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A Novel Black-Scholes Framework: Mean Reverting European Logistic Option Pricing with Jump Diffusion and Transaction Costs

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Abstract- Traditional Black-Scholes models assume frictionless markets with continuous price movements, limiting their applicability in emerging economies where transaction costs and sudden market shocks significantly influence asset pricing dynamics. These limitations became particularly evident during the COVID-19 pandemic when markets experienced unprecedented volatility and liquidity constraints. This study derives a novel Black-Scholes differential equation that incorporates mean reversion, jump diffusion processes, and transaction costs within a European logistic option pricing framework to address real-world market complexities. We extend geometric Brownian motion by integrating Vasicek mean reversion dynamics, Poisson jump processes for market discontinuities, and explicit transaction cost modeling. The enhanced stochastic differential equation is: $dS(t) = (\alpha - \lambda k - \tau)(\bar{S} - \ln S(t))S(t)(S^* - S(t))dt + \sigma S(t)(S^* - S(t))dZt + S(t)dq$, where α represents mean reversion speed, λ is jump intensity, τ denotes transaction costs, and dq captures Poisson jumps. Parameters are estimated using maximum likelihood methods with conditional density functions. Empirical validation using four major Nairobi Securities Exchange companies (2020-2022) demonstrates superior performance. Our model produces consistently lower, more realistic volatility estimates compared to five benchmark models. For Equity Bank, volatility estimates were 0.97 (2020) versus 1.36-1.68 from traditional models. ANOVA analysis confirms statistical significance of transaction cost effects ($F = 1690.54$, $p < 2.8E-276$). The model effectively captured crisis-period dynamics while maintaining stability through mean reversion mechanisms. The enhanced framework provides more accurate asset pricing for emerging markets by simultaneously accounting for bounded growth, price reversions, jump risks, and trading frictions. This offers substantial improvements for portfolio optimization, risk management, and derivative pricing in volatile market environments.

Indexed Terms- Black-Scholes equation, mean reversion, jump diffusion, transaction costs, logistic Brownian motion, emerging markets, volatility estimation

I. INTRODUCTION

The Black-Scholes-Merton model, introduced by Black and Scholes (1973) and extended by Merton (1973), fundamentally transformed option pricing theory by providing the first closed-form solution for European options. However, the model's restrictive assumptions—including constant volatility, continuous price movements, zero transaction costs, and perfect market liquidity—have proven increasingly problematic in contemporary financial markets, particularly in emerging economies where these assumptions are routinely violated.

Classical Black-Scholes models assume that asset prices follow geometric Brownian motion (GBM), characterized by continuous price paths and constant parameters. This assumption implies unlimited exponential growth, contradicting empirical evidence of mean reversion documented by Poterba and Summers (2020) and Balvers et al. (2020). Furthermore, the frictionless market assumption ignores substantial transaction costs that can range from 0.5% to 5% in emerging markets (Lesmond, 2019). The 2008 financial crisis and COVID-19 pandemic further exposed these limitations, as markets experienced extreme volatility clusters, sudden price jumps, and severe liquidity constraints that traditional models failed to capture.

Empirical research has documented several stylized facts challenging traditional assumptions. Asset prices exhibit mean reversion over longer horizons, with Fama and French (2018) showing negative serial correlation in stock returns over 3-5 year periods. Financial markets experience sudden price discontinuities, with Cont and Tankov (2016) demonstrating that 10-15% of price variation stems from jumps, increasing to 25-30% during crisis periods. Transaction costs in emerging markets are typically 3-5 times higher than developed markets (Bekaert & Harvey, 2017), creating substantial arbitrage barriers and affecting price discovery.

The COVID-19 pandemic provided a natural experiment highlighting model inadequacy. During March 2020, the Nairobi Securities Exchange experienced unprecedented volatility, with the NSE 20 index declining over 35% within three weeks. Traditional Black-Scholes models severely underestimated option prices, failing to account for extreme volatility clustering and jump risks. Transaction costs increased significantly as bid-ask spreads widened and liquidity dried up, creating feedback loops that amplified price volatility.

This study addresses these challenges by developing an enhanced Black-Scholes framework integrating four critical features: (1) Vasicek-type mean reversion modeling price tendency toward long-term equilibrium, (2) Poisson jump processes capturing sudden discontinuities, (3) explicit transaction cost modeling reflecting trading frictions, and (4) logistic growth constraints imposing realistic bounds on price evolution. The primary objective is to derive a comprehensive Black-Scholes differential equation incorporating these elements within a European logistic option pricing framework.

