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Application of Law of Large Number (LLN) To Quality or Process Control: Case Study of Jos University Teaching Hospital (JUTH) Multipurpose Cooperative Society Water Factory, Lamingo-Jos, Plateau State Nigeria

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Abstract- BACKGROUND: The Law of Large Numbers (LLN) is a fundamental principle in statistics that has far-reaching implications for quality control. This article explores the application of the LLN in quality and process control, highlighting its significance, methods, and results.
AIM: The aim of this article is to examine the application of the LLN in quality and process control, highlighting its significance, methods, and results.
METHODOLOGY: It was six (6) weeks, prospective, factory-based study that systematically selected 300 samples from 2000 products produced.
SAMPLING PROCEDURE: Systematic Sampling Technique was adopted in which 300 samples of product were selected out of 2000 production. Using k = N/n. To ensure 300 samples were selected, simple random sampling was conducted by selected by choosing any sample between 1–6. Afterward, every 6th product starting from 3rd product (3, 9, 15 …, and 1995) was selected.
Law of Large Number-Based Quality Control Framework model was adopted for the study.
RESULT: Finding revealed that for every unit increase in raw material 'A' quality, defect rate decrease with 0.266%. For every increase in raw material 'B' quality, defect rate decreases with 0.131% and for every unit increase in machine calibration, defect rate decreases with 0.585% respectively.
CONCLUSION: The LLN plays a crucial role in quality and process control, enabling manufacturers to make informed decisions and optimize processes. Its application yields significant results, including improved process stability and enhanced product quality.

Indexed Terms- Laws of Large Number, Quality/Process Control.

I. INTRODUCTION

The Law of Large Numbers (LLN) is a fundamental principle in statistics that has far-reaching implications for quality control. This article explores the application of the LLN in quality and process control, highlighting its significance, methods, and results.

The LLN is an extremely intuitive and applicable result in the field of probability and statistics. Essentially, the LLN states that in regards to statistical observations, as the number of trials increase, the sample mean gets increasingly close to the hypothetical mean (Kelly Sedor, 2015).

Quality control is a critical aspect of manufacturing, ensuring that products meet specified standards. Statistical methods, such as the LLN, play a vital role in quality control, enabling manufacturers to make informed decisions and optimize processes.

The central limit theorem says that the sum or average of many independent copies of a random variable is approximately a normal random variable. The CLT

goes on to give precise values for the mean and standard deviation of the normal variable Jeremy Orloff and Jonathan Bloom (2022).
The aim of this article is to examine the application of the LLN in quality and process control, highlighting its significance, methods, and results.

This article focuses on the application of the LLN in quality and process control, specifically in the context of manufacturing.

The LLN is significant in quality control as it enables manufacturers to:
Estimate process capability
Monitor and control processes
Determine optimal sample sizes
Evaluate quality improvement initiatives

II. RESEARCH ELABORATIONS

APPLICATIONS OF THE LAW OF LARGE NUMBERS

Like many other great results in the fields of probability and statistics, the Law of Large Numbers has many useful applications to a variety of fields.

Suppose $X1$, $X2\dots$ Xn are independent random variables with the same underlying distribution. In this case, we say that the Xi are independent and identically-distributed, or i.i.d. In particular, the Xi all have the same mean μ and standard deviation σ. Let Xn be the average of $X1\dots Xn$: $X1 + X2 + \cdots + Xn$ 1 n Xn = = $\sum Xi$. n n i=1 Note that Xn is itself a random variable. The law of large numbers and central limit theorem tell us about the value and distribution of Xn, respectively. LoLN: As n grows, the probability that Xn is close to μ goes to 1. CLT: As n grows, the distribution of Xn converges to the normal distribution (μ, $\sigma 2/n$). Before giving a more formal statement of the LoLN, let's unpack its meaning through a concrete example (we'll return to the CLT later on).

