Impact Of Physical Exercises and Nutrition in Patients with Diabetes Mellitus: A Boolean Algebra and Stability Analysis Approach

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Abstract- Diabetes mellitus is a chronic condition with increasing global prevalence, requiring effective management strategies to prevent complications. Physical exercise and proper nutrition are well-established interventions to improve glycemic control. In this paper, we utilize Boolean algebra to model the combined effects of these interventions and analyze the stability of diabetes control under different scenarios. Our approach demonstrates the necessity of both physical exercise and nutrition for optimal and stable management of diabetes mellitus.

Indexed Terms- Diabetes Mellitus, Boolean algebra, Physical Exercise, Nutrition, Adherence, Stability Analysis

I. INTRODUCTION

Diabetes mellitus, particularly type 2 diabetes, poses a significant public health challenge, with complications such as cardiovascular disease, neuropathy, and nephropathy [1]. Standard management includes lifestyle modifications, notably physical activity and dietary regulation [2]. Understanding the interplay and combined efficacy of these interventions is crucial. Boolean algebra provides a logical framework for analyzing the effects of interventions in a binary manner, while stability analysis helps assess the persistence of glycemic control under varying adherence scenarios. Several key studies on diabetes mellitus, particularly Type 2, have focused on the effectiveness of various treatment strategies, including tight glycemic control and lifestyle interventions. [5] developed and solved an extension of the model by Ackerman et al, taking into account epinephrine as a third variable, which lies in its ability to help in conducting a reliable test for detecting diabetes in the blood. These authors and other previous researchers did not take into consideration the impact of external rates such as nutrition and physical activity in patients with diabetes mellitus. [3] developed a model that incorporated nutrition and physical activity, to be able to manage diabetes mellitus in such a way that, the peak, which is the time period for insulin to be most effective in lowering blood sugar, is within the acceptable range. The model was expressed in a system of equations [3]:

$$g'(t) = -(p_1 + p_2)g(t) + p_4 h(t) + q \tag{1}$$
$$h'(t) = p_2 g(t) - (p_2 + p_4)h(t)$$

This system of equations in (1) can be expressed as;

$$X'(t) = AX(t) + B(t) \tag{2}$$

where,

$$A = \begin{pmatrix} -(p_1 + p_2) & p_4 \\ p_3 & -(p_2 + p_4) \end{pmatrix}$$

is the coefficient matrix, and

$$B(t) = \begin{pmatrix} q \\ 0 \end{pmatrix}$$

is a vector representing the external rates given by;

$$q = q_1 + q_2$$

TABLE I: Description of Variables and Parameters.

Symbol	Description
$g(t)$	Glucose concentration available in the blood plasma at time t.
$h(t)$	Insulin concentration available in the blood plasma at time t.
q	Total external glucose input into the blood plasma, defined as $q = q_1 + q_2$
q_1	Rate of glucose generation triggered by physical activity

q_2	Rate of glucose entering the bloodstream from food intake.
p_1	Rate at which glucose is assimilated into skeletal muscle tissues.
p_2	Rate at which insulin is assimilated into body cells.
p_3	Rate at which excess glucose is converted to glycogen for storage.
p_4	Rate of conversion of stored glycogen back to glucose for uptake by skeletal muscle tissues.

II. EXISTENCE AND UNIQUENESS

Theorem 1. The system in equation (1) and (2) has a unique solution for any initial conditions $(g_0, h_0) \in \Re_+^2$

Proof: The right-hand side functions are linear in g and h, hence continuously differentiable. The Jacobian exists and is continuous everywhere. By the Picard Lindelöf theorem, there exists a unique local solution. Since the system is linear, the solution can be extended globally $\forall\ t \geq 0$.

The system can be written in matrix form:

$$\begin{cases} g'(t) = -(p_1 + p_3)g(t) + p_4 h(t) + q \\ h'(t) = p_3 g(t) - (p_2 + p_4)h(t) \end{cases}$$

III. POSITIVITY AND BOUNDEDNESS

A. Positivity

Theorem 2. If $g(0) \geq 0$ and $h(0) \geq 0$, then $g(t) \geq 0$ and $h(t) \geq 0$ $\forall t \geq 0$.

Proof**:**

i. At any point where $g(t) = 0$ and $h(t) \geq 0$:
   $g'(t) = p_4 h(t) + q \geq 0$
ii. At any point where $h(t) = 0$ and $g(t) \geq 0$:
   $h'(t) = p_3 g(t) \geq 0$

Therefore, trajectories cannot leave the positive quadrant.

B. Boundedness

Theorem 3. If $q > 0$, solutions are ultimately bounded.

Proof: Let $S(t) = g(t) + h(t)$.