This research contributes theoretically by extending option pricing theory through a unified framework addressing multiple empirical regularities simultaneously. Methodologically, the integration of mean reversion, jump diffusion, and transaction costs within logistic growth represents a novel approach balancing mathematical tractability with empirical realism. Practically, the enhanced model provides more accurate volatility estimates and option prices for emerging market applications, with direct implications for portfolio management and risk assessment.

II. LITERATURE REVIEW

2.1 Evolution of Option Pricing Theory

The Black-Scholes-Merton framework established by Black and Scholes (1973) and Merton (1973) provided the theoretical foundation for modern option pricing under geometric Brownian motion assumptions. However, empirical studies consistently documented systematic biases, leading to numerous extensions. Hull and White (1987) introduced stochastic volatility models to address constant volatility assumptions, while Heston (1993) developed the widely-used model allowing volatility correlation with underlying assets. These advances highlighted the trade-off between model flexibility and computational tractability that continues to challenge option pricing research.

2.2 Mean Reversion in Financial Markets

Mean reversion in asset pricing has substantial theoretical and empirical support. Poterba and summers (1988) provided pioneering evidence of mean reversion in U.S. stock markets, documenting significant negative serial correlation over 3-5 year horizons. Fama and French (1988) extended this research across different portfolio formations, finding stronger mean reversion for smaller firms and growth stocks, attributed to market inefficiencies and behavioral biases. International evidence from Balvers et al. (2000) using 18 OECD countries confirmed robust mean reversion with approximately 3.5-year half-life, particularly strong in emerging markets where informational inefficiencies create greater mispricing opportunities.

The Vasicek (1977) model provided the mathematical framework for incorporating mean reversion, originally for interest rates but subsequently adapted for equity applications. The model describes financial variables reverting toward long-term means at speeds determined by mean reversion parameters, offering more realistic price dynamics than exponential growth models.

2.3 Jump Diffusion Models

Merton (1976) pioneered jump process incorporation into option pricing, recognizing that asset prices experience sudden, large movements unexplainable by continuous diffusion. His jump-diffusion model combines geometric Brownian motion with compound Poisson processes, where jumps arrive according to Poisson distributions with specified size distributions. Economic motivation stems from discontinuous price movements coinciding with earnings announcements, macroeconomic surprises, or geopolitical events.

Empirical studies consistently document jump presence in financial data. Andersen et al. (2007) using high-frequency data found jumps contribute 20-30% of return variation during volatile periods. Recent developments include Kou's (2002) double exponential jump models allowing asymmetric distributions, and Carr and Wu's (2004) time-changed Lévy models combining jumps with stochastic volatility.

2.4 Transaction Costs and Market Frictions

Transaction costs represent fundamental market frictions significantly affecting trading strategies and option pricing. Leland (1985) provided the first rigorous treatment, demonstrating that optimal hedging under transaction costs differs fundamentally from continuous rebalancing strategies of frictionless models. His analysis showed transaction costs lead to wider hedging bands and less frequent rebalancing.

Empirical evidence documents substantial cross-sectional and temporal variation in transaction costs. Lesmond et al. (1999) found emerging market transaction costs 3-5 times higher than developed markets, reflecting lower liquidity and wider spreads. Bekaert et al. (2007) showed costs are particularly high during crisis periods when liquidity providers withdraw and spreads widen dramatically. Amihud and Mendelson (1986) demonstrated that higher transaction cost assets require higher expected returns, establishing the liquidity premium as an important pricing factor.

2.5 Logistic Growth Models and Integrated Approaches

Logistic growth models provide frameworks for bounded asset price evolution, addressing geometric Brownian motion's unlimited growth limitation. Onyango (2003) introduced logistic Brownian motion to financial modeling, incorporating carrying capacity constraints that create natural price bounds. Oduor (2016) extended this to option pricing, developing closed-form solutions for European options under logistic dynamics.

Recent developments focus on integrated models combining multiple empirical regularities. Bates (1996) combined stochastic volatility with jumps for exchange rates, while Eraker et al. (2003) extended this to equity markets. Schwartz (1997) developed mean-reverting jump-diffusion models for commodities. Mulambula et al. (2020) integrated logistic growth with jump diffusion, demonstrating superior empirical fit compared to individual components. These integrated approaches aim to capture complex market feature interactions while maintaining mathematical tractability.

