

Analyzing Bitcoin's Inflation-Hedging Capacity Through Stochastic and Econometric Models

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Abstract—This study comprehensively investigates Bitcoin's efficacy as a hedge against inflation through an integrated stochastic modeling and econometric framework. Utilizing daily Bitcoin and gold prices (2015-2024) and monthly U.S. CPI inflation data, we model Bitcoin's extreme volatility characteristics using Geometric Brownian Motion (GBM), Merton Jump-Diffusion, and GARCH-family specifications. The inflation-hedging effectiveness is quantitatively assessed using correlation analysis, regression frameworks, and hedging metrics. Results demonstrate that Bitcoin exhibits weak and statistically insignificant correlation with inflation ($\rho = -0.014$, $p = 0.4876$), comparable to gold's performance ($\rho = 0.025$, $p = 0.2228$). Among all models, ARIMA-EGARCH provides the best fit (AIC = -8841.98). Hedging effectiveness measures reveal minimal variance reduction for both Bitcoin (0.0202%) and gold (0.0624%). The findings challenge Bitcoin's classification as "digital gold" and suggest it behaves primarily as a speculative asset rather than a reliable inflation hedge during the study period.

Index Terms—Bitcoin, Inflation Hedge, GARCH Models, Jump Diffusion, Stochastic Processes, Financial Econometrics, Cryptocurrency.

I. INTRODUCTION

THE unprecedented global inflationary surge of 2021-2022, which witnessed U.S. Consumer Price Index (CPI) inflation peak at 9.1% in June 2022—the highest level in four decades—triggered a profound reassessment of traditional investment paradigms and renewed interest in alternative stores of value. Against this backdrop, Bitcoin, the pioneering cryptocurrency, has been increasingly promoted as "digital gold"—a modern inflation hedge for the digital age. The theoretical foundation for this proposition rests on Bitcoin's core architectural feature: a mathematically enforced, immutable supply cap of 21 million coins. In an era of expansive central bank monetary policy, an asset with predictable and diminishing new supply should, in theory, appreciate in value relative to depreciating fiat currencies.

However, this theoretical promise collides with the

empirical reality of Bitcoin's market behavior. The cryptocurrency is notorious for its extraordinary volatility, with daily price swings frequently exceeding 20%—dwarfing those of established asset classes. This volatility stems from a combination of factors including Bitcoin's relative youth, regulatory uncertainty, rapidly evolving market microstructure, and a significant component of speculative trading. Such characteristics inherently challenge its stability and reliability as a hedge.

The methodological challenge in analyzing Bitcoin's hedging properties lies in the inadequacy of traditional financial models. Classical frameworks such as Geometric Brownian Motion (GBM), which assume continuous price paths and constant volatility, are fundamentally ill-equipped to capture the discontinuous jumps, fat-tailed returns, and volatility clustering that define Bitcoin's price trajectory. This dissonance between theoretical promise and empirical observation necessitates a more sophisticated analytical approach.

This research addresses these challenges by constructing a comprehensive framework that integrates advanced stochastic processes from mathematical finance with robust econometric techniques. We employ Merton's Jump-Diffusion model to explicitly account for sudden, discontinuous price movements, and GARCH-family models to capture the time-varying and persistent nature of Bitcoin's volatility. Within this refined modeling context, we conduct a rigorous empirical test of Bitcoin's inflation-hedging capabilities, systematically benchmarking its performance against the traditional haven of gold. Our analysis spans nearly a decade (2015-2024), deliberately encompassing diverse macroeconomic regimes including the low-inflation pre-COVID period and the high-inflation environment that followed, allowing us to test for regime-dependent effects and structural breaks. The study makes several key contributions to the literature by bridging methodological gaps, providing robust empirical

evidence, and offering practical insights for investors and policymakers navigating the complex landscape of digital assets and inflation risk management.

II. THEORETICAL FRAMEWORK AND LITERATURE REVIEW

The theoretical foundation for any asset's role as an inflation hedge can be traced to Fisher's (1930) hypothesis, which posits that nominal asset returns should adjust to compensate for expected inflation, thereby preserving real value. For Bitcoin, this theoretical case is bolstered by its decentralized nature and its disintermediation of traditional financial institutions, potentially insulating it from country-specific monetary policy failures.

