Mathematical Modeling for Geometric Design and Calibration of Open Frustum Tank for Expandable Liquid

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Abstract- In order to proffer solution to the sudden failure of liquid tanks, which has been an industrial challenge, this paper presents a proposed mathematical modeling for geometric design and calibration of frustum tank that can hold expandable liquid at elevated temperature and accommodate increase in liquid volume due to temperature rise. In the design, the tank is presented in three segments; the first segment represents the reserved liquid volume below the datum line, the second segment represents the actual liquid volume and the third segment represents the minimum volume of free space above the liquid level, provided to compensate for liquid expansion. Another design condition is that the liquid levels must not fall below the datum. Based on these conditions, mathematical equations have been derived for calculating tank dimensions, volumes of the various tank segments, total tank volume and calibration of the actual liquid volume. By iteration method, the proposed model was used for geometric design of an open frustum tank of 30000litres actual liquid capacity. The dimensions obtained, were used to calculate the total tank volume, which yielded approximately 35000litres. The result of the calibration shows a non-linear relationship between the tank instantaneous height and the corresponding instantaneous volume. It was concluded that open frustum tank of expandable liquid can be designed if the actual liquid capacity and the dimensions of the tank below the datum are known.

Indexed Terms- Geometric, Design, Calibration, Frustum tank

I. INTRODUCTION

1.1 Background of Study

Liquid tanks are used in many industries and homes for processing and storage of liquids. Industries that make use of liquid tanks include Iron and steel Making industries [1], Bottling companies [2], Pharmaceutical industries [3], Water utility [4, 5] and Petroleum industries [6]. Previous works have shown the importance and wide application of liquid tanks but unfortunately, cases of tank failure have been reported frequently and explosion has more often been reported as the failure mode. According to Varkey and Reigh [7, 8], tank explosion can be caused by expansion of the fluid content of the tank.

Another reported cause of liquid tank explosion is overfilling of the tank with the liquid content [9]. Overfilling of tanks cause excessive internal pressure and explosion [9]. Expansion of liquid in a tank increases tank internal pressure and can as well lead to spill or liquid explosion [7, 8]. It is deemed that proper tank design, which factors in the expansion behaviour of the liquid will solve the problem of tank explosion due to liquid expansion.

Also, calibration of liquid tanks will help in regulating the volume of liquid in the tank; thereby solve the problem of tank explosion due to overfilling. Tank calibration makes way for easy accountability of the liquid and remedies the

dispensing issues often recorded in industries. Therefore, this study is focused on geometric design and calibration of frustum tank of expandable liquid, considering the expansion behaviour of the liquid.
Tank design may be viewed in two aspects: the geometric design and thickness design [10] Geometric design involves dimensional detailing of the tank considering some factors, such as the expansion behaviour of the fluid while thickness design involves the determination of the most appropriate shell thickness that can withstand harsh environmental factors, fluid factors and standards of operation (SOP) within the service life of the tank.

Tank calibration is another measure that helps in efficient running of the tank and enhances productivity. Unfortunately, many industries and homes use liquid tanks made out of design. The consequences of such practice include structural failure, explosion and fire outbreak [11]. The shape of liquid tank may be cube, sphere, cone, cylinder or frustum.

The causes of tank failure, the associated consequences and preventive measures have been discussed in several published articles. Kletz [12] reported the failure of fibre-reinforced-plastic (FRP) liquid tanks. From the investigation reports, poor design was among the causes of failure. The stated consequences include structural damage and loss of the liquid content. According to Kletz [12] tank failure can be preventive by proper design, proper usage, regular inspection and proper identification and control of failure factors. In Trebuna et al. [13], the failure of hot water tank was studied. The investigation reports show that overload, poor design and corrosion are the causes of failure.

In another published article [14], the failure modes of storage tanks stated include:

i. Failure based on pumping,
ii. Failure based on material,
iii. Failure based on service,
iv. Mechanical, civil, or electrical failure, etc.

In the article [14], the listed causes of tank failure include:

i. Corrosion
ii. Improper construction
iii. Poor maintenance
iv. Incompatibility of fluid with tank wall
v. Dispensing problems
vi. Lack of physical safety
vii. Excessive pressure due to over filling
viii. Violent weather change
ix. Seismic design failure and others.