III. METHODOLOGY

3.1 Model Framework

Our approach begins with the standard logistic Brownian motion and progressively incorporates mean reversion, jump diffusion, and transaction costs. The base logistic model is given by:

$$dS(t) = \alpha S(t)(S^* - S(t))dt + \sigma S(t)(S^* - S(t))dZ_t$$

where $S(t)$ is the asset price, S^* is the market equilibrium, α is the growth parameter, σ is volatility, and dZ_t is a Wiener process.

3.2 Mean Reversion Integration

We incorporate mean reversion by modifying the drift term to include the difference between the long-term mean and current log price:

$$dS(t) = \alpha(\bar{S} - \ln S(t))S(t)(S^* - S(t))dt + \sigma S(t)(S^* - S(t))dZ_t$$

where $\bar{S}$ represents the long-term equilibrium log price.

3.3 Jump Diffusion and Transaction Costs

The complete model incorporates jump diffusion through a Poisson process and transaction costs as a drift adjustment:

$$dS(t) = (\alpha - \lambda k - \tau)(\bar{S} - \ln S(t))S(t)(S^* - S(t))dt + \sigma S(t)(S^* - S(t))dZ_t + S(t)dq$$

where λ is the jump intensity, k is the average jump size, τ represents transaction costs, and dq is the Poisson jump process.

3.4 Solution Methodology

Using the transformation $y_t = \ln S(t)$ and applying Itô's lemma, we derive the stochastic differential equation for the log price. The solution is obtained through integration and involves:

1. Exponential transformation: Converting the SDE to a more tractable form
2. Integration by parts: Solving the resulting integral equation
3. Maximum likelihood estimation: Estimating model parameters
4. Variance calculation: Deriving the conditional variance structure

3.5 Parameter Estimation

Parameters are estimated using maximum likelihood methods with the conditional density function:

$$f(y_t) = (2\pi)^{(-0.5)} * [\sigma^2/2(\alpha-\lambda k-\tau) * (1-e^{(-2(\alpha-\lambda k-\tau)t)})]^{(-0.5)} * \exp[-(y_t - mean)^2 / (2 * variance)]$$

3.6 Empirical Validation

We test the model using data from four major companies listed on the Nairobi Securities Exchange:

- Equity Group Holdings
- KCB Bank
- East African Breweries Limited (EABL)
- Kenya Power and Lighting Company (KPLC)

The sample period covers 2020-2022, capturing the market volatility during the COVID-19 pandemic.

IV. RESULTS

4.1 Parameter Estimation Results

Maximum likelihood estimation of the enhanced model yields the following parameter estimates for the four NSE companies over 2020-2022. The mean reversion speed (α) ranges from 0.85 to 1.24 across companies, indicating moderate to strong reversion tendencies. Jump intensity (λ) varies from 0.12 to 0.28, suggesting 12-28 jumps per year on average. Transaction cost estimates (τ) range from 0.008 to 0.035, reflecting 0.8% to 3.5% trading frictions. All parameters are statistically significant at the 1% level, with robust standard errors accounting for heteroskedasticity.

Company	$\hat{\alpha}$ (s.e.)	$\hat{\lambda}$ (s.e.)	$\hat{k}$ (s.e.)	$\hat{\tau}$ (s.e.)	$\hat{\sigma}$ (s.e.)	Log-likelihood
Equity	1.14(0.08)	0.18(0.03)	0.045(0.01)	0.012(0.003)	1.29(0.05)	-892.4
KCB	0.96(0.07)	0.15(0.02)	0.038(0.01)	0.010(0.002)	1.28(0.04)	-886.7
EABL	1.24(0.11)	0.28(0.05)	0.062(0.02)	0.025(0.006)	4.02(0.18)	-1156.3

KPLC	0.85(0.09)	0.22(0.04)	0.071(0.02)	0.035(0.008)	3.78(0.21)	-1189.6

4.2 Volatility Estimation Comparison

Our enhanced model (Model 5) consistently produces more conservative and realistic volatility estimates compared to benchmark models. The comparison across five models for 2020-2022 demonstrates systematic reductions in volatility estimates, particularly during crisis periods.

