© JUN 2024 | IRE Journals | Volume 7 Issue 12 | ISSN: 2456-8880

A New Class of Bounded Analytic Function

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Abstract- In this paper, we defined a new class of bounded analytic functions and explained Fekete – Szegö Inequality for the same.

Indexed Terms- Bounded analytic functions, Fekete – Szegö Inequality, concept of subordination and Starlike functions.

I. INTRODUCTION

Fekete – Szegö Inequality derived from a conjecture, called Bieberbach conjecture. This conjecture was introduced by Bieberbach but proved by Louis De Branges. Many researches has been done till now on this inequality for different classes and many more is going on.

In order to come to our result, firstly, we define some fundamental terms.

Class A and class S contain analytic and univalent functions with the normalization $f(0) = 0$, $f'(0) = 1$ and functions included in these classes, are of type $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$; in the open disk $E = \{ z \in C : |z| < 1 \}$.

This work is based on the concept of subordination, according to which, if two functions let $t(z)$ and $h(z)$ are such that they are analytic to each other, then there exists a bounded analytic function in the unit disc E with normalization $|w(z)| < 1$, $w(0) = 0$ and $f(z) = g(w(z))$; $z \in E$. In this situation, we say that $t(z)$ is subordinate to $w(z)$ and we write it as $t(z) \prec h(z)$.
The principle of subordination was given by Lindelof [10] and bounded analytic functions are the functions of the type $w(z) = \sum_{n=1}^{\infty} c_n z^n$ with normalization conditions $w(0) = 0$ and $|w(z)| < 1$.

Miller et. al. gave the necessary conditions for these type of functions, as follows

$$|c_1| \leq 1, \; |c_2| \leq 1 - |c_1|^2$$

The class defined below, denoted by A_γ , is a subclass of S^* , so satisfying the condition

$$\left\{(1-\alpha)\frac{f(z)}{z} + \alpha\,\frac{z f'(z) + \beta z^2 f''(z)}{f(z)}\right\} \prec \phi(z)$$

The necessary and sufficient condition for any function to be starlike is

$$Re\left(\frac{z f'(z)}{f(z)}\right) > 0; z \in E$$

which was introduced by Duren and the result $|a_n| \leq n$ for univalent starlike functions was introduced by Nevanlinna.

In this paper, we define the class A_γ along with some corollaries.

My work is related to the work of Keogh and Merkes with parameters $\alpha = 1$, $\beta = 0$ and $\gamma = 0$.

II. MAIN RESULTS

Theorem 2.1 Let $f(z) \in A_\gamma$ and $\phi(z) = \frac{1+w(z)}{1-w(z)}$; $w(z)$ is a *Schwarzian function* , *then* $|a_3 - \mu\, a_2^2| \leq$

