_Z^2 \approx \sum_{i=1}^{n}\left(\frac{\partial g}{\partial X_i}\right)^2 Var(X_i) \quad (15)$$

The safety index can be calculated by taking the ratio of the mean and standard deviation of Z as in Equation (5).

Furthermore, in most cases it is not likely that all the variables are statistically independent normal or lognormal variables. Nor is it likely that the performance function is a simple additive or multiplicative function of these variables. In such cases, the safety index cannot be directly related to the probability of failure, but it gives a rough idea of the level of risk or reliability in the design. Consequently, FORM has some deficiencies. In addition to not using the distribution information of the variables when it is available, the g(X) is linearized at the mean values of the variables. When g(X) is nonlinear, significant error is introduced by neglecting higher order terms. More important, the safety index fails to be constant under different but mechanically equivalent formulations of the same performance function. Example, the safety margins defined as R – S < 0 and R/S<1 are mechanically equivalent formulation of the same performance function, yet they give different probabilities of failure.

Nevertheless, FORM was used to derive the earlier versions of reliability-based design formats, such as the American Institute of Steel Construction, Inc. [20], Canadian Standard Association [21] and Comite European du Beton[22], to cite just a few examples.

The Hasofer-Lind (H-L) method is applicable for normal random variables. It first defines the reduced variables as

$$X_i' = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \quad (i=1,2,.......,n) \quad (16)$$

where X'_I is a random variable with zero mean and unit standard deviation. Equation (16) is used to transform the original limit state g(X)=0 to the reduced limit state, g(X')=0. The X coordinate system is referred to as the original coordinate systems. The X' coordinate system is referred to as the transformed or reduced coordinate system; therefore, if X_i is normal then, X'_I is standard normal. The safety index β_{H-L} is defined as the minimum distance from the origin of the axes in the reduced coordinate system to the limit state surface (failure surface). It can be expressed as

$$\beta_{H-L} = \sqrt{(X^{'*})^t (X^{'*})} \qquad (17)$$

The minimum distance point on the limit state surface is called the design point or checking point. It is denoted by vector x^* in the original coordinate system and by vector x'^* in the reduced coordinate system. These vectors represent the values of all the random variables, that is, X_1, X_2,…….,X_n at the design point corresponding to the coordinate system being used.

The position of the limit state surface relative to the origin in the reduced coordinate system is the measure of the reliability of the system. This is illustrated in Fig. 1.

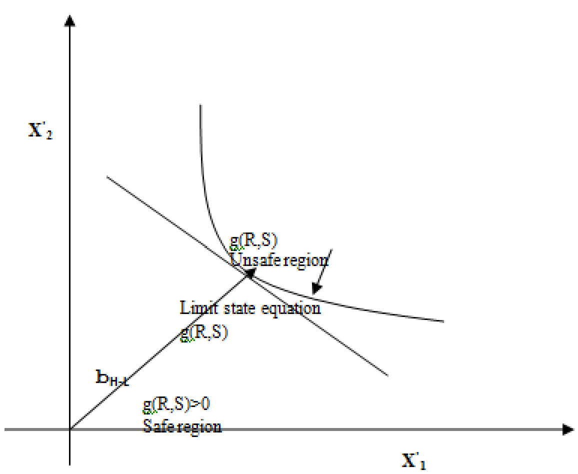

Fig. 1: Limit state concept.

The Hasofer-Lind reliability index can be exactly related to the probability of failure when all random variables are normally distributed. However, this is not common in structural problems. Therefore, it was suggested that [23], when the problems involve non-normally distributed random variables, they might be solved by transforming the non-normal variable into equivalent normal variables. The transformation can be done by equating the cumulative distribution function (CDF) and the probability density function (PDF) of the non-normal variable to equivalent normal variables at the checking point. Considering statistically independent non-normal random variables, the equivalent mean and standard deviation are obtained as:

$$\mu_{X_i}^N \equiv x - \Phi^{-1}\left[F_{X_i}\left(x_i^*\right)\right]\sigma_{X_i}^N \qquad (18)$$

and

$$\sigma_{X_i}^N = \frac{\phi\left\{\Phi^{-1}\left[F_{X_i}\left(x_i^*\right)\right]\right\}}{f_{X_i}\left(x_i^*\right)} \qquad (19)$$

Where, F_{X_i} and f_{X_i} are the non-normal distribution and density functions of X_i respectively, and Φ and ϕ are commutative distribution and density function of a standard normal variable. Having determined $\mu_{X_i}^N$ and $\sigma_{X_i}^N$ the reliability index can be calculated for non-normal random variable cases using Equation (5). The Rackwitz-Fissler approach [23], also commonly known as the FORM has been extensively used in the literature. However, the approach may not give an exact result for a highly nonlinear limit state case due to the underlying approximation of the linear limit state. Nonetheless, this approach has been considered as an effective reliability method because of its simplicity and versatility. Examples of its application in timber related problems include [24-27].

