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Statistical Modelling of Bladder Cancer Remission Data of Fluctuating Hazard Rate: An Inverse Power Pranav Distribution Approach

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Abstract- ***This study introduces the Inverse Pranav Distribution (IPD) as a flexible model for survival data with both monotone and non-monotone hazard rate behaviors. The Pranav distribution was inversely transformed to create the model, and its main statistical characteristics were determined. Using data on bladder cancer patients' remission times, the IPD's performance was contrasted with that of the Inverse Rama, Inverse Lindley, Inverse Exponential, and Inverse XGamma distributions. Based on likelihood values, information criteria, and goodness-of-fit tests, the results indicated that the IPD offered the best overall fit, outperforming rival models. The distribution was appropriate for survival and reliability studies because it could capture intricate hazard structures, including decreasing, increasing, and bathtub-shaped patterns.***

Index Terms- ***Bladder Cancer, Inverse Power Pranav***

I. INTRODUCTION

Bladder cancer is the ninth most diagnosed and widespread disease in medical science, see Gurung. Among various cancers worldwide, bladder cancer accounts for 3%, see Riester *et al*. Moreover, it is the sixth and seventeenth most prevalent disease among men and women, see Bray et al. There are a lot of types and infection percentage as we can find that bladder-transitional cell carcinoma in the patients of bladder cancer are about 90%, bladder-squamous cell carcinoma is greater than 5%, and bladder-squamous cell carcinoma is less than 2% see Kim et al. According to pathological histology. In the first diagnostic patient with bladder cancer, 70% to 85% have quasi bladder cancer and about 15% −30% suffers from cancer of the bladder muscle-invasive, see Witjes *et al*. Real-world phenomena of survival times in medical studies, financial risk assessment, and reliability engineering demand models that can capture complex behavior, such as skewness, heavy-tailed, or non-monotonic hazard rates. Rigby and Stasinopoulos (2005) stated that classical distributions, such as the exponential, Weibull, and gamma, have long been foundational tools. Without a doubt, the parametric models mentioned above are extensively employed in survival research. Regrettably, these models still have certain shortcomings as their rigidity tends to limit their suitability for modern data sets under typical patterns. A more flexible statistical distribution known as Inverse Power Pranav developed by Nwankwo et al. (2021) was aimed at broadening its applicability to inversely related datasets that display strong right-skewed behavior or increasing function in their hazard function. The Probability Density Function (PDF) of the Inverse Pranav Distribution as

$$f(x;\alpha,\beta)=\frac{\alpha\beta^{\alpha}}{x^{(\alpha+1)}},x>\beta,\alpha,\beta>0 \qquad (1.1)$$

where α is the shape parameter, which governs the steepness and concentration of the distribution around the threshold β, and β is the scale parameter that controls the lower bound of the distribution and shifts it along the horizontal axis. The Cumulative Distribution Function (CDF) of the distribution is expressed as

$$F(x;\alpha,\beta)=1-\left(\frac{\beta}{x}\right)^{\alpha},x>\beta \qquad (1.2)$$

This CDF is strictly increasing and ranges from 0 to 1 as x progresses from just above β toward infinity. At x=β, the CDF is equal to 0, reflecting the fact that the

distribution is not defined for values less than or equal to β, and as x→∞, the CDF asymptotically approaches 1, indicating that the entire probability mass is concentrated within the interval (β, ∞). This study focuses on the applicability of the Inverse Power Pranav distribution in modelling breast cancer data with a fluctuating hazard rate

II. LITERATURE REVIEW

Literature establishes the fact that describes the milestone contributions, novel developments, and related models that underpin and put into perspective the application of the inverse Pranav distribution.

Abed al-Kadim and Hantoosh (2014) introduced the Double Weighted Inverse Weibull Distribution, enhancing the tail flexibility of the inverse Weibull model through a double-weighting approach. The authors derived expressions for key properties such as the probability density function (PDF), cumulative distribution function (CDF), and hazard rate function (HRF), and implemented the maximum likelihood estimation (MLE) for parameter fitting. Empirical results from reliability data demonstrated superior fitting performance over standard inverse distributions.