A similar study was conducted by Dev [16] and he gave consequences of tank failure as structural damage and loss of the liquid tank. He gave preventive measures as proper design, proper material selection, proper construction and regular inspection.

1.2 Aim/Objectives of Study

The aim of this study is to develop mathematical model for geometric design and calibration of open frustum capable of holding liquid at high temperature, accommodate rise in liquid volume due to increase in temperature. The specific objectives include:
1.To derive equations for calculating the dimensions of frustum tank of given capacity.
2.To derive equation for calculating the instantaneous volume of the tank.
3.To design and calibrate a 30000litres frustum tank for holding expandable at elevated temperature.

1.3 Tank Design

Previous studies have shown that one of the most common causes of tank failure is poor design. Several factors are considered for proper tank design. High environmental temperature will result in liquid expansion whereas high wind load can affect the structural stability of the tank especially for overhead tanks. Environmental temperature and pressure varies with altitude, therefore, based on usage locations, tanks are classified as overhead (above ground surface), surface and underground (beneath the earth) tanks [15, 16].

Steckler [17], gave seven considerations for tank design as follows:

1. Design Configurations
2. Manufacturing process
3. Coating
4. Tank construction
5. Maintenance/Life time value
6. Flexibility
7. Expandability

Different researchers have shown various methods adopted for tank design and the factors considered are based on design objectives. Azzuni and Guzey [18] showed the use of linear elastic analysis for design of cylindrical steel tank. Mohamed [19] gave the design of concrete cylindrical and rectangular tanks in which the total cost of the tank was considered. The design variables were length and width (for the rectangular tank), water depth for (cylindrical tank) and unit weight of water and floor slab thickness for both the cylindrical and rectangular tank. The tanks were designed in accordance to Indian standard (IS: 456-200) using a developed computer program for analysis.

The Design of cylindrical concrete water tank has been illustrated by Thevendran and Thambiratnam [20]. In the design, the internal radius and height of water tank were kept constant while the thickness of the tank was varied along the axis to determine the allowable bending and hoop stresses. The analysis was done using numerical method. The design of cylindrical tanks using STAAD pro Software and Intergraph software and have also been presented in other studies [16, 21].

Liquid in a tank subjects the internal walls of the tank to hydrostatic pressure (or stress) which can lead to yielding of the shell. This mode of failure can be prevented by calculating the thickness of the shell which can withstand the hydrostatic pressure. This can be achieved using One-Foot Method (IFM) or Variable Design point Method (VDM) [22]. One-foot method is applicable to tanks whose height is up to 200ft but less in diameter [22].

Designs of cylindrical and rectangular tanks have been discussed in previous publications [18 - 21] but none of the cited works presented frustum tank design. Frustum tank seem to have some advantages over other shapes. For instance, where there are environmental constraints, frustum tank will occupy a smaller space at its base compared to cylindrical or rectangular tank of the same capacity. Farshad et al. [23] illustrated the use of conical frustum container for heat transfer fluid for enhanced thermal energy storage in a diesel engine cogeneration system. Another study [24] illustrated the theoretical modeling of flow for frustum-shaped ceramic water filters. More applications of frustum shaped containers such as boilers and oil reservoirs are envisaged, which is the motivation for this work.

II. METHODS

2.1 Modeling of Frustum Tank

The modeling of frustum tank presented in this paper is based on the geometry of frustum. Liquid tanks of various shapes have been discussed but there was no mention of frustum tank design in the previous works.

Marco and Lemay [25] presented various types of frustum as given in Figure 1 but only conical frustum (type A) is considered in this work with schematic diagram given as Figure 2.