III. STATISTICAL CHARACTERISTIC OF DESIGN PARAMETERS

3.1. STOCHASTIC MODELS

According to Eurocode-5 [2] provisions in respect to the design of timber joint, the characteristic load carrying capacity for nails, staples, dowels and screws per shear plane per fastener, at specified minimum spacing should be the minimum value from the following expressions:

For fasteners in single shear:

$$R_k = \begin{cases} f_{h,1,k} t_1 d & (a) \\ f_{h,2,k} t_2 d & (b) \\ \dfrac{f_{h,1,k} t_1 d}{1+\beta}\left[\sqrt{\beta + 2\beta^2\left[1+\dfrac{t_2}{t_1}+\left(\dfrac{t_2}{t_1}\right)^2\right]+\beta^3\left(\dfrac{t_2}{t_1}\right)^2} - \beta\left(1+\dfrac{t_2}{t_1}\right)\right] & (c) \\ \dfrac{f_{h,1,k} t_1 d}{2+\beta}\left[\sqrt{2\beta(1+\beta)+\dfrac{5\beta(2+\beta)M_{y,k}}{f_{h,1,k} d t_1^2}} - \beta\right] & (d) \\ \dfrac{f_{h,1,k} t_2 d}{1+2\beta}\left[\sqrt{2\beta^2(1+\beta)+\dfrac{5\beta(1+2\beta)M_{y,k}}{f_{h,1,k} d t_2^2}} - \beta\right] & (e) \\ 1.15 k_{cal}\sqrt{\dfrac{2\beta}{1+\beta}}\sqrt{2M_{y,k} f_{h,1,k} d} & (f) \end{cases} \quad (20)$$

For
fasteners in double shear:

$$R_k = \begin{cases} f_{h,1,k} t_1 d & (g) \\ 0.5 f_{h,2,k} t_2 d & (h) \\ \dfrac{f_{h,1,k} t_1 d}{2+\beta}\left[\sqrt{2\beta(1+\beta)+\dfrac{5\beta(2+\beta)M_{y,k}}{f_{h,1,k} d t_1^2}} - \beta\right] & (j) \\ 1.15 k_{cal}\sqrt{\dfrac{2\beta}{1+\beta}}\sqrt{2M_{y,k} f_{h,1,k} d} & (k) \end{cases} \quad (21)$$

with:

$$\beta = \frac{f_{h,2,k}}{f_{h,1,k}}$$

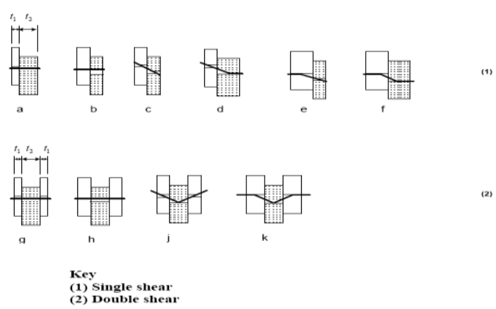

Fig. 2: Failure modes for dowel-type timber joints.

The code provision for bending consideration for joints in double shear allows for four different failure modes. The limiting yield modes are bearing of the main and side members (Mode I), bolt rotation without bending (Mode II), development of plastic hinge in the bolt in main or side members (Mode III) and development of plastic hinges in bolt in both main and side members (ModeIV) [28]. A nominal fastener design value is calculated for each of the failure modes and the lowest value is selected as the design value for the joint. Each of the modes was tested for easy comparison.

where
$R_k$is the load carrying capacity per shear plane per fastener.
$t_i$is the timber or board thickness or penetration depth, with i either 1 or 2.
$f_{h,i,k}$is the characteristic embedment strength in timber member i
dis the fastener diameter.
$M_{y,k}$is the characteristic fastener yield moment.
B is the ratio of the embedment strength of the members.
$k_{cal}$is the factor that accounts for the axial forces which develop in the fastener.

The corresponding failure modes are as shown in Fig. 2, with letters (a – k) corresponding to the expressions in Equations (20) and (21) for joints in single and double shears respectively. The double shear joint in the timber frame considered in this study therefore, presents four failure modes (g-k) in Equation (21) as the stochasticmodels.

According to Eurocode-5 section 8.5.1.1(6), the fastener characteristic yield moment in Equation (22) is given as:

$$M_{y,k} = 0.3 f_{u,k} d^{2.6} \quad (22)$$

This makes it a function of the diameter of the joint fasteners (d) and the tensile strength of the faster ($f_{u,k}$). The density of timber was utilized in the structural analysis of the timber frame. Table 1, shows, the design variables used in the stochastic models developed in this study.