Ali *et al.*, (2021) proposed the Alpha-Power Exponentiated Inverse Rayleigh Distribution, which incorporated the alpha-power transformation to create a highly flexible distribution capable of modeling a range of skewed data structures. The model's statistical properties were derived analytically, and performance was evaluated through simulations and survival data applications, where it consistently outperformed existing models in flexibility and accuracy.

Similar to what has been stated before, Alkarni (2015) expanded the inverse Lindley distribution by including one more parameter, and thus caused it to have more adequate capability to show the development of the hazard rate asymmetrically. The author presented very detailed mathematical deductions and examined the model, as well as engineering data. According to the result, the developed version performed excellently in positively skewed situations compared to the original method.
Barreto-Souza *et al.*, (2011) suggested the Weibull-Geometric distribution. This hybrid model incorporated the geometric and Weibull distributions to overcome the problem of dispersion in lifetime data. Estimation was carried out through the EM algorithm and MLE, and further, validation using empirical data showed notable improvements related to fit in failure time data.

Shukla and Shanker formulated the Weighted Pranav Distribution in 2020, where a weight function was used to increase model flexibility for survival data real-world applications. The scholarly articles' authors furnished a good number of the model's moments and order statistics. They also did numerical illustrations that showed that the newly proposed model using cancer survival data decreased AIC and K–S statistics more than existing models, even though the study did not use cross-validation techniques to evaluate predictive performance was not conducted.

Nwankwo *et al.* (2021) proposed the Inverse Power Pranav Distribution (IPPD), which features an additional shape parameter, providing greater flexibility in curve fitting and hazard rate control. In an article in the Journal of Advances in Mathematics and Computer Science, they rigorously examine the structural advantages of the model using methods including maximum likelihood estimation (MLE), Akaike information criteria (AIC), and comparative goodness-of-fit analysis. Applied to actual data involving fatigue life and system reliability, the IPPD was found to be more efficient than conventional inverse models such as the inverse Weibull and inverse exponential distributions.

As far as Silva and Cordeiro's research in 2013 is concerned, the Beta-Weibull Geometric Distribution was a model that garnered extreme attention due to the fact that, above all, it combined the beta and Weibull-geometric distributions. The study which was published really evoked a number of issues of the reliability and survival of the distribution and this also confirmed that the distribution could be used for different death probability situations. The paper also indicated its modeling capacity with a possibility of

encountering the above-mentioned challenge during sparse data conditions.

In the year 2022, the idea of Chaudhary, Telee, and Kumar (2022) was to put forward the Inverse Exponentiated Odd Lomax Exponential Distribution as a new class of odd exponential models. The process also involved the distribution of reliability methods in closed form to demonstrate the applicability of the new type of probability density function to data. The outcomes of the paper gave proof that the model indeed can be used to real-life medical applications as China Telemedicine was expanded to villages for professional expertise and more extensive validation was suggested.

Shukla (2018) outlined the Pranav distribution, where he presented the key statistical properties followed by illustrating the use of the distribution in lifetime and medical data. MLE was used by the author to estimate the parameters and compared the model's fit through graphical methods. The conclusion of the study was that the Pranav model's performance was good in terms of both tail and hazard shape.

Shukla and Shankar (2020) have extended the work on Weighted Pranav and using the empirical analysis, they have confirmed the performance of the new model as compared to the classical lifetime models.

Their work was limited to the survival data of industrial applications with a weighting mechanism that facilitated the tail adjustments of better performance.

Shukla (2024) in his paper brought forward the Truncated Pranav Distribution to address the boundary-related modeling issues. He also derived the distribution of the theory and gave evidence of the improvement in the modelling of the truncated data in the reliability context. However, the truncation method made it to be more efficient in practice but it was not having a Bayesian estimation framework.

Wani, Hassan, and Shafi (2020) brought forward the Pranav Quasi Gamma Distribution which is basically gamma plus the with the rich and flexible character of the Pranav distribution. By moments-based estimation and simulation study, the proposed model was found to be able to provide good fits even in cases of very high variability in medical data.

Enogwe *et al.*, (2020) suggested a new power Akash Distribution, as in their model, and successfully represented the sharp point-changing nature of the lifetime data. They showed the flexibility of their model over classical forms via data analysis and parameter estimation.

Ghitany *et al.,* (2008) presented the new Lindley Distribution and examined its applications in medical reliability data. Their model, which was also less complicated, provided a quite significant improvement for data that had slight skewness as compared to the exponential ones.