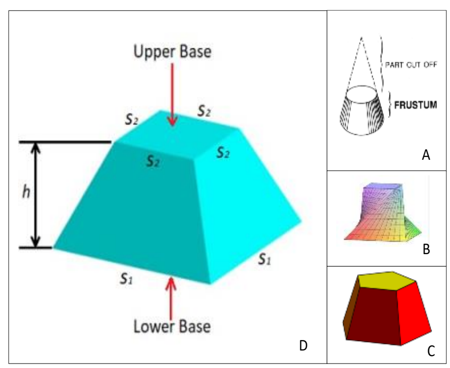


Figure 1: Various types of frustum [25]

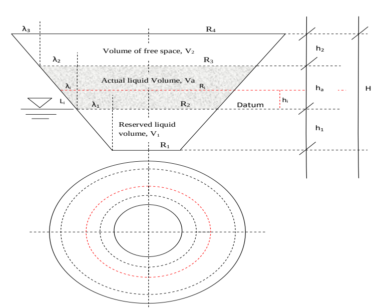


Figure 2: Schematic diagram of frustum dimensions

From Figure 2, the following relationships were derived:

$$\lambda_1 = R_2 - R_1; \lambda_2 = R_3 - R_2$$

$$\lambda_3 = R_4 - R_3; \lambda_i = R_i - R_2$$

$$\frac{\lambda_1}{h_1} = \frac{\lambda_2}{h_a} = \frac{\lambda_3}{h_2} = \frac{\lambda_i}{h_i} \quad (1)$$

$$R_2 = R_1 + \lambda_1 \quad (2)$$

$$R_3 = R_2 + \lambda_2 = R_2 + \frac{h_a}{h_1}\lambda_1 \quad (3)$$

$$R_4 = R_3 + \lambda_3 = R_3 + \frac{h_2}{h_1}\lambda_1 \quad (4)$$

$$R_i = R_2 + \lambda_i = R_2 + \frac{h_i}{h_1}\lambda_1 \quad (5)$$

Where R_i is instantaneous radius of liquid surface h_i is instantaneous height of liquid from the datum level, h_1 is the height of liquid below the datum line h_a is the maximum height of liquid from the datum line h_2 is the minimum height of free space above the liquid surface (which compensates for liquid expansion) and H_t is the total height of the tank. According to Marco and Lemay [25], volume of frustum is given by

$$V = \frac{\pi H}{3}\left(R^2 + Rr + r^2\right) \quad (6)$$

Where H the height and R is the radius of the larger cross-section and r is the radius of the smaller cross-section.

In Figure 2, V_1 is the volume bounded by R_1 and R_2 lines; V_a is the actual liquid volume of bounded by R_2 and R_3 lines; V_2 is the volume of free space bounded by R_3 and R_4 lines; V_t is the total tank volume bounded by R_1 and R_4 lines and V_i is the instantaneous volume bounded by R_2 and R_i,

Substituting r and R as shown in Eq. (6) respectively with R_1 and R_2 as shown in Figure 2 gives Eq. (7).

$$V_1 = \pi\frac{h_1}{3}(R_1^2 + R_1R_2 + R_2^2) \quad (7)$$

Substituting r and R as shown in Eq. (6) respectively with R_2 and R_3 as shown in Figure 2 gives Eq. (8).

$$V_a = \pi\frac{h_a}{3}(R_3^2 + R_3R_2 + R_2^2) \quad (8)$$

When the tank is filled to its actual liquid capacity, , at minimum running temperature, , the total liquid volume in the tank is . When the liquid is heated, its volume increases in accordance to the kinetic theory of matter. Also, when the liquid is heated to the maximum running temperature its volume increases to a maximum point, given by Eq. (9), where γ is coefficient of cubic expansion of the liquid.

$$\begin{aligned} V_2 &= \gamma(V_1 + V_a)(T_{max} - T_{min}) \\ &= \pi\frac{h_2}{3}(R_3^2 + R_3R_4 + R_4^2) \end{aligned} \quad (9)$$

Substituting h, r and R in Eq. (6) respectively with H_t, R_1 and R_4 as shown in Figure 2 gives Eq. (10).

$$\begin{aligned} V_t &= \pi\frac{H_t}{3}(R_4^2 + R_4R_1 + R_1^2) \\ &= V_1 + V_a + V_2 \end{aligned} \quad (10)$$

Substituting h, r and R in Eq. (6) respectively with h_i, R_2 and R_i in Figure 2 gives Eq. (11).

$$V_i = \pi\frac{h_i}{3}(R_i^2 + R_iR_2 + R_2^2) \quad (11)$$

From Figure 2 the tank total height is deduced using Eq. (12).

$$H_t = h_1 + h_a + h_2 \qquad (12)$$

Calibration*:* To calibrate the actual liquid volume, h_a is divided into n equal intervals, which gives instantaneous height, h_i , i.e.

$$h_i = i\frac{h_a}{n} \qquad (13)$$

Where n is a positive whole integer, $i = 1, 2, 3, \ldots n$ and h_i is instantaneous liquid height.