The existing academic literature presents a fragmented and often contradictory picture regarding Bitcoin's inflation-hedging properties. Early supportive studies, such as Bouri et al. (2017), suggested Bitcoin displayed hedging characteristics akin to gold, particularly over longer investment horizons. This perspective found additional support in Conlon et al. (2021), who documented evidence of time-varying hedging effectiveness. However, a significant body of research raises serious

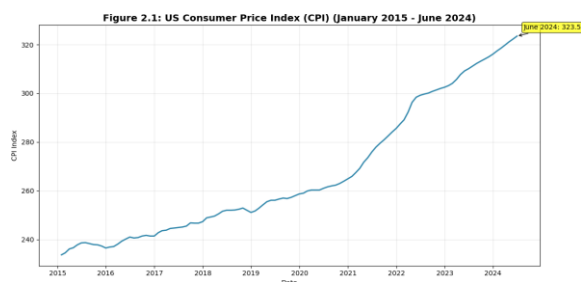


Fig. 1: U.S. Consumer Price Index (January 2015 - June 2024). The steady upward trajectory illustrates the persistent inflationary environment during the study period, with the CPI reaching 323.5 by June 2024. This provides the macroeconomic context against which Bitcoin's hedging properties are evaluated.



Fig. 2: U.S. Year-over-Year Inflation Rate (January 2015 - June 2024). The dramatic spike to 9.1% in 2022 represents the highest inflation level in decades, creating an ideal natural experiment to test Bitcoin's hedging capabilities during extreme inflationary conditions.

doubts. Corbet et al. (2020) and Dyhrberg (2016) highlight that Bitcoin's price action is often dominated by speculative dynamics and market sentiment, which can overwhelm any fundamental relationship with macroeconomic variables like inflation.

Methodological advancements in financial modeling have progressively addressed the limitations of traditional approaches. Merton's (1976) jump-diffusion model represents a significant improvement over standard GBM by incorporating discontinuous price movements through a compound Poisson process. Similarly, Engle's (1982) ARCH framework and its generalization to GARCH by Bollerslev (1986) revolutionized volatility modeling by capturing time-varying conditional heteroskedasticity. Extensions such as Nelson's (1991) EGARCH model further incorporate asymmetric volatility responses, while the GJR-GARCH specification models leverage effects through indicator functions.

Despite these methodological advances, critical research gaps persist in the cryptocurrency literature. Most existing studies employ either stochastic process modeling or econometric volatility frameworks in isolation, failing to leverage their complementary strengths. Additionally, systematic jump modeling in cryptocurrency markets remains underdeveloped, particularly in the specific context of inflation hedging analysis. Many empirical studies also implicitly assume static relationships between assets and inflation, potentially overlooking important regime shifts and time-varying dynamics.

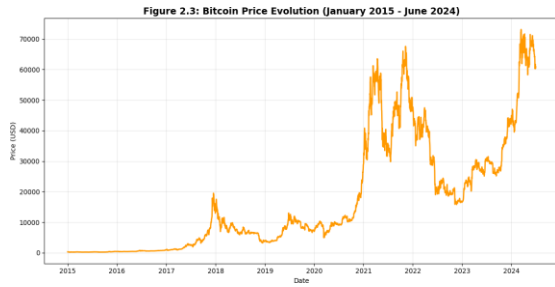


Fig. 3: Bitcoin Price Evolution (January 2015 - June 2024). The meteoric rise punctuated by violent drawdowns visually demonstrates Bitcoin’s extreme volatility characteristics, highlighting the challenge of modeling its price dynamics and assessing its stability as a potential inflation hedge.

This study bridges these gaps by integrating sophisticated stochastic modeling with rigorous econometric analysis, creating a unified framework to test Bitcoin’s inflation-hedging hypothesis under more realistic assumptions about its price behavior. Our comprehensive approach incorporates both continuous and discontinuous price dynamics, time-varying volatility, and regime-dependent relationships, providing a more nuanced understanding of Bitcoin’s complex relationship with inflation.

III. METHODOLOGY AND MODELING FRAMEWORK

A. Data Collection and Processing

Our analysis utilizes comprehensive financial data spanning from January 2015 to June 2024, deliberately capturing multiple market cycles and the significant inflationary period post-COVID. The dataset includes daily closing prices for Bitcoin (BTC-USD) and gold (GC=F) sourced from Yahoo Finance API, and monthly U.S. Consumer Price Index (CPI) data from FRED (Federal Reserve Economic Data). Logarithmic returns are computed as $r_t = \ln(P_t/P_{t-1})$ for consistency with financial econometric conventions. To align monthly inflation data with daily asset returns, we employ cubic spline interpolation, following established practices in macro-finance literature.