Sharma and Khandelwal (2021) proposed an expansion of the inverse Lindley distribution that has more capacity for hazard modeling. The results of their paper concluded the superiority of the proposed distribution as they conducted treatments on simulated and empirical data. However, they mentioned the difficulty of estimating the parameters under censored data as a major constraint of the model.

Ross (2023), in his publication "Monte Carlo Statistical Methods," outlined the idea of continuous random variables generation and showed visualization methods for future empirical modeling

Rashid *et al.*, (2022) share the Three-Parameter Quasi Gamma Distribution, aiming it at the biomedical field. The estimation process used moment-based and MLE methods, and the suitability of the real-world applications substantiated the approach's better performance with tail-heavy distributions.

III. DATA PRESENTATION

The analysis in this study utilizes secondary data on remission times (in months) for 128 bladder cancer patients, as presented in Gayus et al (2022)

0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46, 3.64, 5.09,

7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69

IV. RESULT

4.1 Estimates

Table 1: Estimates of the Inverse Pranav Distribution

Model	Estimate	Error	LL	AIC	CAIC	BIC	HQIC	K-S	P
IP	θ =10.76	0.4461	-162.631	-323.263	-323.231	-320.411	-322.104	0.72656	0.5581
IR	θ = 3.441	0.1967	459.8496	921.6992	921.7952	924.551	922.858	0.73438	0.5426
IL	θ = 3.09	0.2286	468.4544	938.9089	938.9406	941.7609	940.0677	0.76562	0.4806
IE	θ = 2.484	0.2196	460.3823	922.7646	922.7963	925.6166	923.9234	0.83594	0.3411
IXG	θ = 3.377	0.2464	469.3456	940.6912	940.7872	943.5432	941.85	0.73438	0.5426

The estimate (θ) represents the estimated parameter of each distribution, as shown in Table 1. Error is a measure of the standard error in the estimation. In the Log-Likelihood (LL), higher values indicate better model fit. To examine the model performance, Akaike Information Criterion (AIC), Corrected AIC (CAIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQIC) were employed. Lower values suggest a better model fit. The Kolmogorov-Smirnov (K-S) Statistic was used to verify if the model fits the data well. Lower values indicate better goodness-of-fit. For p-value (P), higher values suggest a better fit in the K-S test. The Inverse Pranav (IP) distribution has the lowest LL (-162.631), while the others have significantly higher values, suggesting a different likelihood behavior.

For the performance criteria, AIC, CAIC, BIC, and HQIC were used. The Inverse Pranav distribution has the lowest values across all these criteria, indicating a better overall model fit compared to the other distributions. Examining goodness of fit using Kolmogorov-Smirnov (K-S) and p-value, the IE (Inverse Exponential) distribution has the highest K-S value (0.83594) and the lowest p-value (0.3411), suggesting it fits the data less well. The Inverse Pranav distribution has a moderately low K-S statistic (0.72656) and a relatively high p-value (0.5581), suggesting a good fit.

The Inverse Pranav distribution performed the best overall based on AIC, CAIC, BIC, HQIC, and the p-value from the K-S test. It has the lowest AIC and BIC values, indicating that it provides the best balance of model complexity and goodness-of-fit.

4.2 PDF and CDF plot of the Inverse Pranav distribution

Figure 4.1a, 4.1b, 4.1c and 4.1d presents the Probability Density Function (PDF) and Cumulative Distribution Function (CDF) of the Inverse Pranav distribution for different values of θ.

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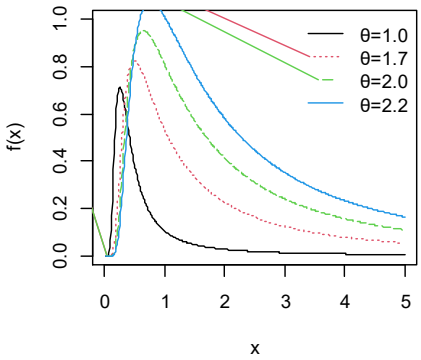