Substituting the h_i given in Eq. (13) into Eq. (5) gives instantaneous liquid radius, R_i expressed by Eq. (14)

$$R_i = R_2 + i\frac{h_a}{nh_1}\lambda_1 \qquad (14)$$

Now, eliminating h_i and R_i in Eq. (11) using Eq. (13) and Eq. (14) gives Eq. (15).

$$V_i = i\frac{\pi h_a}{3n}\left(\begin{array}{l}\left(R_2 + i\frac{h_a}{nh_1}\lambda_1\right)^2 + \\ \left(R_2 + i\frac{h_a}{nh_1}\lambda_1\right)R_2 \\ + R_2^2\end{array}\right) \qquad (15)$$

2.1 Application of the Proposed Model

The proposed model has been used for design and calibration of 30000litres frustum tank. The design procedure is given as follows:

Assumed parameters:

$$T_{max} = 464^o C;\ T_{min} = 25^o C; \gamma = 2.142\times10^{-4} K^{-1}$$

$$R_1 = 137\,cm;\ R_2 = 147\,cm;\ h_1 = 30\,cm;$$

$$V_a = 30000\,litres$$

Iteration 1

$$\lambda_1 = R_2 - R_1 = 10cm$$

Substituting the values of R_1, R_2 and h_1 into Eq. (7) gives

$$V_1 = 1901963cm^3 \approx 1901.96 litres$$

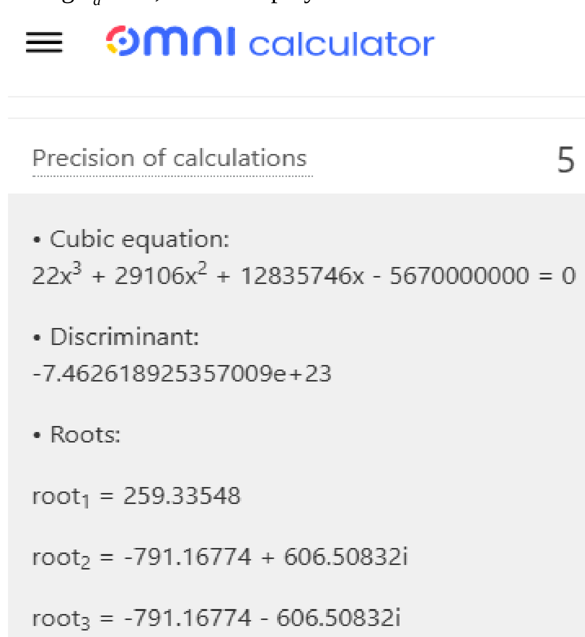
Substituting the vales of h_1 and λ_1 into Eq. (3) gives

$$R_3 = 147 + \frac{h_a}{3}$$

Substituting the values of V_a, R_2, R_3 and $\pi = 22/7$ into Eq. (8) gives

$$30000000 = \frac{22}{21}h_a\left(\begin{array}{l}\left(147 + \frac{h_a}{3}\right)^2 + \\ 147\left(147 + \frac{h_a}{3}\right) \\ + 147^2\end{array}\right)$$

The above equation was simplified and solved using Online Omni Cubic Equation Calculator [26], taking $h_a = x$, which displayed results as follows:



The only positive root is 259.33548cm.

$$\therefore\ h_a \approx 259.34cm \text{ and } R_3 \approx 233.445cm.$$

Iteration 2

Equation (9) was used to calculate V_2, i.e.