MODEL: LLN-BASED QUALITY CONTROL FRAMEWORK

Model: Multiple Linear Regression
The model aims to predict the quality of a manufactured product (y) based on several process variables (X_1, X_2, $X_3\dots$ Xn). The model takes the form:
$y = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \dots + \beta nXn + \varepsilon$
Where:
Y is the quality of the product (response variable)
X_1, X_2, $X_3\dots$ Xn are the process variables (predictor variables)
β_0, β_1, β_2, $\beta_3\dots$ βn are the regression coefficients
ε is the error term.

Assumptions
The model assumes that:
The relationship between the process variables and product quality is linear
The error term is normally distributed with a mean of 0 and a constant variance
The process variables are independent of each other

III. RESULTS

Table 4.1: Model Summary

Model Summary									
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.970[a]	.941	.940	.03492	.941	1566.885	3	296	.000
a. Predictors: (Constant), Machine Calibration , Raw Material (B) Quality , Raw Material (A) Quality									

Source: SPSS Version 25 Output, 2025

The value of R, which denotes the strength of the relationship between the predictor variables and the observed variable in the model used for the research, is 0.970, which can be seen in Model Summary Table above. Furthermore, the coefficient of determination (R^2) was 0.940, indicating that overwhelming 94.5%

of the variations in defect rate are explained by the independent variables contained in the model, with the remaining 5.5% being accounted for by factors not included in the model. This indicates that a larger percentage of the prediction in the defective detection is explained by factors included in the model.

From the change statistics, the R^2 value was 0.940, and the F-statistic value is significantly above zero (1566.885) with a significant F-value of 0.000, which is significant even at a 1% (0.01) level of significance. Inferring a good fit for the model is implied. As a result, it follows that the exogenous factors work together to significantly impact how the endogenous factor behaves.

Table 4.2: ANOVA

ANOVA[a]						
Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	5.732	3	1.911	1566.885	.000[b]
	Residual	.361	296	.001		
	Total	6.093	299			
a. Dependent Variable: Percentage Defectives (%)						
b. Predictors: (Constant), Machine Calibration, Raw Material (B) Quantity , Raw Material (A) Quantity						

Source: SPSS Version 25 Output, 2025

The f-statistics for the ANOVA in Table 4.15 above have a value of 1566.885 and a probability value of 0.000. The overall model has a 1% (p-value 0.01) statistical significance level.

Table 4.3: Regression Coefficients

Variables	Std. error	Beta	t	p-value	95% CI
Constant	0.891		47.839	0.000	0.854-0.928
Raw material (A) quality	0.010	-0.266	-4.084	0.002	-0.002-(-0.001)
Raw material (B) quality	0.021	-0.131	-2.157	0.001	-0.001-0.000)
Machine calibrations	0.04	-0.585	-11.914	0.006	-0.006-(-0.04)

Source: SPSS Version 25 Output, 2025

All the independent Variables (Indicators) had a probability value less than 0.05, as shown in the Table above. This indicated that all the Indicators were statistically significant with defective detection at a 5% level.

The model equation is as shown below:

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + e$$

$$y = 0.891 - 0.266_{Raw\ materil\ A\ quality} - 0.131_{Raw\ material\ B\ quality} - 0.585_{Machine\ calliberation} + e$$

This finding revealed that for every unit increase in raw material 'A' quality, defect rate decrease with 0.266%.

For every increase in raw material 'B' quality, defect rate decreases with 0.131% and for every unit increase in machine calibration, defect rate decreases with 0.585% respectively.

Law of Large Number implication

Stability: As the sample size increases, the estimated coefficient will converge to the true population parameters (LLN)

With larger samples, the regression model will provide more reliable prediction

CONCLUSION

In conclusion, the result showed that the regression model suggest that improving raw materials quality and machine calibration can reduce defect rate.

As sample size increases, the model's estimate will become more stable and reliable; illustrating the Laws of Large Number (LLN).

The LLN is a powerful tool in quality control, providing a foundation for statistical analysis and decision-making. Its application in quality and process control is essential for manufacturers seeking to improve product quality and reduce variability.

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