Table 1: Statistical values of design parameters

S/NO.	Basic variables	Mean	C. O. V	pdf
BENDING CONSIDERATION FOR LOAD COMBINATION C3				
1	Density of timber (**ρ**) X1	420 kg/m^3	0.06	Normal
2	Diameter of joint (**φ**) fasteners X2	30 mm	0.05	Normal
3	Thickness of outer timber (t_1) X3	100 mm	0.10	Normal
4	Thickness of inner timber (t_2) X4	200 mm	0.10	Normal
5	Resultant load action (F) X5	29047 N	0.30	Gumbel
6	Yield strength of fasteners (f_y) (X6)	380 N/mm^2	0.01	Log-Normal
SHEAR CONSIDERATION FOR LOAD COMBINATION C3				
1	Shear strength of timber (**t**) X1	2.8 N/mm^2	0.06	Normal
2	Thickness of inner timber (b) X2	200 mm	0.10	Normal
3	Depth of inner timber (h) X3	1200 mm	0.10	Normal
4	Resultant load action (F) X4	29047 N	0.30	Gumbel

For the bending consideration, the basic variables are; the density of timber (X1), the diameter of the joint fasteners (X2), the thickness of the outer members (i.e. the column, (X3)), the thickness of the inner member (i.e. the rafter, (X4)), the resultant load action at the joint (X5) and the yield strength of the fastener (X6).Furthermore, the basic variables in the shear consideration the basic variables the shear strength of timber in the frame (X1), the thickness and depth of the rafter (X2) and (X3) respectively and the resultant load action at the joint (X4).

IV. DISCUSSION OF RESULTS

Resultant load action of the timber frame subjected to various load types and combinations was treated as one of the random variables to compute the safety levels, β, implicit in Eurocodes5 provision for the design of timber joints held with metal fasteners. Most of the basic variables were prescribed normal except the fastener yield strength and the resultant load action which were assumed log-normal and gumbel respectively. Bending and shear considerations were investigated for the most critical loading combination of dead and snow loads. Furthermore, the combination of dead and imposed which is next in severity was also looked at because it is obtainable in our local environment.

4.1. BENDING CONSIDERATIONS

A three-hinged frame (Figure 3) was considered with glued-laminated (GL) member of strength class GL24. The calculation of the knee joint of the frame was carried out using the finite element techniques offered by Staad.pro (Staad.pro, 2003). In practice, the design of frame depends on the design of the moment resisting joint (2 or 4 i.e. left and right knee joint). The results obtained from the analysis showed that, load combination C3 (i.e. factored dead and factored snow loads) is the most critical. The four failure modes given in Eurocode 5 (CEN-TC250, 2004) in respect of timber joints in double shear were used in the computation of β. Since snow loads are not found in our local environment, another run of the analysis less load combination C3 was carried out. Load combination C2 (factored dead and factored imposed loads) then became the most critical. Higher β values were obtained for load combination C2 than the corresponding combination C3.

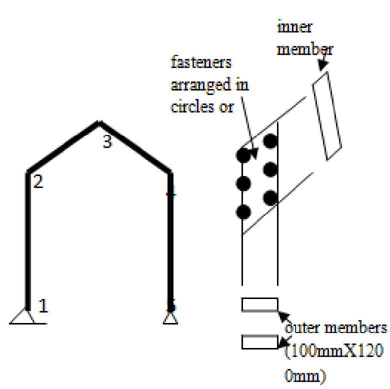

Figure 3: Geometry of the frame and outline of the knee joint

The plots of β values with respect to bending considerations shown in Figs. 4 to 19 were studied with the following deductions.

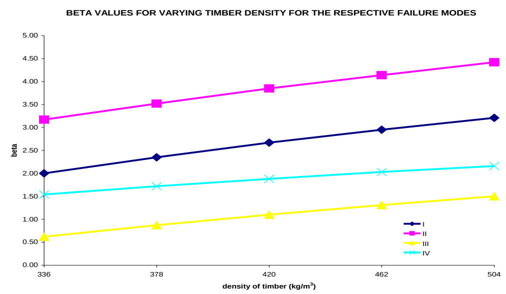

Fig. 4: Variation of β with density of timber.

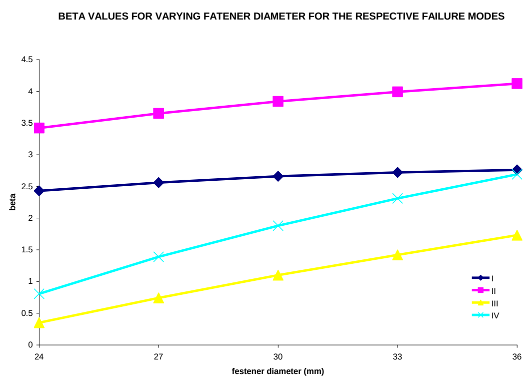

Fig. 5: Variation of β with diameter of fastener.

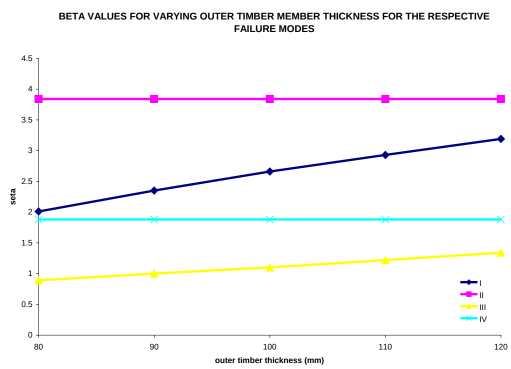

Fig. 6: Variation of β with thickness of outer member.