$$V_2 = \gamma(V_1 + V_a)(T_{max} - T_{min})$$

$$\Delta T = T_{max} - T_{min} = 464 - 25 = 439\,K$$

$$V_2 = \frac{2.142}{10000} \times 439 \times (1901.96 + 30000)$$

$$\approx 3000 \text{ litres}$$

Substituting the values of R_3, h_1 and λ_1 into Eq. (4) gives

$$R_4 = 233.445 + \frac{h_2}{3}$$

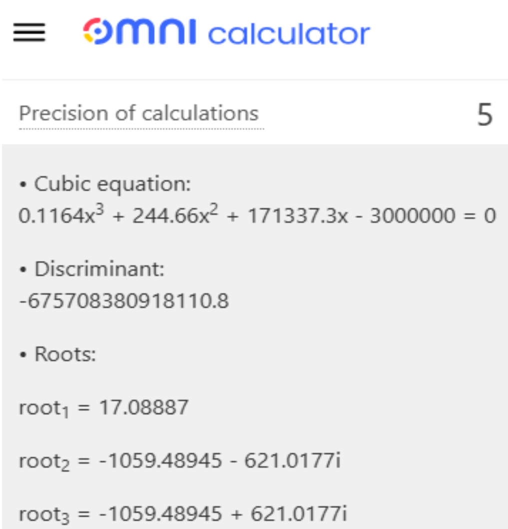
Eq. (9) also gives

$$V_2 = \pi \frac{h_2}{3}(R_3^2 + R_3 R_4 + R_4^2)$$

Therefore, substituting the values of R_3 and R_4 into Eq. (9) gives

$$3000000 = \frac{22}{21} h_2 \left(\left(233.445 + \frac{h_2}{3}\right)^2 + 233.445\left(233.445 + \frac{h_2}{3}\right) + 233.445^2 \right)$$

The equation above was simplified and also solved with Omni Cubic Equation Calculator [26], taking $h_2 = x$, which displayed results as follows:



$$\therefore h_2 \approx 17.1cm \text{ and } R_4 \approx 239.145cm$$

The total tank volume, V_t, was calculated using Eq. (10), while the total tank height, H_t, was calculated using Eq. (12)

i.e.

$$H_t = 30 + 259.336 + 17.1 \approx 306.44cm$$

$$V_t = 1901963 + 30000000 + 3000000$$

$$= 34901963\ cm^3 \approx 35000\ litres.$$

Calibration: To calibrate the 30000litres actual volume of the tank, h_a was divided into 25 equal intervals in accordance to Eq. (15), using Version 2010 Microsoft Excel.

The instantaneous heights and the corresponding instantaneous volumes is obtained is presented graphical by Figure 3.

III. RESULTS AND DISCUSSIONS

3.1 Results

The results obtained from the design and calibration of 30000litres frustum tank are presented in this section. The instantaneous dimensions and volumes are shown in Table 1 while Table 2 shows the dimensions and volumes of various segments of the tank. Furthermore, the instantaneous heights and the corresponding instantaneous volumes in Table 1 are presented graphically in Figure 3 while Figure 4 is the schematic illustration of the dimension of the designed tank.

Table 1: Instantaneous dimensions and volumes of the designed tanks

Dimension and Volume				
i	h_i(cm)	R_i (cm)	L_i (cm)	V_i (litres)
1	10	150	11	721
2	21	154	22	1476
3	31	157	33	2266
4	41	161	44	3091
5	52	164	55	3952
6	62	168	66	4850
7	73	171	77	5787
8	83	175	87	6761

9	93	178	98	7775
10	104	182	109	8830
11	114	185	120	9925
12	124	188	131	11062
13	135	192	142	12241
14	145	195	153	13464
15	156	199	164	14731
16	166	202	175	16042
17	176	206	186	17399
18	187	209	197	18803
19	197	213	208	20254
20	207	216	219	21752
21	218	220	230	23300
22	228	223	241	24896
23	239	227	251	26544
24	249	230	262	28242
25	259	233	273	29992

Table 2: designed Specifications

Volumes				Dimensions		
V_1 (ltr)	V_a (ltr)	V_2 (ltr)	V_t (ltr)	R_1 (cm)	R_4 (cm)	H_t (cm)
1901.96	30000	3000	35000	137	239.15	306.44

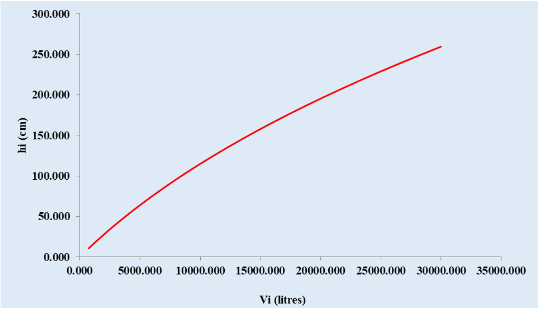


Figure 3: Showing the instantaneous volume against the instantaneous volume of the designed frustum tank

