

Leveraging Deep Learning Techniques for the Stability Principles of Current Artificial Neural Networks Are Emerging Into Their Activation Functions.

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Abstract— Continuous-time recurrent neural network stability issues have been thoroughly researched. This paper aims to present a thorough analysis of the literature on the stability of continuous-time recurrent neural networks, encompassing models such as Cohen-Grasberg [1] and Hopfield neural networks. The stability results of recurrent neural networks with various classes of time delays are thoroughly examined, as time delays are an inherent part of real-world applications. The findings of dealing with the constant/variable delay in recurrent neural networks for the case of delay-dependent stability are compiled. Different forms, including algebraic inequality forms, -matrix forms, and linear forms, are produced by the relationship between stability. It is addressed and compared with Lyapunov diagonal stability forms and matrix inequality [2] forms. Additionally covered are certain adequate and essential stability requirements for recurrent neural networks in the absence of time delays. Finally, some thoughts are shared on the stability analysis of recurrent neural networks going forward.

Indexed Terms—Balance points, continuous neural networks, monotonic conduct and fixed point hypothesis.

I. INTRODUCTION

The case when activation functions in recurrent neural networks are continuous, limited, and strictly monotonically rising is addressed by the stability concepts of existence, uniqueness, and global asymptotic/exponential[3] stability of the equilibrium point. However, unbounded activation functions approximated by diode-like exponential-type functions are required to impose restrictions when recurrent neural networks are designed to solve optimization[4] issues in the presence of constraints (linear, quadratic, or more general programming problems). The literature proposes

expansions of the results with bounded activation functions [5] to reactivation functions in recurrent neural networks due to the differences between the bounded and unbounded

$$g'_i(\zeta) = d_{g_i}(\zeta)/d\zeta > 0, \lim_{\zeta \rightarrow +\infty} g_i(\zeta) = 1,$$

activation functions. Take note that artificial neural[6] networks can perform much better when they have an appropriate and more generalized activation function.

For recurrent neural networks, for instance, the activation function feature affects their capacity. showed that substituting a no monotonic activation function for the typical sigmoid activation function[7] can significantly increase the associative memory model's absolute capacity. Consequently, creating a new neural network with a more broadly applicable activation function is crucial. We will next go over a few different kinds of recurrent neural network activation functions.

Naturally, there are various kinds of activation functions in neural networks. The signum function, hyperbolic tangent function, piecewise linear functions, hard-limiter nonlinearity [8], and so forth are a few examples. The primary focus of the following is on Lipschitz-continuous activation functions and their variations.

1) The following sigmoidal activation functions have been used in

$$\lim_{\zeta \rightarrow -\infty} g_i(\zeta) = -1, \lim_{|\zeta| \rightarrow \infty} g'_i(\zeta) = 0.. (1)$$

Where $g_i(\cdot)$ the activation is function of th neuron, and is the number of neurons. Obviously, it is differentiable, monotonic, and bounded.

2) The following activation functions have been used in:

$$|g_i(\zeta) - g_i(\xi)| \leq \delta_i |\zeta - \xi|.. (2)$$

$$p_n = g(p_{n-1}) \quad p$$

Regardless of whether or not the activation function is bounded. As previously mentioned, this kind of activation function isn't always smooth and monotone.

3) The following activation functions have been employed:

$$0 < \frac{g_i(\zeta) - g_i(\xi)}{\zeta - \xi} \leq \delta_i \dots (3)$$

4) The following activation functions have been employed:

$$0 \leq \frac{g_i(\zeta) - g_i(\xi)}{\zeta - \xi} \leq \delta_i \dots (4)$$

5) The following activation functions are developed:

$$\delta_i^- < \frac{g_i(\zeta) - g_i(\xi)}{\zeta - \xi} \leq \delta_i^+ \dots (5)$$

As mentioned in δ_i^- and δ_i^+ , and can have a value of zero, negative, or positive. Those are then exceptional circumstances, based on the previously utilized Lipschitz conditions. comparisons between different continuous activation function classes.

The existence and uniqueness of the equilibrium point in recurrent neural networks is one of the key concepts related to activation functions $|g_i(x_i)| \leq \frac{M}{p}$. We will now make a quick observation

regarding this issue. Mainly, the fixed-point theorem is used to establish the existence of the equilibrium point for the bounded activation function $|g_i(x_i)| \leq \delta_i^0 |x_i| + \sigma_i^0$ or the quasi-Lipschitz activation[9] function (which may be unbounded), where δ_i^0 and σ_i^0 are constants.