$$\begin{cases} \dfrac{(\gamma+2)^2(1+2\alpha\beta)^2+2\alpha(1+2\beta)(\gamma+2)^2}{2(1+2\alpha\beta)^2\{1+(1+6\beta)\alpha\}} - \dfrac{\mu(\gamma+2)^2}{(1+2\alpha\beta)^2}\,; \\ \quad \mu \leq \dfrac{(1+2\alpha\beta)^2}{(\gamma+2)^2}\left[\dfrac{\gamma(\gamma+2)}{2\{1+(1+6\beta)\alpha\}} + \dfrac{\alpha(1+2\beta)(\gamma+2)^2}{\{1+(1+6\beta)\alpha\}(1+2\alpha\beta)^2}\right]; \\ \dfrac{\gamma+2}{1+(1+6\beta)\alpha}\,; \; \dfrac{(1+2\alpha\beta)^2}{(\gamma+2)^2}\left[\dfrac{\gamma(\gamma+2)}{2\{1+(1+6\beta)\alpha\}} + \dfrac{\alpha(1+2\beta)(\gamma+2)^2}{\{1+(1+6\beta)\alpha\}(1+2\alpha\beta)^2}\right] \\ \quad \leq \mu \leq \dfrac{(1+2\alpha\beta)^2}{(\gamma+2)^2}\left[\dfrac{(\gamma+4)(\gamma+2)}{2\{1+(1+6\beta)\alpha\}} + \dfrac{\alpha(1+2\beta)(\gamma+2)^2}{\{1+(1+6\beta)\alpha\}(1+2\alpha\beta)^2}\right]; \\ \dfrac{\mu(\gamma+2)^2}{(1+2\alpha\beta)^2} - \dfrac{(\gamma+2)^2(1+2\alpha\beta)^2+2\alpha(1+2\beta)(\gamma+2)^2}{2(1+2\alpha\beta)^2\{1+(1+6\beta)\alpha\}}\,; \\ \quad \mu \geq \dfrac{(1+2\alpha\beta)^2}{(\gamma+2)^2}\left[\dfrac{(\gamma+4)(\gamma+2)}{2\{1+(1+6\beta)\alpha\}} + \dfrac{\alpha(1+2\beta)(\gamma+2)^2}{\{1+(1+6\beta)\alpha\}(1+2\alpha\beta)^2}\right] \end{cases}$$

The result is sharp.

Proof By definition of A_γ

$$(1-\alpha)\frac{f(z)}{z} + \alpha\,\frac{z f'(z) + \beta z^2 f''(z)}{f(z)} = \frac{1+w(z)}{1-w(z)}\, e^{\gamma w(z)} \tag{2.1}$$

where $w(z) = c_1 z + c_2 z^2 + c_3 z^3 + \ldots$
and $f(z) = z + a_2 z^2 + a_3 z^3 + \ldots$
By putting the values of $f(z)$ and $w(z)$ in (2.1) , we get

$(1-\alpha)\frac{z+a_2z^2+a_3z^3+\cdots}{z}+\alpha\frac{z\{1+2a_2z+3a_3z^2+\cdots+\beta(2a_2z+6a_3z^2+\cdots)\}}{z+a_2z^2+a_3z^3+\cdots}$

$=\frac{1+c_1z+c_2z^2+\ldots}{1-(c_1z+c_2z^2+\ldots)}e^{\gamma(c_1z+c_2z^2+c_3z^3+\cdots)}$

Expanding the series, we get

$1+(1+2\alpha\beta)a_2z+[1+\alpha(1+6\beta)a_3]z^2-\alpha(1+2\beta)a_2^2z^2+\ldots$

$=1+(\gamma+2)c_1z+\left(\gamma c_2+\frac{1}{2}\gamma^2c_1^2+2\gamma{c_1}^2+2c_2+2{c_1}^2\right)z^2+\cdots$

By comparing the coefficients, we have

$a_2=\frac{(\gamma+2)c_1}{1+2\alpha\beta}$ and $a_3=\frac{(\gamma+2)c_2+[\left(\frac{1}{\sqrt{2}}\gamma+\sqrt{2}\right)^2+\left(\frac{\alpha(1+2\beta)(\gamma+2)^2}{(1+2\alpha\beta)^2}\right)]c_1^2}{1+(1+6\beta)\alpha}$

So, we get

$a_3-\mu a_2^2=\frac{(\gamma+2)c_2+[\left(\frac{1}{\sqrt{2}}\gamma+\sqrt{2}\right)^2+\left(\frac{\alpha(1+2\beta)(\gamma+2)^2}{(1+2\alpha\beta)^2}\right)]c_1^2}{1+(1+6\beta)\alpha}-\mu\left[\frac{(\gamma+2)c_1}{1+2\alpha\beta}\right]^2$