Fig.4.1a: pdf of IP distribution

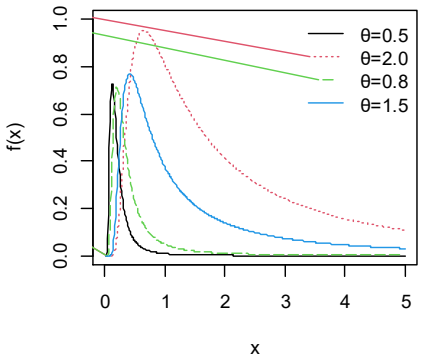

Fig.4.1b: pdf of IP distribution

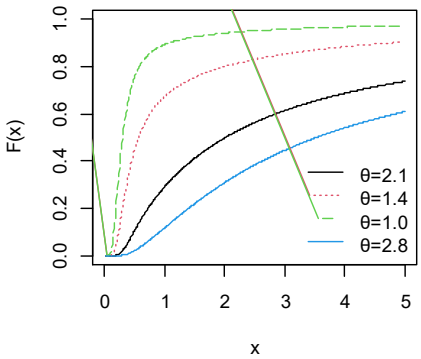

Fig.4.1c: CDF of IP distribution

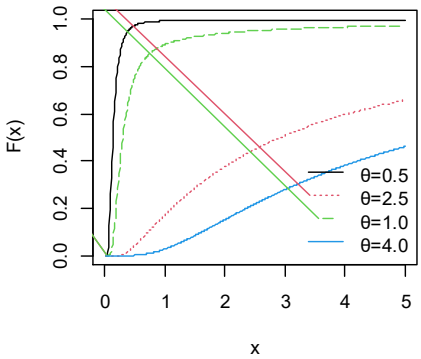

Fig.4.1d: CDF of IP distribution

The Probability Density Function (PDF) describes the likelihood of different values of the random variable occurring. In the graph, the shape of the PDF indicates how the probability mass is distributed. The PDF is right-skewed. It suggests that lower values of the random variable are more likely. The peak of the PDF represents the mode, the value where the distribution is most concentrated. Examining the shape of the PDF, the distribution is unimodal. The tail behavior shows how the probability decreases as values increase, which is important in reliability and survival analysis.

For the Inverse Pranav distribution, the PDF likely starts high and decreases gradually, reflecting the nature of an inverse-type distribution where smaller values are more probable.

The Cumulative Distribution Function (CDF) represents the probability that a random variable is less than or equal to a given value. In the graph, the CDF is an increasing function because probabilities accumulate as the variable increases. For the Inverse Pranav distribution, the CDF likely starts near zero and rises towards one, indicating that lower values are more common while higher values become increasingly rare. Also, the CDF increases slowly at first and then rapidly, suggesting a heavy-tailed behavior, meaning some extreme values are possible.

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4.3 Survival and hazard functions of the Inverse Pranav distribution