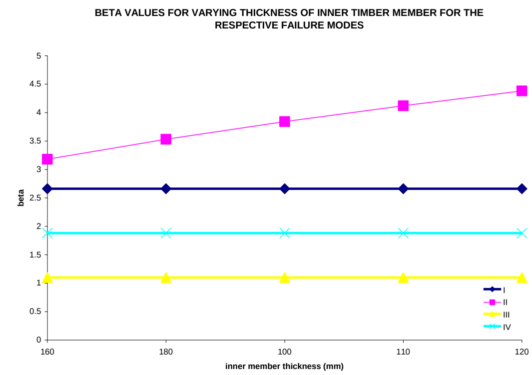

Fig. 7: Variation of β with thickness of inner member.

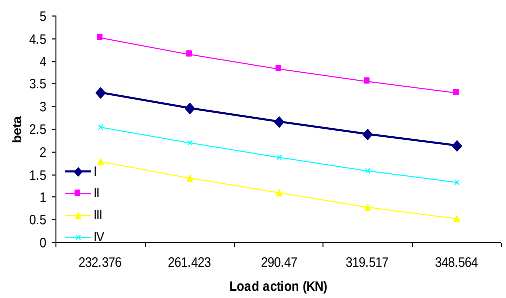

Fig. 8: Variation of β with resultant load action.

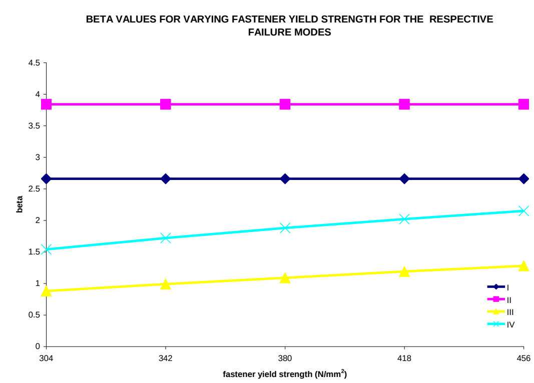

Fig. 9: Variation of β with yield strength of fastener.

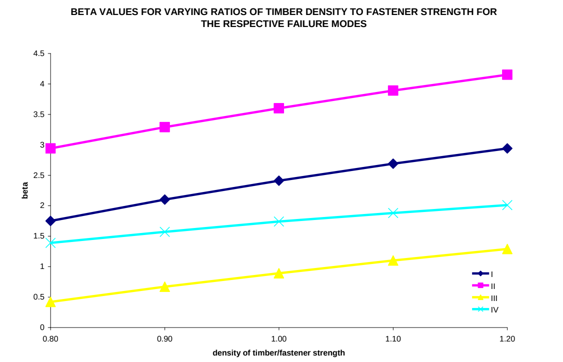

Fig. 10: Variation of β with ratios of timber density to fastener yield strength.

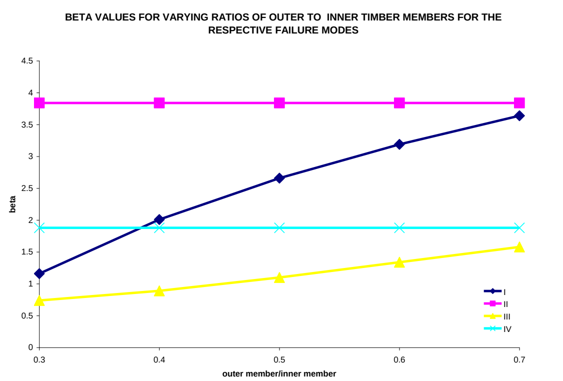

Fig. 11: Variation of β with ratios of thickness of outer to inner member.

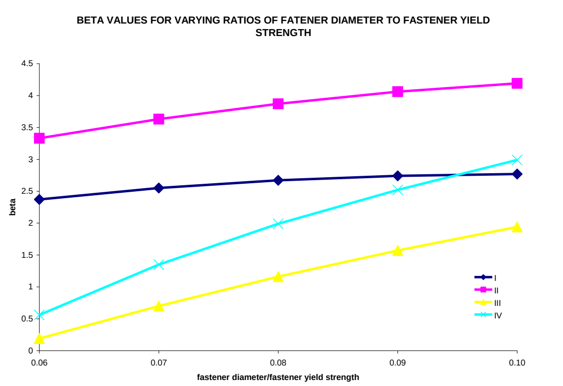

Fig. 12: Variation of β with ratios of fastener diameter to fastener yield strength.