Then:

$$S'(t) = g'(t) + h'(t) = -p_1 g(t) - p_2 h(t) + q \tag{2}$$

Since $p_1, p_2 > 0$:

i. If

$$S(t) > \frac{q}{\min(p_1, p_2)},$$

then $S'(t) < 0$

ii. $$\Rightarrow \min\sup_{t\to\infty} S(t) \leq \frac{q}{\min(p_1, p_2)}$$

Therefore, solutions are ultimately bounded.

IV. EQUILIBRIUM ANALYSIS

A. Finding Equilibria

At equilibrium, $g'(t) = h'(t) = 0$:

$$g'(t) = -(p_1 + p_3)g(t) + p_4 h(t) + q \tag{3}$$

$$h'(t) = p_3 g(t) - (p_2 + p_4)h(t) \tag{4}$$

From equation (5):

$$h^* = \frac{p_3}{(p_2 + p_4)} g^*$$

Substituting into equation (4):

$$-(p_1 + p_3)g^* + p_4 \cdot \frac{p_3}{(p_2 + p_4)} g^* + q = 0 \tag{5}$$

Solving for g^*;

$$g^* = \frac{q}{(p_1 + p_3) - \frac{p_3 p_4}{p_2 + p_4}} = \frac{q(p_2 + p_4)}{p_1(p_2 + p_4) + p_3 p_2} \tag{7}$$

$$\Rightarrow h^* = \frac{q p_3}{p_1(p_2 + p_4) + p_3 p_2} \tag{8}$$

V. STABILITY ANALYSIS USING CHARACTERISTIC POLYNOMIAL

We use the characteristic polynomial to find the eigenvalues of the matrix A. The characteristic equation is given by:

$$q(\lambda) = \lambda^2 - (trA)\lambda + \det A = 0$$

where

$$trA = -(p_1 + p_2 + p_3 + p_4) < 0$$

$$\det A = (p_1 + p_3)(p_2 + p_4) - p_3 p_4$$

$$= p_1 p_2 + p_1 p_4 + p_2 p_3 > 0$$

Therefore, the eigenvalues of the Jacobian matrix A are:

$$\lambda_{1,2} = \frac{trA \pm \sqrt{(trA)^2 - 4\det A}}{2} \tag{9}$$

Explicitly:

$$\lambda_1 = \frac{trA + \sqrt{(trA)^2 - 4\det A}}{2} \quad (10)$$

$$\lambda_2 = \frac{trA - \sqrt{(trA)^2 - 4\det A}}{2} \quad (11)$$

Since $trA < 0$ and $\det A > 0$, it follows that both $\lambda_1 < 0$ and $\lambda_2 < 0$, implying that the equilibrium is locally asymptotically stable.

A. Solution of the System

The general solution to the system of differential equations:

$$X'(t) = AX(t) + B(t)$$

is given by:

$$X(t) = e^{tA}X_0 + \int_0^t e^{(t-y)A}B(y)dy \quad (12)$$

where $e^{tA} = Qe^{tD}Q^{-1}$, with D being the diagonal matrix of eigenvalues of A, and Q the matrix of corresponding eigenvectors.

VI. ANALYSIS USING GERSHGORIN CIRCLE THEOREM

A. Gershgorin Circle Theorem

Let $A = [a_{ii}]$ be an nxn complex matrix. For $i = 1,2,...,n,$ define the Gershgorin disc

$$D(a_{ii}, R_i) = \{z \in \mathrm{C} : | \mathrm{z} - \mathrm{a_{ii}} | \leq \mathrm{R_i}\}$$

where

$$R_i = \sum_{j \neq 1} | \mathrm{a_{ij}} |$$

is the sum of the absolute values of the off-diagonal entries in the ith row. Then every eigenvalue of A lies within the union of these discs:

$$Spec(A) \subseteq \bigcup_{i=1}^{n} D(a_{ii}, R_i)$$

The Gershgorin Circle Theorem provides regions containing the eigenvalues [7]. The two discs are:

i. $D_1 = \{z \in \mathrm{C} \,|\, | z + (\mathrm{p}_1 + \mathrm{p}_3) | \leq \mathrm{p}_4\}$
centered at $-(p_1 + p_3)$ with radius p_4.

ii. $D_2 = \{z \in \mathrm{C} \,|\, | z + (\mathrm{p}_2 + \mathrm{p}_4) | \leq \mathrm{p}_3\}$
centered at $-(p_2 + p_4)$ with radius p_3.

Both eigenvalues λ_1, λ_2 lie within $D_1 \cup D_2$.