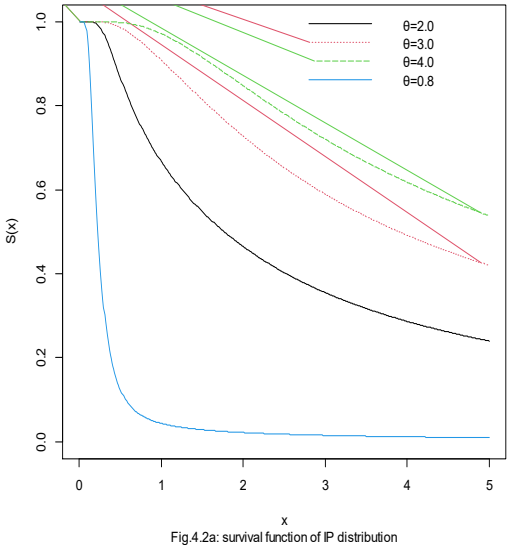

Fig.4.2a: survival function of IP distribution

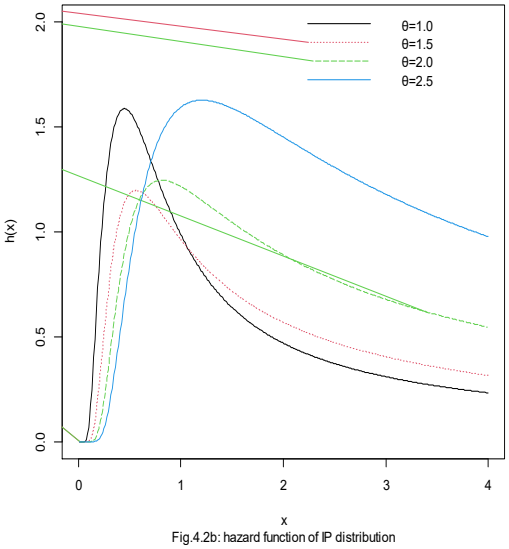

Fig.4.2b: hazard function of IP distribution

The survival function (Figure 4.2a) represents the probability that a system or individual survives beyond a given time. It is a non-increasing function that starts at 1 (100% survival) and gradually decreases to 0 as time progresses, indicating that failure or death becomes inevitable. A slower decline in the survival curve suggests greater longevity, while a steeper drop indicates higher failure rates. Different curves correspond to different values of θ. As x increases, $S(x)$ decreases, meaning the probability of survival declines.

The hazard function (Figure 4.2b) describes the instantaneous rate of failure at any given time, given that survival has occurred up to that point. It provides insight into the risk dynamics over time. A constant hazard function suggests an exponential-like failure pattern, while an increasing hazard function indicates a growing risk over time, common in aging systems. Conversely, a decreasing hazard function suggests early failures dominate, followed by improved reliability. In fig. 4.2b, the hazard function initially increases and then either levels off or decreases, depending on the value of θ, which indicates different risk patterns over time.

4.4 Cumulative proportional failure and TTT Plots of the data

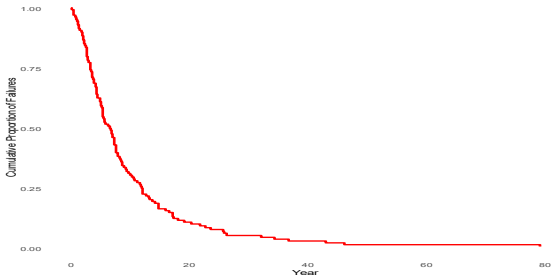

Fig. 4.3: Cumulative proportional failure

The graph in fig. 4.3 represents a cumulative failure function or a Kaplan-Meier failure curve of the data used in this research. It shows the cumulative incidence of cancer recurrence (failure) over time. The X-axis represents time in years (since remission). It indicates how long patients remain in remission before experiencing recurrence. The Y-axis (Cumulative Proportion of Failures) shows the cumulative probability of cancer recurrence over time. It ranges from 0 (no recurrences) to 1 (all patients have relapsed). The Red Stepwise Line represents the cumulative proportion of patients who have experienced a recurrence at each time point. The steep initial increase indicates a high recurrence rate in the early months after remission. The curve

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flattens out later, suggesting that patients who remain in remission for a longer period have a lower risk of recurrence.

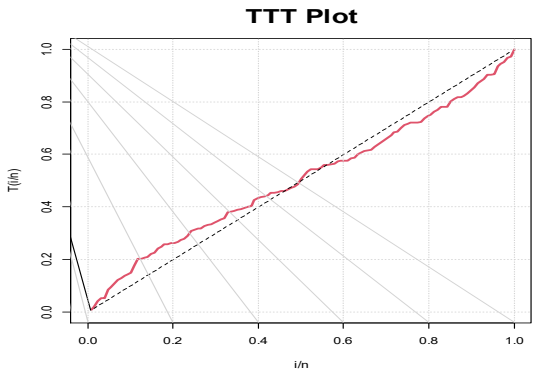

Fig. 4.4: Total Time on Test (TTT) Plot for Bladder Cancer Remission Data

The Total Time on Test (TTT) plot (fig. 4.4) is a graphical method used to assess the shape of the failure rate function (hazard function) of a dataset. It provides insights into whether the failure rate is increasing, decreasing, or constant over time. The X-axis (i/n) represents the proportion of the total number of observations (normalized rank of failures). The Y-axis (T(i/n)) represents the cumulative sum of observed failure times, scaled by the total failure time. The Convex (below the diagonal line, early flattening) represents decreasing failure rate (DFR). It indicates an initially high failure rate that decreases over time. It also suggests early failures due to the initial recurrence of bladder cancer soon after remission. The concave (above the diagonal line, late rise) represents an increasing failure rate (IFR). It indicates that the failure rate increases over time. It also suggests ageing effects, meaning that the longer a patient remains in remission, the higher their risk of recurrence over time. The linear (close to a diagonal line) represents a constant failure rate (CFR). It suggests that failures occur randomly over time, similar to an exponential distribution. This indicates that the recurrence of bladder cancer occurs at a roughly constant rate, independent of remission duration. The TTT curve starts below the diagonal, then rises above it, indicating a bathtub-like shape. This suggests that the hazard rate is initially decreasing but later increases. Initially, recurrence is high, possibly due to aggressive cancer relapses in some patients. Later, the risk decreases, meaning some patients experience long remission periods. After a certain point, the hazard rate increases again, possibly due to the natural progression of cancer in ageing patients.