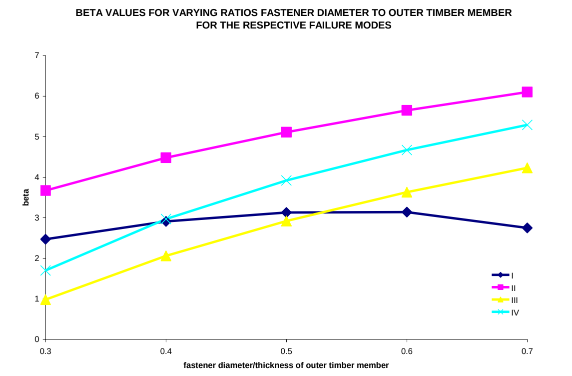

Fig. 13: Variation of β with ratios of fastener diameter to the thickness of outer member of timber.

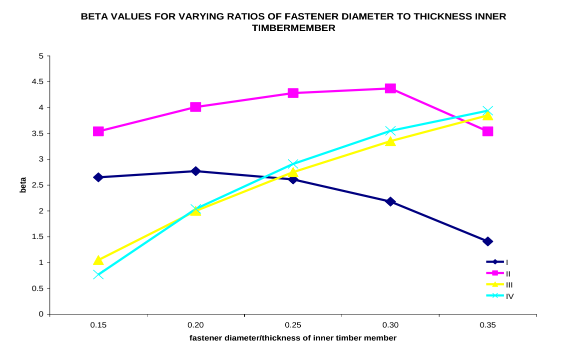

Fig. 14: Variation of β with ratios of fastener diameter to the thickness of inner member of timber.

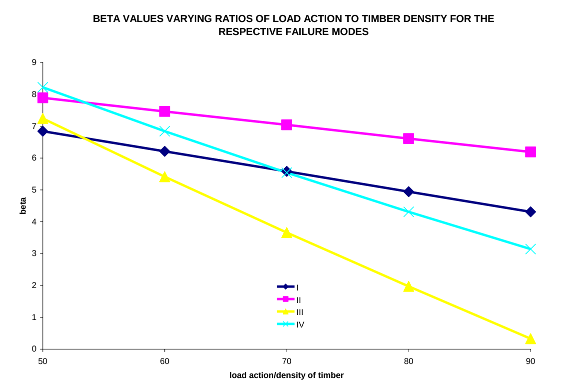

Fig. 15: Variation of β with ratios of load action to timber density.

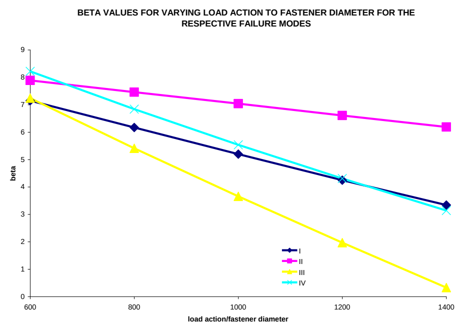

Fig. 16: Variation of β with ratios of load action to fastener diameter.

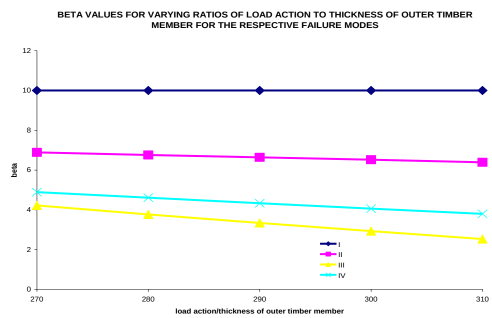

Fig. 17: Variation of β with ratios of load action to the thickness of outer member of timber.

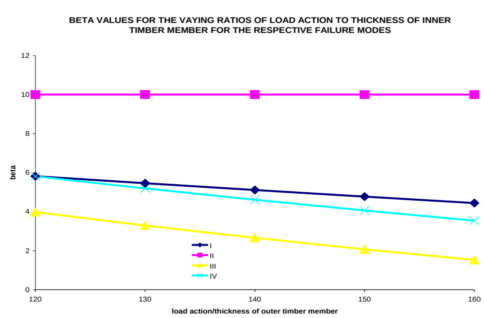

Fig. 18: Variation of β with ratios of load action to the thickness of inner member of timber.

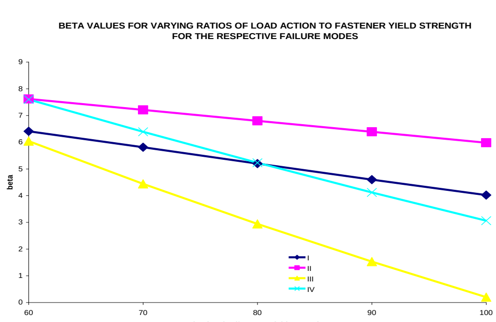

Fig. 19: Variation of β with ratios of load action to the fastener yield strength.