II. FIXED POINT THEOREM

$$\frac{g(p) - g(q)}{p - q} = g'(\xi)$$

Let $g \in C[a, b]$ be such that $g(x) \in [a, b]$ for all $x \in [a, b]$. Suppose, in addition, that g' existence on $[a, b]$. And $0 < k < 1$

$$|p_n - p| = |g(p_{n-1}) - g(p)| = |g'(\xi)| |p_{n-1} - p| \leq k |p_{n-1} - p|$$

$$p_n = g(p_{n-1})$$

(a, b) and that a constant exists with

$$|p_n - p| \leq |p_{n-1} - p| \leq k^2 |p_{n-2} - p| \leq \dots k^n |p_0 - p|$$

$$|g'(x)| \leq k \text{ for all } x \in (a, b)$$

Then, $g(x)$ has a unique fixed point in $[a, b]$. Further, for any number in $[a, b]$, the sequence defined by

Converges to the unique fixed point in $[a, b]$.

Evidence:

III. EVIDENCE

First, we demonstrate that has at least one $g(x)$ fixed point in $[a, b]$. We are done if or. If not, present $g(a) = a$ $g(b) = b$

Observe that $h(a) = g(a) - a > 0$ and because and $h(b) = g(b) - b < 0$ is confined within. For some x in, according to the intermediate value theorem. Keep in mind that a fixed point of is a root of.

We then demonstrate that there can never be more than one fixed point in. Let us assume, for the sake of contradiction, that and are both fixed points for in with. The Mean Value Theorem[10] gives us the following

For some ξ between ξ and q . Now note that

$$|p - q| = |g(p) - g(q)| = |g'(\xi)| |p - q| < |p - q|$$

For some ξ between ξ and \cdot . Now note that

Which is a contradiction. Thus any fixed point in $[a, b]$ must be unique.

p_n Now consider the sequence of iterated points p_n . Since maps $[a, b]$ to $[a, b]$, there is no problem with the sequence wandering[5] out of the interval. To show that it converges to p we use the fact that $|g'(x)| \leq k$ for all $x \in (a, b)$ and the Mean Value Theorem to show

Iterating this observation leads to

$$\lim_{n \rightarrow \infty} |p_n - p| \leq \lim_{n \rightarrow \infty} k^n |p_0 - p| = 0$$

In general, for the case of bounded activation functions satisfying Lipschitz continuous conditions, the existence of the

solution can be guaranteed by the existence theorem of ordinary differential equations.

The presence of the equilibrium point for the unbounded activation function in the general form is primarily determined by the Leray-Schauder principle, homeomorphism mapping, and other related concepts.

Whether the existence, uniqueness, and global asymptotic/exponential stability must be addressed simultaneously within the stability analysis of recurrent neural networks is another crucial issue related to activation functions. In the early days of the stability theory of neural networks, this subject was frequently asked and there was no consensus on it. Two groups of methods for the stability analysis of recurrent neural networks[12] are derived from this problem

- 1) To provide the worldwide asymptotic/exponential stability results directly, without requiring the equilibrium point's existence and uniqueness to be proven; and
- 2) To provide comprehensive evidence supporting the equilibrium point's existence, uniqueness, and global asymptotic/exponential stability. It is obvious that this issue needs to be resolved before moving further with the stability analysis of recurrent neural networks.

From a mathematical perspective, the proof of stability requires proving the existence (and, if relevant, uniqueness) of equilibrium point or points. However, one may choose a somewhat alternative treatment plan for the equilibrium point's stability proof based on varying criteria for the activation function.

Since the existence of the equilibrium[11] point is always guaranteed by the bounded activation function, we may immediately offer the proof of the global asymptotic/exponential stability for the general case of bounded activation functions in recurrent neural networks. As in the case of bounded activation functions, the existence of an equilibrium point is likewise ensured for the quasi-Lipschitz scenario. For recurrent neural networks with bounded activation functions, it is therefore sufficient to demonstrate the global asymptotic/exponential stability of the equilibrium point; the uniqueness of the equilibrium point follows immediately from this global asymptotic/exponential stability.

CONCLUSION

In conclusion, over the past thirty years, a lot has been accomplished in the field of stability studies for recurrent

neural networks with or without time delays. There are still a lot of unsolved issues, though. The advancement of mathematical theory will be accompanied by all of these upcoming discoveries, particularly in computational and applied mathematics. Remember that each type of stability criterion has a viable range of its own, and it is unrealistic to think that a small number of stability outcomes will address every stability issue that arises in recurrent neural networks. Each class of stability results has advantages of its own, such as algebraic inequality, LDS, -matrix, and LMI. It has taken into account various trade-offs between the efficiency of stability outcomes and computational complexity. Different elements of the recurrent neural networks involved are reflected in the stability results, and no form of stability results is inherently better than any other form. Thus, the growth of the stability theory of recurrent neural networks is aided by several ways in which stability results can be expressed.

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