V. CONCLUSION

This study reveals that through the comparative analysis with actual medical data, the research formulates that the IPD is better at capturing data sets with heavy tails, right skewness, and non-monotonic hazard behavior; also, it reasonably models failure patterns where risk diminishes as time goes on, which are poorly modeled by standard exponential or Weibull distributions; and finally the shape (α) and scale (β) parameter values significantly influence the flexibility of the distribution as it adjusts to a large variety of survival patterns.

In all, the IPD excels over inverse lifetime distributions both in statistical fit and interpretability for medical remission data. The research confirms the applicability of the IPD in reliability modeling, biomedical survival analysis, and other fields where lifetime behavior is never exponential. Future work should involve the extension of IPD to handle censored data, typical in clinical trials, and to multivariate survival situations where there is more than one covariate affecting the failure time. Also, inference can also be improved using Bayesian estimation techniques for IPD parameters, either in small samples or with prior knowledge.

REFERENCES

[1] Abed al-kadim, Kareema & Hantoosh, Ahmad. (2014). Double Weighted Inverse Weibull Distribution.https://www.researchgate.net/publication/268447208_Double_Weighted_Inverse_Weibull_Distribution.

[2] Abebe, B., & Shukla, K. (2019). A discrete Pranav distribution and its applications. Biometrics & Biostatistics International Journal. https://www.semanticscholar.org/paper/f887f78851d6202c58763155ad7755ae7a70ab06

[3] Adamidis, K., & Loukas, S. (1998). A lifetime distribution with decreasing failure rate. *Statistics & Probability Letters*, *39*(1), 35–42.

[4] Alkarni, S. (2015). Extended inverse Lindley distribution: properties and application. https://springerplus.springeropen.com/articles/10.1186/s40064-015-1489-2

[5] Ali M, Khalil A, Ijaz M, Saeed N (2021) Alpha-Power Exponentiated Inverse Rayleigh distribution and its applications to real and simulated data. plos one 16(1): e0245253. https://doi.org/10.1371/journal.pone.0245253

[6] Alsultan, R. (2023). The Marshall-Olkin Pranav distribution: Theory and applications. Pakistan Journal of Statistics and Operation Research. https://www.semanticscholar.org/paper/c1e6d7b8cdfad762cca7e1d63e7c1a9605811549

[7] Badmus, Nofiu & Amusa, Saidi & Ajiboye, Yemisi. (2021). Weibull-Extended Pranav Distribution: Application To Lifetime Data Sets. 1. 104-120.

[8] Barreto-Souza, W., de Morais, A. L., & Cordeiro, G. M. (2011). The Weibull-geometric distribution. *Journal of Statistical Computation and Simulation*, *81*(5), 645–657. [1]

[9] Bhati, Deepesh & Malik, Mohd & Kanichukattu Jose (2016). A new 3-parameter extension of generalized Lindley distribution.http://arxiv.org/abs/1601.01045.

[10] C. C. Odom, E. C. Nduka, M.A. Ijomah, (2019). A Modification of Pranav Distribution using Quadratic Rank Transmutation Map Approach. International Journal of Scientific Research in Mathematical and Statistical Sciences, 6(2), 193-202.

[11] Chaudhary, A., Sah Telee, L. B., & Kumar, V. (2022). Inverse Exponentiated Odd Lomax Exponential Distribution: Properties and Applications. Nepalese Journal of Statistics. https://www.semanticscholar.org/paper/f27d71f6f4cdfa0344502e945c73bc01db8464e5

[12] Chrisogonus, Onyekwere & Osuji, George & Enogwe, Samuel. (2020). Inverted Power Rama Distribution with Applications to Life Time Data. Asian Journal of Probability and Statistics. 9. 1-21. 10.9734/AJPAS/2020/v9i430231.