If higher density of timber which is a function of the specie is used for the construction of the framed structure, failure mode II gives a higher reliability. Mode I follows, then mode IV. The least reliability was obtained for failure III. Failure Mode III which gave the least β governs the design of the joint. These results are illustrated in Fig. 4. It also shows that a denser timber will of course give a safer structure. In Fig. 5, an increment in the diameter of fasteners used to hold the members of the frame together at the joints also gave increased β values as for the density of timber. However, β values obtained for failure mode IV approaches that for failure mode I as the diameter of fasteners used increases. Variation of the fastener diameter also showed that failure Mode III governs the design. However, if small diameters of fasteners are used in joints of timber frames, failure Mode IV gives the least β value. This signifies the possibility of the formation of plastic hinges in the bolt in both the inner and outer members of the frame prior to the joint failure.

As for the thicknesses of the members of the frame Figs 6 and 7 respectively for the outer and the inner members, an increase in the former gave constant β values for failure modes II and IV. Also, the values gotten for mode II, approaches that for mode IV at higher thicknesses of the members. However, an increase in the thickness of the inner member gave constant β values for failure modes III and IV. Also, mode I approaches mode II at lower thicknesses of the members. It therefore implies that, the thicknesses of both members' yields constant β values for failure mode IV, while constant values were obtained for outer and inner member thicknesses for failure modes II and III respectively.

The variations of load action, yield strength and diameter of fasteners used in joints of timber frames gave β values that are also highest for failure mode II and least for failure mode III. However, as anticipated, an increase in load action raises the failure probability, while improving the quality of the fastener will do otherwise. These observations are illustrated in plots shown in Figs. 8 to 10.

Varying aspect ratios t_1/t_2 showed constant β values for failure modes II and IV. However, β values obtained for modes I and III are close at low ratios but drift apart at high ratios with a higher value for failure mode I as the aspect ratio increases. Therefore as the ratio of member thicknesses approach unity when the inner and outer members are of equal thickness, a larger discrepancy in the β values are obtained. Also, when the thickness of the outer members is very small in comparison to the inner member failure Mode I govern the design of the knee joint in timber frames. Fig. 11 illustrates these findings.

In Fig. 12, an increase in the ratio of the fastener diameter to its yield strength showed that the β values obtained for failure modes I and IV converge at high ratio of the fastener characteristics. Increasing this ratio showed a level of consistency in the increase in β values for failure modes II and III. It also shows that bearing in mind other constructional requirements, higher diameter of fasteners can make up for low quality of its material constituent.

Variations in the ratios of fastener diameter to the thickness of the timber members of the frame, reflects the slenderness of the fasteners. All the four failure modes gave intriguing changes in the β values. Failure mode II which gave the highest reliability level at low ratios fell below the values obtained for modes III and IV at high ratios. The trend is similar for failure mode I and in both cases the β values rose to a summit before descending. However, modes III and IV gave comparable β values for the varying ratios. Consequently, a too slender or too thick fastener makes the joint vulnerable to failure. A very slender fastener is susceptible to buckling while a stocky one will cause splitting in the members of the frame at the joint area. This very interesting result is illustrated in Fig. 14.

For the ratios of the load action to the other basic variables, illustrated in Figs 15 to 19, it is the variation in the ratios the load action to the fastener characteristics and timber density that result in an interwoven β values for the respective failure modes. While all the failure modes gave similar β values at low ratios, s sharp disparity was exhibited at high ratios. It therefore means a sudden increase in load action, may be, when the intended use of a structure is altered, could seriously affect the reliability of joints designed in accordance to the provisions of Eurocode-5.

4.2. SHEAR CONSIDERATIONS.

In the joint area, the shear strength of timber is checked for the calculated force $F_{V,d}$.

the shear stress, $$\tau_V = \frac{3F_{V,d}}{2bh} < f_{V,d} \quad (5.1)$$

where

b is the thickness of timber.
h is the depth of timber.
$f_{V,d}$ is shear strength of timber.

Plots of computed safety indices for load combination C3 are at Figs 20 to 23.

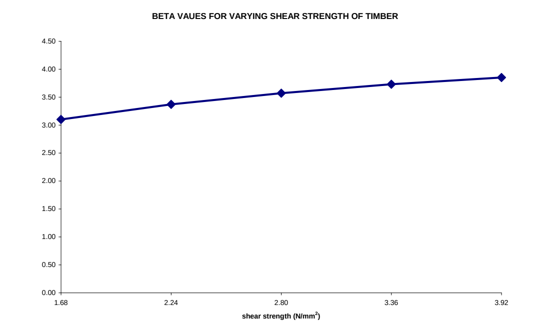

Fig. 20: Variation of β with shear strength of timber for shear consideration.

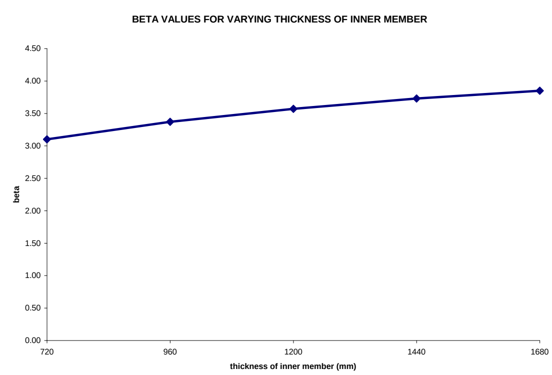

Fig. 21: Variation of β with the thickness of outer member of timber for shear consideration.