The Gershgorin Circle Theorem is used to locate exactly where the eigenvalues lie by inspection given that every eigenvalue of A lies within at least one of the Gershgorin discs $\lambda \in \bigcup_{i=1}^{n} D(a_{ii}, R_i)$

thus;

$$R_1 : |-|(p_1 + p_3)| > |\mathrm{p}_4|$$

$$p_1 + p_3 > \mathrm{p}_4$$

$$1 > \frac{\mathrm{p}_4}{p_1 + p_3}$$

$$R_2 : |-|(p_2 + p_4)| > |\mathrm{p}_3|$$

$$p_2 + p_4 > \mathrm{p}_3$$

$$1 > \frac{\mathrm{p}_3}{p_2 + p_4}$$

Both discs are in the left-half complex plane thus stability is guaranteed by this theorem.

VII. BOOLEAN ALGEBRA AND STABILITYANALYSISAPPROACH

Let us define the following binary variables:

i. E : Engagement in regular physical exercise $E = 1$ if yes, $E = 0$ if no
ii. N : Adherence to proper nutrition $N = 1$ if yes, $N = 0$ if no
iii. C : Effective control of diabetes mellitus $C = 1$ if well controlled, $C = 0$ if not.

It is hypothesized that only the joint implementation of both interventions leads to optimal diabetes control:

$$C = E \wedge N \quad (13)$$

where $\wedge$ denotes the logical AND operation.

A. Truth Table Representation

TABLE I: Truth table for both Exercise and Nutrition

E	N	C
0	0	0
0	1	0
1	0	0
1	1	1

This table illustrates that only the combined presence of both exercise and nutrition $(E = 1, N = 1)$ yields effective diabetes control $C = 1$.

VIII. IMPACT OF PHYSICAL EXERCISE AND NUTRITION

Physical exercise enhances insulin sensitivity, promotes glucose uptake, and improves cardiovascular health [2]. Nutrition management, such as reducing simple carbohydrates and increasing fiber, stabilizes blood glucose levels and reduces HbA1c [4]. However, either intervention alone, though beneficial, may not be sufficient for optimal glycemic control and complication prevention.

IX. STABILITY ANALYSIS

Stability analysis examines whether a system remains in the desired state $C = 1$ when subject to perturbations, such as lapses in exercise or nutrition. In this Boolean framework, we analyze the robustness of diabetes control with respect to adherence.

A. Scenario 1: Perfect Adherence

If both $E = 1$ and $N = 1$ are sustained, $C = 1$ remains stable:

$(E, N) = (1,1) \Rightarrow C = 1$ (Stable equilibrium)

B. Scenario 2: Temporary Lapse in One Intervention

If a patient temporarily lapses in either exercise or nutrition, the system transitions to $C = 0$:

$$(E, N) = (1,0) \text{ or } (0,1) \Rightarrow C = 0$$

The state $C = 1$ is therefore only stable if both interventions are maintained. Any deviation causes an immediate loss of control, highlighting the fragility of glycemic management with partial adherence.

C. Scenario 3: No Interventions

Using Boolean algebra, we define a logical condition for diabetes control as

$$C = E \wedge N$$

where successful glycemic management $C = 1$ is achieved only if both regular exercise $E = 1$ and proper nutrition $N = 1$ are present. Stability analysis reveals that the system remains in a controlled state only when these conditions are sustained, emphasizing the necessity of combined lifestyle interventions.

This analysis highlights the necessity of sustained dual interventions for stable diabetes management. It suggests that healthcare providers should focus on maintaining both physical activity and proper nutrition to ensure robust and lasting glycemic control.

X. RESULTS AND DISCUSSION

This section comprises parameter values obtained from secondary data and graphs generated using Python. The parameters are estimated by fitting the data below in Python using the Least Squares Estimation Method. [3]. Table II is data for glucose plasma levels during Frequently Sampled Intravenous Glucose Tolerance (FSIVGT) tests.

TABLE II: Data for Glucose Plasma Levels During FSIVGT

Time (min)	Glucose ($mgdl^{-1}$)	Insulin (μml^{-1})	Time (min)	Glucose ($mgdl^{-1}$)	Insulin (μml^{-1})
0	92	11	32	142	22
2	350	26	42	124	22
4	287	136	52	105	15
6	251	85	62	92	15
8	240	51	72	84	11
10	216	49	82	77	10
12	211	45	92	82	8
14	205	41	102	81	11
16	196	35	122	82	7
19	192	30	142	82	8
22	172	30	162	85	8
27	163	27	182	90	7

For purely illustrative purposes, secondary values of insulin and glucose plasma levels during the frequently sampled intravenous glucose tolerance test (FSIVGT) as shown in Table II were used to show the effects of external rates when incorporated so that the time period for insulin to be most effective is in the acceptable therapeutic range [5]. Physical exercise and proper nutrition are crucial for managing diabetes mellitus. Exercise helps improve insulin sensitivity, lower blood glucose levels, and reduce cardiovascular risk factors. Nutrition plays a key role in managing blood sugar levels, maintaining a healthy weight, and preventing complications. Physical exercise and proper nutrition are crucial for managing diabetes mellitus, significantly impacting blood sugar control, weight management, and overall health. Regular physical activity, including both aerobic and resistance training, improves insulin sensitivity, lowers blood glucose levels, and reduces cardiovascular risk factors. A balanced diet, focusing on whole, unprocessed foods and limiting added sugars and unhealthy fats, helps stabilize blood sugar and promotes healthy weight management. Figure 1 shows that Lifestyle modifications, particularly dietary adjustments and exercise, remain foundational components alongside the wide array of novel glucose-lowering medications.