[13] Cordeiro, G. M., & Ortega, E. M. M. (2010). The beta generalized exponential distribution. Journal of Statistical Computation and Simulation, 80, 159–172.

[14] Edith, Umeh & Henrietta, A.. (2019). A Two-Parameter Pranav Distribution with Properties and Its Application. Journal of Biostatistics and Epidemiology. 5. 10.18502/jbe.v5i1.1909.

[15] Enogwe, S. U., Obiora-Ilouno, H., & Onyekwere, C. K. (2020). Inverse Power Akash Probability Distribution with Applications. European Journal of Mass Spectrometry. https://www.semanticscholar.org/paper/aba5acdb348b9e1b4ee3c63299987ac549d78bde

[16] Eraikhuemen, I. B., Onwuka, G., Yakura, B. S., & Allahde, H. (2021). An Exponentiated Odd Lindley Inverse Exponential Distribution and its Application to Infant Mortality and HIV Transmission Rates in Nigeria. Asian Journal of Probability and Statistics. https://www.semanticscholar.org/paper/dabfff95cc152e8508734f85ab90fc86aef4fbdd

[17] Fatima, K., & Ahmad, S. P. (2017). The Exponentiated Inverted Exponential Distribution. Department of Statistics, University of Kashmir. Retrieved from kawsarfatima@gmail.com. http://www.publishingindia.com/GetBrochure.aspx?query=UERGQnJvY2h1cmVzfC80MjczLnBkZnwvNDI3My5wZGY=

[18] Ganaie, R., & Rajagopalan, V. (2020). A New Generalization of Quasi Gamma Distribution with Properties and Applications. International Journal of Engineering and Advanced Technology.https://www.semanticscholar.org/paper/0a7f4fd647e96bf731629bc50ff8d6051cac6c97

[19] Ganaie, R., & Rajagopalan, V. (2020). Length Biased Weighted Quasi Gamma Distribution with Characterizations and Applications. Conference Proceedings. https://www.semanticscholar.org/paper/b10f87e23f0ebc81a5c0322080959dc77acd6852

[20] Gusmão, Felipe & Ortega, Edwin & Cordeiro, Gauss. (2011). The generalized inverse Weibull distribution. Statistical Papers. 52. 591-619. 10.1007/s00362-009-0271-3.

[21] Ghitany, M., Atieh, B. and Nadadrajah, S. (2008) Lindley Distribution and Its Applications. Mathematics and Computers in Simulation, 78, 493-506. http://dx.doi.org/10.1016/j.matcom.2007.06.007

[22] Hogg, R.V. and Craig, A. T (2013). Introduction To Mathematical Statistics. Pearson Education, Inc., 7th edition.

[23] Ibrahim, S., Doguwa, S., Isah, A., & Jibril, H. M. (2020). On the Flexibility of Topp Leone Exponentiated Inverse Exponential Distribution. Journal of Data Science, 6, 83. https://www.semanticscholar.org/paper/9c6f7a215decee9fad8dde6fae7f650e770c561f

[24] Kartsonaki, C. (2016). Survival analysis. *Diagnostic Histopathology, 22*(7), 263–270. https://doi.org/10.1016/j.mpdhp.2016.06.005

[25] Kargbo, M., Gichuhi, A. W., & Wanjoya, A. (2023). A novel extended inverse-exponential distribution and its application to COVID-19 data. Engineering eports.

[26] https://www.semanticscholar.org/paper/f832eebf64310b143f0dbaf3a12134a057a27fa0

[27] Kayid, M., & Shrahili, M. (2023). Rényi Entropy for Past Lifetime Distributions with Application in Inactive Coherent Systems. Symmetry, 15(7), 1310. https://doi.org/10.3390/sym15071310

[28] Kamlesh Kumar Shukla A Truncated Two Parameter Pranav Distribution and its Applications // RT&A. 2020. No3 URL: https://cyberleninka.ru/article/a-truncated-two-parameter-pranav-distribution-and-its-applications (last date: 26.04.2025).

[29] Khan, Muhammad Shuaib & King, Robert. (2012). Modified Inverse Weibull Distribution. Journal of Statistics Applications and Probability. 1. 115-132. 10.12785/jsap/010204.