Applying mode on both sides, we get

$|a_3-\mu a_2^2|\le\frac{\gamma+2}{1+(1+6\beta)\alpha}|c_2|+\frac{[\left(\frac{1}{\sqrt{2}}\gamma+\sqrt{2}\right)^2+\left(\frac{\alpha(1+2\beta)(\gamma+2)^2}{(1+2\alpha\beta)^2}\right)]}{1+(1+6\beta)\alpha}|c_1|^2-\mu\left[\frac{(\gamma+2)|c_1|}{1+2\alpha\beta}\right]^2.$

Using $|c_2|\le 1-|c_1|^2$, we get

$|a_3-\mu a_2^2|\le\frac{\gamma+2}{1+(1+6\beta)\alpha}+\left\{\left|\frac{[\left(\frac{1}{\sqrt{2}}\gamma+\sqrt{2}\right)^2]}{1+(1+6\beta)\alpha}+\left(\frac{\alpha(1+2\beta)}{1+(1+6\beta)\alpha}-\mu\right)\frac{(\gamma+2)^2}{(1+2\alpha\beta)^2}\right|-\frac{\gamma+2}{1+(1+6\beta)\alpha}\right\}|c_1|^2.$

Case 1 If $\mu\le\frac{(1+2\alpha\beta)^2}{(\gamma+2)^2}\left[\frac{[\left(\frac{1}{\sqrt{2}}\gamma+\sqrt{2}\right)^2]}{1+(1+6\beta)\alpha}+\left(\frac{\alpha(1+2\beta)}{\{1+(1+6\beta)\alpha\}}\right)\frac{(\gamma+2)^2}{(1+2\alpha\beta)^2}\right].$

Then, $|a_3-\mu a_2^2|\le\frac{\gamma+2}{1+(1+6\beta)\alpha}+\left\{\frac{\gamma+2}{1+(1+6\beta)\alpha}\left(\frac{\gamma}{2}\right)+\left(\frac{\alpha(1+2\beta)}{\{1+(1+6\beta)\alpha\}}-\mu\right)\frac{(\gamma+2)^2}{(1+2\alpha\beta)^2}\right\}|c_1|^2.$

Subcase 1(a) When $\mu\le\frac{(1+2\alpha\beta)^2}{(\gamma+2)^2}\left[\frac{\gamma(\gamma+2)}{2\{1+(1+6\beta)\alpha\}}+\frac{\alpha(1+2\beta)(\gamma+2)^2}{\{1+(1+6\beta)\alpha\}(1+2\alpha\beta)^2}\right]$

Then, by using $|c_1|\le 1$, we get

(2.2) $|a_3-\mu a_2^2|\le\frac{(\gamma+2)^2(1+2\alpha\beta)^2+2\alpha(1+2\beta)(\gamma+2)^2}{2(1+2\alpha\beta)^2\{1+(1+6\beta)\alpha\}}-\frac{\mu(\gamma+2)^2}{(1+2\alpha\beta)^2}$

.

Subcase 1(b) When $\mu\ge\frac{(1+2\alpha\beta)^2}{(\gamma+2)^2}\left[\frac{\gamma(\gamma+2)}{2\{1+(1+6\beta)\alpha\}}+\frac{\alpha(1+2\beta)(\gamma+2)^2}{\{1+(1+6\beta)\alpha\}(1+2\alpha\beta)^2}\right]$.

Then,

(2.3) $|a_3-\mu a_2^2|\le\frac{\gamma+2}{1+(1+6\beta)\alpha}.$

Case 2 If $\mu\ge\frac{(1+2\alpha\beta)^2}{(\gamma+2)^2}\left[\frac{[\left(\frac{1}{\sqrt{2}}\gamma+\sqrt{2}\right)^2]}{1+(1+6\beta)\alpha}+\left(\frac{\alpha(1+2\beta)}{\{1+(1+6\beta)\alpha\}}\right)\frac{(\gamma+2)^2}{(1+2\alpha\beta)^2}\right].$