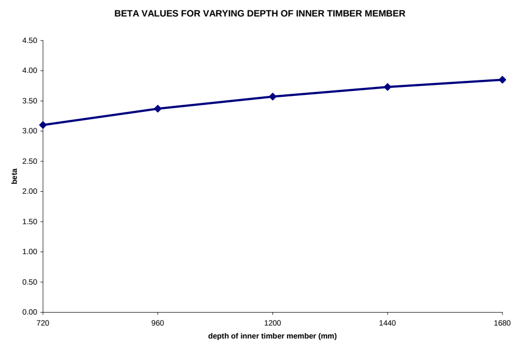

Fig. 22: Variation of β with the thickness of inner member of timber for shear consideration.

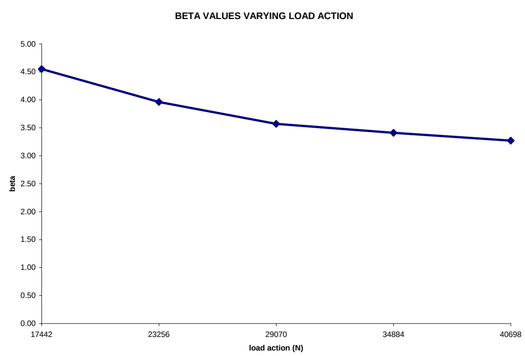

Fig. 23: Variation of β with the resultant load action for shear consideration.

The shear consideration showed some degree of consistency in the implied safety levels. This was observed with respect to variation in strength and geometrical properties. Also shear consideration gave a slightly higher reliability level. Therefore bending requirement is more critical than that of shear in the design of timber joints.

4.3 SIGNIFICANCE OF DESIGN PARAMETERS

All the basic variables except the load action are the parameters taken into consideration and the graphical plots for the sensitivity studies are shown I figs 24 to 27 for the respective failure modes. For each of this parameters considered, a central reliability level is taken as β_o and the corresponding value of the parameter is noted (reference point). For an increase above and below the reference point of the design parameter the reliability ratio β/β_0 is computed from the varying β values. Comparisons made in the plots for the design parameters follows.

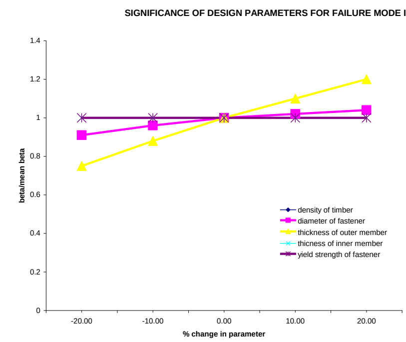

Fig. 24: Sensitivity Plot for Failure Mode I

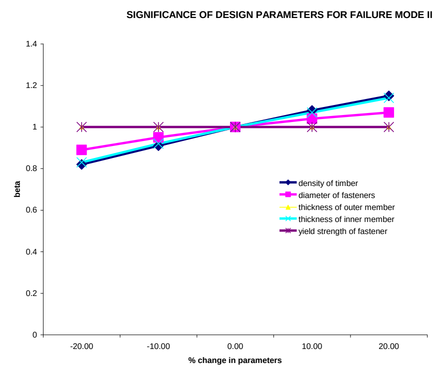

Fig. 25: Sensitivity Plot for Failure Mode II

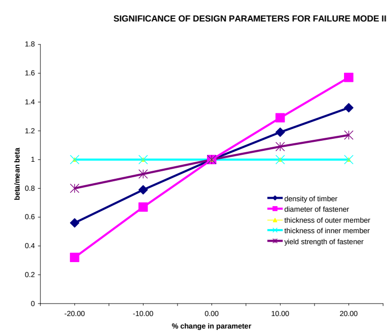

Fig. 26: Sensitivity Plot for Failure Mode III

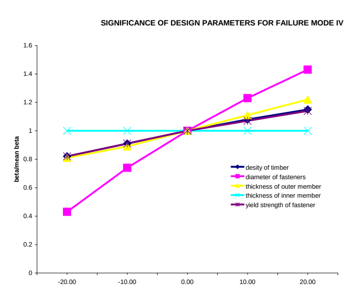

Fig. 27: Sensitivity Plot for Failure Mode IV

Considering the results obtained for failure mode I (fig. 24), 10% increase in the density of timber showed 10% change in reliability, an equal increase also showed 10% change in reliability with respect to the thickness of the outer member. The reliability levels of the thickness of the inner member and the yield strength of the fasteners remained constant while a change of 2% was observed with respect to the diameter of the fasteners. Therefore failure mode 1 provision revealed that density of timber and thickness of outer member are the most significant design parameters.