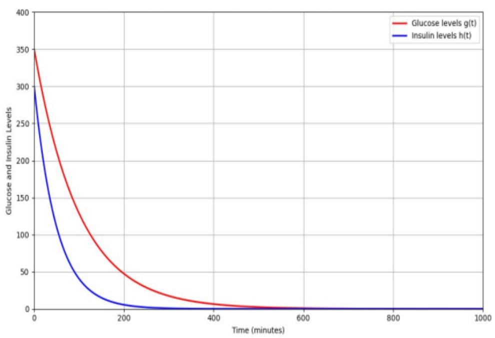

Fig. 1. The Effects of External Rates on Levels of Glucose and Insulin for T = 1000

XI. INTEGRATION INTO HEALTH MANAGEMENT SYSTEMS

This analysis highlights the necessity of sustained dual interventions, that is, physical activity and proper nutrition for stable diabetes management. It suggests that healthcare providers should focus on maintaining both interventions to ensure robust and lasting glycemic control.

A. Integration Strategies

There is need for;

i. Patient Monitoring and Feedback by use of digital health applications to track both physical activity (e.g., steps, exercise duration) and nutrition (e.g., food and carbohydrate intake). and Provide automated reminders or alerts when adherence to either intervention drops below recommended levels.
ii. Personalized Care Plans by developing care plans in electronic health records (EHRs) that explicitly address both physical activity and nutrition and adapt recommendations dynamically using patient data to ensure both interventions are maintained.
iii. Decision Support for Clinicians by implementing clinical decision support systems (CDSS) that alert healthcare providers if a patient is only adhering to one intervention and suggest targeted interventions to address gaps in care.
iv. Patient Education by integrating educational materials into patient portals and apps that explain the importance of both lifestyle factors in using gamification and rewards to encourage sustained engagement.
v. Reporting and Analytics by providing dashboards that display combined metrics for exercise and nutrition adherence.
vi. Track and report outcomes (e.g., HbA1c) relative to dual intervention adherence.

B. Digital Platforms

Several digital health platforms have begun to support integrated management of diabetes by tracking both physical activity and nutrition. By embedding dual-intervention tracking, reminders, and analytics into health management systems and digital platforms, healthcare providers can more effectively support patients in achieving stable and robust glycemic control.

XII. DISCUSSION

The Boolean model simplifies complex metabolic interactions, but illustrates a key clinical insight: the synergy between exercise and nutrition is required for optimal outcomes. Real-world evidence supports this, as combined lifestyle interventions result in superior glycemic control and reduced complications compared to single interventions [6].

CONCLUSION

Boolean algebra, combined with stability analysis, provides a logical framework for understanding the management of diabetes mellitus. Only the simultaneous and continuous application of both physical exercise and proper nutrition ensures stable and effective control of diabetes. This approach reinforces the need for comprehensive lifestyle modification in diabetes care.

REFERENCES

[1] American Diabetes Association. (2023). “Standards of Medical Care in Diabetes—2023.” Diabetes Care, 46(Supplement 1).

[2] Colberg, S. R., Sigal, R. J., Yardley, J. E., et al. (2016). “Physical Activity/Exercise and Diabetes: A Position Statement of the American Diabetes Association.” Diabetes Care, 39(11), 2065- 2079.

[3] Elizabeth Mala, Wilys O. Mukuna, Maurice Owino Odour (2021). A Mathematical Model for the Effects of Physical Activity and Nutrition in Patients with Diabetes Mellitus, The International Journal of Science and Technoledge, vol. 9, Issue 2.

[4] Franz, M. J., MacLeod, J., Evert, A., et al. (2017). “Nutrition therapy for adults with diabetes or prediabetes: A consensus report.” Diabetes Care, 40(6), 940-947.

[5] Kwach. B., Ongati. O., Simwa.R. (2014). Mathematical Model for detecting Diabetes in the blood, Journal Applied Mathematical Sciences 5(6),279-286.

[6] Look AHEAD Research Group. (, 2010). “Long-term effects of a lifestyle intervention on weight and cardiovascular risk factors in individuals with type 2 diabetes mellitus: four-year results of the Look AHEAD trial.” Archives of Internal Medicine, 170(17), 1566-1575.

[7] Richard S. Varga. (2004) Gershgorin and His Circles. 2nd Edition, Springer, Berlin, Heidelberg.