[30] Nwankwo, K. C., Onyeagu, I. S., Nwankwo, C.H., Osuji, G. A., & Onyekwere, C. K. (2021). On the Characteristics and Application of Inverse Power Pranav Distribution. Journal of Advances in Mathematics and Computer Science. https://www.semanticscholar.org/paper/054e71df2c66825bdd470d34e20783fba5f3bbc3

[31] Rashid, Q., Yaseen, H. M., Uzair, M., & Jamshaid, M. T. (2022). Three Parameters Quasi Gamma Distribution and with Properties and Applications. Pakistan Journal of Statistics and Operation Research. https://www.semanticscholar.org/paper/fc0053bdd9e9ec3f5b216e0927e197ccc17615b3

[32] Rigby, R. A., and Stasinopoulos, D. M. (2005). *Generalized additive models for location, scale and shape (with discussion)*. Journal of the Royal Statistical Society: Series C (Applied Statistics), 54(3), 507–554. https://doi.org/10.1111/j.1467-9876.2005.00510.x

[33] Ross, S. M. (2023). Generating continuous random variables. In S. M. Ross (Ed.), Simulation (6th ed., pp. 69-98). Academic Press. https://doi.org/10.1016/B978-0-32-385738-3.00010-9

[34] Shahbaz, Muhammad & Shahbaz, Shahbaz & Butt, Nadeem. (2012). The Kumaraswamy-Inverse Weibull Distribution. SSRN Electronic Journal. 10.2139/ssrn.2168680.

[35] Sharma, V., & Khandelwal, P. (2021). On the extension of inverse Lindley distribution. Journal of Data Science. https://www.semanticscholar.org/paper/fd2928a5f3d356e6b1c0e1681859ef36a63cf4a1

[36] Shanker, R. and Shukla, K.K. (2017) Ishita Distribution and Its Applications. Biometrics & Biostatistics International Journal, 5, 1-9. https://doi.org/10.15406/bbij.2017.05.00126

[37] Shukla, K. (2018). Pranav distribution with properties and its applications. Biometrics & Biostatistics International Journal, 7. https://www.semanticscholar.org/paper/d9820eb1a3173a91e3a8bed38ef6c4e517e816ed

[38] Shukla, K. K. (2024). Truncated Pranav distribution: Properties and applications. *Reliability: Theory & Applications, 18*(1), 178–192. https://gnedenko.net/Journal/2023/012023/RTA_1_2023-15.pdf

[39] Shukla, K., & Shanker, R. (2020). A Weighted Pranav Distribution and its Application to Survival Data. Indian Journal of Industrial and Applied Mathematics, 11, 79. https://www.semanticscholar.org/paper/1690f05fb7f2fe86e41675a807d43ccb505488ec

[40] Silva, G. O., & Cordeiro, G. M. (2013). The beta-Weibull geometric distribution. Statistics, 47, 817–834. https://www.semanticscholar.org/paper/db045e3421ce83b41c890c25c27ad5c40651fb65

[41] Treng Kirnan Gayus Sani Ibrahim Doguwa (2022). Bayesian Estimation of the Parameters of the Odd Generalized Exponentiated – Inverse Exponential Distribution. Asian Journal of Probability and Statistics. 16(2), 25-36

[42] Uwaeme, O. R., Akpan, N., & Orumie, U. C. (2019). An Extended Pranav Distribution. Asian Journal of Probability and Statistics. https://www.semanticscholar.org/paper/57eccc0036cd320f3b5a2931720de16986c1ede2

[43] Wani, S. A., Hassan, A., & Shafi, S. (2020). Pranav Quasi Gamma Distribution: Properties and Applications. Journal of Statistical Theory and Applications. https://www.semanticscholar.org/paper/ad488ba137860328e4d40005c7f2d52a2a344cef

[44] Wani, Sameer. (2020). Poisson-Pranav Distribution and Its Applications. 57-72.

[45] PRANAV_DISTRIBUTION_AND_ITS_APPLICATIONS/link/609fac63458515c265911b68/download?_tp=eyJjb250ZXh0Ijp7InBhZ2UiOiJwdWJsaWNhdGlvbiIsInByZXZpb3VzUGFnZSI6bnVsbH19 https://www.researchgate.net/publication/351601240_POISSON