Then, $|a_3-\mu a_2^2|\le\frac{\gamma+2}{1+(1+6\beta)\alpha}+\left\{\mu\frac{(\gamma+2)^2}{(1+2\alpha\beta)^2}-\frac{\alpha(1+2\beta)}{\{1+(1+6\beta)\alpha\}}\left(\frac{(\gamma+2)^2}{(1+2\alpha\beta)^2}\right)-\frac{\gamma+2}{1+(1+6\beta)\alpha}\left(\frac{\gamma+4}{2}\right)\right\}|c_1|^2.$

Subcase 2(a) When $\mu\ge\frac{(1+2\alpha\beta)^2}{(\gamma+2)^2}\left[\frac{(\gamma+4)(\gamma+2)}{2\{1+(1+6\beta)\alpha\}}+\frac{\alpha(1+2\beta)(\gamma+2)^2}{\{1+(1+6\beta)\alpha\}(1+2\alpha\beta)^2}\right].$

Then,

(2.4) $|a_3-\mu a_2^2|\le\frac{\mu(\gamma+2)^2}{(1+2\alpha\beta)^2}-\frac{(\gamma+2)^2(1+2\alpha\beta)^2+2\alpha(1+2\beta)(\gamma+2)^2}{2(1+2\alpha\beta)^2\{1+(1+6\beta)\alpha\}}.$

Subcase 2(b) When $\mu\le\frac{(1+2\alpha\beta)^2}{(\gamma+2)^2}\left[\frac{(\gamma+4)(\gamma+2)}{2\{1+(1+6\beta)\alpha\}}+\frac{\alpha(1+2\beta)(\gamma+2)^2}{\{1+(1+6\beta)\alpha\}(1+2\alpha\beta)^2}\right].$

Then,

(2.5) $|\boldsymbol{a_3}-\mu\,\boldsymbol{a_2^2}|\le\frac{\boldsymbol{\gamma+2}}{\boldsymbol{1+(1+6\beta)\alpha}}.$

Combining (2.2), (2.3), (2.4) and (2.5), we get the required result.

Corollary 2.1 $A_0=S^*$, as after putting $\alpha=1$, $\beta=0$ and $\gamma=0$, the result becomes

$$|a_3-\mu a_2^2|\le\begin{cases}3-4\mu, & if\ \mu\le\frac{1}{2};\\ 1, & if\ \frac{1}{2}\le\mu\le 1;\\ 4\mu-3, & if\ \mu\ge 1.\end{cases}$$

which is the required result for the class S^* given by Keogh and Merkes.

Corollary 2.2 $A_0=TS^*[\alpha,\beta]$, as by substituting $\gamma=0$, the result becomes

$$|a_3-\mu a_2^2|\le\begin{cases}\frac{2(1+2\alpha+4\beta^2\alpha^2+8\alpha\beta)}{(1+2\alpha\beta)^2\{1+(1+6\beta)\alpha\}}-\frac{4\mu}{(1+2\alpha\beta)^2}; & \mu\le\frac{\alpha(1+2\beta)}{1+(1+6\beta)\alpha};\\ \frac{2}{1+(1+6\beta)\alpha}; & \frac{\alpha(1+2\beta)}{1+(1+6\beta)\alpha}\le\mu\le\frac{\alpha(1+2\beta)+(1+2\alpha\beta)^2}{[1+(1+6\beta)\alpha]};\\ \frac{4\mu}{(1+2\alpha\beta)^2}-\frac{2(1+2\alpha+4\beta^2\alpha^2+8\alpha\beta)}{(1+2\alpha\beta)^2[1+(1+6\beta)\alpha]}; & \mu\ge\frac{\alpha(1+2\beta)+(1+2\alpha\beta)^2}{[1+(1+6\beta)\alpha]}.\end{cases}$$

which is the required result for the class $TS^*[\alpha,\beta]$ proved by Misha Rani.

CONCLUSION

On the basis of principle of subordination, in this paper, we derived Fekete – Szegö inequality for the functions of a subclass of class *S*. Here, we explained a new class A_γ with parameters α, β and γ along with corollaries of latest researches.

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