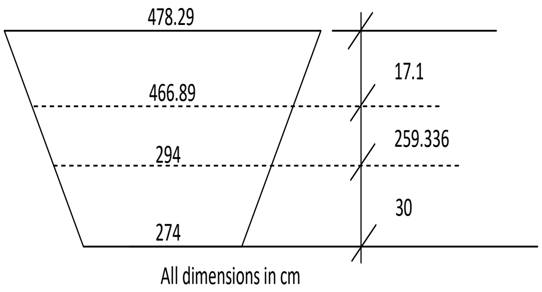


Figure 4: Schematic diagram showing the dimensions of the designed frustum tank

3.2 Discussion

This work is focused on the geometric design of frustum and tank for expandable liquid products. The modeling was based on principles of solid geometry and menstruation. The mathematical equations used for tank design were derived considering liquid expansion due to rise in temperature as noted by Elert [27].

In the design, reserved liquid volume was provided below datum line. It is design condition that the liquid level must not fall below the datum unless discharged during maintenance. The reserved volume is provided to collect debris and tiny solid particle in the liquid. It is the sedimentation zone of the tank.

The actual liquid zone of the tank was provided to hold a given capacity of the liquid at elevated temperature and pressure, but within the design specifications.

Tank actual height is calibrated for easy determination of the instantaneous volume of liquid in the tank. Tank calibration helps in prevention of over filling which has been noted the major cause of tank explosion [9]. It will also enable the tank operator regulate the liquid volume to prevent spills especially for a liquid that expand appreciably when heated. Calibration of tank volume also helps in making proper accountability of the liquid [28]. The free space above the actual liquid height was provided to compensate for liquid expansion. Figure 4 is a schematic diagram showing the dimensions of the designed frustum tank. The dimensions were used to calculate the total tank volume, which yielded 35120.9litres. The accuracy of this model was verified by summing the volumes of the various sections of the tank, which also gives 35120.9litres.

The proposed model can be recommended for geometric design of frustum container for the following areas of applications:

1.Water treatment tanks
2.Palm oil boiler,
3.Liquid steel crucible

Although the integrity of the proposed model has been shown by the design and calibration of 30000litres open frustum tank using the model. Factor of safety is recommended in the application of the model. A further research is recommended, which will include the thickness design of frustum tank. Modeling of closed frustum tank is also recommended in which internal pressure of the liquid at different points in the frustum tank will be factored into the design.

3.3 Conclusions
Based on the findings from this study, the following conclusions have been drawn:

1.Open frustum tank of expandable liquid can be designed if the actual liquid capacity and the dimensions of the tank below the datum are known.
2.The height-volume-relationship of frustum tank is none linear.
3.Increase in the instantaneous vertical height of frustum tank results in non linear increase in radius, slant height, and volume of the tank.
4.Expansion behaviour of a liquid influence the geometric design of the tank used to hold the liquid.

3.4 List of abbreviations

V_1 = Volume of reserved liquid

V_i = Instantaneous volume

V_a = Actual liquid volume

V_2 = Volume of free space

V_t = Total tank volume

h_1 = Height of liquid below the datum

h_a = Actual liquid height

h_2 = Height of free space

h_i = Instantaneous height

L_i = Instantaneous slant height

H_t = Total tank height

γ = Cubic expansivity of the liquid

$T_{\min}$ = Minimum running temperature

$T_{\max}$ = Maximum running temperature

3.5 Contribution to Knowledge
In addition to existing methods of tank design in previous publications, mathematical model has been proposed in this paper for geometric design and calibration of frustum tanks.

IV. DECLARATIONS

Availability of data and materials
All data generated in this study are included in this paper.

Competing interests
The authors of this article declare no form of competing interest on this paper.

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This work is just mathematical modeling. There was no experimentation or field work. Therefore funding is not applicable.

Authors' contributions
KO initiated this work and did the mathematical derivation of the equations and calculations. HO prepared the tables and plotted the graphs. LC sourced for related works and included relevant citations. CO did the type setting. All the authors contributed in technical editing and proofreading of the work.

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