Company	Year	Craine(2000)	Oduor(2022)	Mulambula(2021)	Opondo(2021)	Our Model
Equity	2020	1.36	1.68	1.48	1.15	0.97
Equity	2021	1.64	2.22	1.92	1.50	1.29
Equity	2022	2.28	1.96	1.61	1.20	1.05
KCB	2020	1.71	1.68	1.48	1.15	0.94
KCB	2021	2.26	2.22	1.92	1.50	1.28
KCB	2022	1.98	1.96	1.61	1.20	1.01
EABL	2020	4.01	4.15	4.21	3.93	3.68
EABL	2021	4.46	4.28	4.62	4.30	4.02
EABL	2022	2.98	2.42	2.12	2.81	1.95
KPLC	2020	6.18	6.79	6.45	6.03	5.61
KPLC	2021	4.27	5.01	4.68	4.15	3.78
KPLC	2022	3.82	4.42	3.98	3.47	3.12

The results show our model reduces overestimation by 10-30% compared to traditional approaches, with larger reductions during volatile periods. For EABL in 2022, our model estimated volatility at 1.95 compared to 2.12-2.98 from other models, demonstrating superior crisis-period normalization.

4.3 Statistical Validation

ANOVA analysis confirms the statistical significance of transaction cost effects on asset price behavior. For Equity Bank 2020 data, the F-statistic of 1690.54 with p-value of 2.8E-276 strongly rejects the null hypothesis that transaction costs have no effect on price dynamics. Similar results hold across all companies and years.

ANOVA Results - Equity Bank 2020:

Source	SS	df	MS	F	p-value	F-critical

Between Groups	161,107	2	80,553.5	1690.54	2.8E-276	3.008
Within Groups	35,165	738	47.65			
Total	196,272	740				

Likelihood ratio tests comparing our enhanced model against restricted versions (without mean reversion, jumps, or transaction costs) yield test statistics exceeding critical values at 1% significance levels, confirming that all model components contribute significantly to explanatory power.

4.4 Mean Reversion Evidence

Unit root tests (Augmented Dickey-Fuller) on log price series reject the null hypothesis of unit roots for all companies, with test statistics ranging from -3.45 to -4.82 (critical value: -2.86 at 5% level). This provides strong evidence for mean reversion in NSE equity prices, supporting our modeling approach.

Mean Reversion Test Results:

Company	ADF Statistic	p-value	Half-life (months)	Reversion Speed
Equity	-4.12	0.001	7.3	1.14
KCB	-3.89	0.002	8.7	0.96
EABL	-4.82	0.000	6.7	1.24
KPLC	-3.45	0.009	9.8	0.85

Half-lives range from 6.7 to 9.8 months, indicating relatively fast mean reversion consistent with emerging market characteristics where informational inefficiencies create temporary mispricing that corrects relatively quickly.

4.5 Jump Detection and Analysis

Jump detection using the Barndorff-Nielsen and Shephard (2006) test identifies significant jump components in all series. EABL and KPLC exhibit higher jump frequencies during 2020-2021, corresponding to COVID-19 market stress. Jump contributions to total return variation range from 15% (banks) to 35% (KPLC), validating the inclusion of jump processes.

Jump Analysis Summary:

Company	Jump Days (%)	Avg Jump Size	Jump Contribution (%)	Largest Jump
Equity	8.2	4.1%	18.3	-12.4%
KCB	7.6	3.8%	15.7	-11.8%
EABL	12.4	5.8%	28.9	-18.7%
KPLC	14.1	6.9%	34.6	-22.3%

4.6 Transaction Cost Impact

The empirical results demonstrate that transaction costs significantly dampen the positive effects of volatility on asset prices. Scenarios with 5% transaction costs show 25-40% lower price growth compared to 0.5% cost scenarios, even under identical volatility conditions. This validates the explicit modeling of trading frictions in emerging markets where such costs are substantial.

Model-implied bid-ask spreads, derived from transaction cost parameters, range from 1.6% to 7.0%, consistent with observed NSE spreads during the sample period. The correlation between estimated transaction costs and observed market liquidity measures is 0.73, providing external validation of our cost estimates.

4.7 Model Performance during COVID-19

The enhanced model's superior performance is particularly evident during the COVID-19 crisis period (March-May 2020). While traditional models failed to capture the rapid volatility changes and subsequent normalization, our model's mean reversion and transaction cost components provided stability. During the peak crisis week (March 16-20, 2020), our model's volatility estimates averaged 15% below benchmark models while maintaining predictive accuracy, as measured by option pricing errors on NSE-listed derivatives.

Out-of-sample forecasting tests for the post-COVID recovery period (June 2020-December 2020) show our model achieves 23% lower mean absolute percentage error compared to the best-performing benchmark model, demonstrating practical superiority for risk management applications.

CONCLUSION

This research contributes to both academic understanding and practical application of option pricing theory, offering a more realistic and empirically validated framework for emerging market financial modeling. The integration of multiple market features within a coherent mathematical structure represents a meaningful advance in quantitative finance research with direct practical benefits for market participants and policymakers.

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