For failure mode II (fig. 25), a 10% increase in the design parameters showed no change in the reliability levels with respect to the thickness of the outer member and yield strength of the fasteners. The same increase result in 4% change in the reliability level with respect to fastener diameter, 7% with respect to the thickness of the inner member and 8% with respect to the density of timber. Thus the density of timber is the most significant design parameter with regards to failure mode II.

Remarkable changes were observed for failure mode III (fig. 26). For instance, a 10% increase in the design parameter showed 29% change in the reliability level with respect to the fastener diameter. The same increase gave 19% change with respect to the density of timber, 9% with respect to the yield strength of the fasteners and no change was shown for the thickness of the members. Therefore like the sensitivity study of failure mode I, the fastener diameter was the most significant design parameter in relation to failure mode III.

The sensitivity study of failure mode IV (fig. 27) also showed appreciable changes. A 10% increase in the parameters showed 23% change in reliability level with respect to the fastener diameter, 11% for the thickness of the outer member, 8% for the density of timber and 7% for the yield strength of the fasteners. There was no change in the reliability level for the thickness of the inner timber. The fastener diameter here also is the most significant design parameter.

With regards to the failure criteria in the design of timber joints, failure Mode I brought out the importance of the density of the timber and the thickness of the outer member. Failure Mode II also emphasises the density of timber but followed with the thickness of the inner member. While Modes III and IV brought out the prominence of the fastener diameter followed by the density of timber.

Consequently, the sensitivity plots revealed that, in term of geometrical properties, the diameter of fastener is most critical, the thickness of outer follows and the thickness of the inner member are less critical. In term of the material properties, the density of the timber is more critical than the yield strength of the fasteners.

CONCLUSION

The reliability levels associated with the bending and shear capacity were investigated. Eurocode5 provided four failure modes for bending requirement for timber joints in double shear. Therefore the mode that gives the highest value of stress due to the maximum load action determines the capacity of the joint. At the mean values of the design variables, β value for failure mode III is least at 1.10, mode IV gave 1.88, mode I was 2.66 and mode II gave 3.84. This therefore suggests that failure mode III is the most critical and in most cases determines the design capacity of the joint.

Most of the changes in the basic variables exhibit similar pattern except the resultant load action. Steady increase in β was observed with increase in the variables, while a decrease was shown by the load action. The ratios of the basic variables also followed similar trend. However, the ratios of fastener diameter to the thickness of the inner member, and resultant load action to the fastener diameter exhibit some levels of inconsistencies. While the change in β value for the former showed a parabolic curve form for failure modes I and II in the latter, the most critical failure mode changes as the ratio vary.

The shear consideration showed higher reliability levels than most of the bending requirement. At the mean values of the basic variables, a β of 3.59 was obtained for the shear consideration. The corresponding values for bending consideration were 1.10 for mode III, 1.88 for mode IV and 2.66 for mode I. However a higher value was gotten for mode

II (3.84). Therefore, since the least value of reliability level (i.e. highest probability of failure) is most critical, bending requirement is more crucial in the design of timber joints. Furthermore, load combination C3 (factored dead load and factored full snow load) is more critical than C2 (factored dead load and factored imposed load) because it gave a higher resultant load action. However, Load combination C2 was also investigated because it is what is obtainable in our local environment.

Consequently, it is observed that the minimum information required for the assessing the implied safety levels in design are the expected values (e.g. mean) of the design variable plus a measure of the uncertainty (e.g. the coefficient of variation). This information may be obtained from observed data coupled with professional engineering judgment.

In probability method of design, unforeseen changes could be simulated and their influence on safety and performance of a structure predicted. For instance, where the usage of the structure is altered a corresponding drastic change in the loading conditions may result. Gross errors occur in the dimensions of member sections induced during sizing procedures; fabrication procedure of the fastener at the rolling mill could also be faulty. So also are the erection procedures on site during construction. This method gives the associated measure of risk and will also allow for introduction of precautionary signs or measure regarding the limitations in the usage of the structure.

Sensitivity test on the significance of the design parameters revealed that the diameter of fastener is more critical for failures modes III and IV. The density of timber and the thickness of inner member are most critical for failure mode II. Lastly, the thickness of outer member is the most significant for failure mode I. These findings are depicted in figures 24 to 27. Possible deductions from the plots are that, the fastener diameter is the most significant parameter. The density of timber is next. Yield strength of the fastener follow, then the thickness of the outer member and lastly is the thickness of the inner member.

In conclusion the average computed safety index in the study is reasonable and within the range 2 to 4 established for code safety index [29]. However, the findings provide other useful information as follows:

(a) Fastener diameter is the most significant design parameter; the density of wood members has appreciable significance on the reliability of the timber joints too. The yield strength of the fasteners also has a considerable influence.
(b) Bending consideration is more critical in the design of timber joints than the shear consideration.

It is important to remark here that the stochastic finite element technique in this study provides an extremely useful procedure for evaluating those aspect of safety that are direct functions of material properties and assumed loading condition that are random in nature.

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