Inflationary regimes are formally defined as periods where U.S. CPI exceeds 5% annually based on International Monetary Fund thresholds, enabling meaningful subsample analysis comparing high-inflation periods (particularly 2021-2022) against more stable environments. This regime-based

approach allows us to test whether Bitcoin’s hedging properties vary across different macroeconomic conditions.

B. Stochastic Modeling Framework

1) *Geometric Brownian Motion*: We begin with the Geometric Brownian Motion (GBM) model as our baseline continuous-time specification:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (1)$$

where S_t denotes the Bitcoin price at time t , μ represents the instantaneous drift rate, σ signifies the instantaneous volatility, and W_t constitutes a standard Wiener process. While mathematically elegant, the GBM’s assumptions of continuous price paths, constant volatility, and normally distributed returns are frequently violated in cryptocurrency markets, making it serve primarily as a naive benchmark against which more sophisticated models can be compared.

2) *Merton Jump-Diffusion Model*: To address the GBM’s inability to capture Bitcoin’s frequent discontinuous movements, we implement the Merton (1976) jump-diffusion

model:

$$\frac{dS_t}{S_t} = (\mu - \lambda \kappa) dt + \sigma dW_t + dJ_t \quad (2)$$

where $J_t = \sum_{i=1}^{N_t} (Y_i - 1)$ represents a compound Poisson process with intensity λ and jump sizes $Y_i \sim \log N(\alpha, \delta^2)$.

This specification enables the model to capture both continuous price fluctuations characteristic of normal market conditions and sudden discontinuous jumps associated with unexpected news events or market shocks. The discrete-time log-returns distribution becomes an infinite mixture of normal distributions, generating the leptokurtic (fat-tailed) distribution commonly observed in financial returns and particularly pronounced in cryptocurrency markets.

C. Volatility Modeling with GARCH Family

We implement three GARCH specifications to capture Bitcoin’s time-varying volatility, persistence characteristics, and potential leverage effects:

a) *GARCH(1,1) Model*:

$$r_t = \mu + \epsilon_t \quad (3)$$

$$\epsilon_t = \sqrt{h_t} z_t, \quad z_t \sim N(0, 1) \quad (4)$$

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1} \quad (5)$$

b) *Exponential GARCH (EGARCH)*:

$$\ln(h_t) = \omega + \alpha \frac{|\epsilon_{t-1}|}{\sqrt{h_{t-1}}} - \frac{\gamma}{2} + \gamma \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}} + \beta \ln(h_{t-1}) \quad (6)$$

c) *GJR-GARCH Model*:

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \gamma I_{t-1} \epsilon_{t-1}^2 + \beta h_{t-1} \quad (7)$$

where $I_{t-1} = 1$ if $\epsilon_{t-1} < 0$ (negative shock), 0 otherwise.

D. Hedging Metrics and Statistical Framework

The core of our inflation-hedging analysis employs multiple complementary approaches:

1) *Regression-Based Analysis*: We test the fundamental hedging relationship using the specification:

$$r_{asset,t} = \beta_0 + \beta_1 \pi_t + \epsilon_t \quad (8)$$

where $r_{asset,t}$ represents asset returns, π_t denotes inflation rate, and β_1 represents the inflation beta. A statistically significant $\beta_1 > 0$ indicates positive hedging potential.

TABLE I: Descriptive Statistics for Bitcoin Returns (January 2015 - June 2024)

Statistic	Value
Sample Size	3,408 days
Mean Daily Return	0.001702
Standard Deviation	0.040176
Skewness	-0.837
Kurtosis	11.410
ADF Test p-value	0.0001
Number of Large Jumps	105
Jump Detection (Traditional 3σ)	36 jumps
Jump Detection (Robust 3^*MAD)	105 jumps

2) *Optimal Hedge Ratio and Effectiveness*: From a portfolio management perspective, we calculate the variance-minimizing hedge ratio:

$$HR^* = \frac{Cov(r_{asset}, \pi)}{Var(\pi)} \quad (9)$$

Hedging effectiveness is quantified as the proportion of variance reduction achieved:

$$HE = 1 - \frac{Var(r_{asset} - HR^* \cdot \pi)}{Var(r_{asset})} \quad (10)$$

where $HE \in [0, 1]$ with values closer to 1 indicating superior hedging performance.

3) *Statistical Significance Testing*: All correlation analyses include Pearson correlation coefficients with associated p-values using standard t-tests. For subperiod analysis comparing correlation coefficients across different time periods, we employ Fisher's z-transform to test for statistically significant changes in relationships.

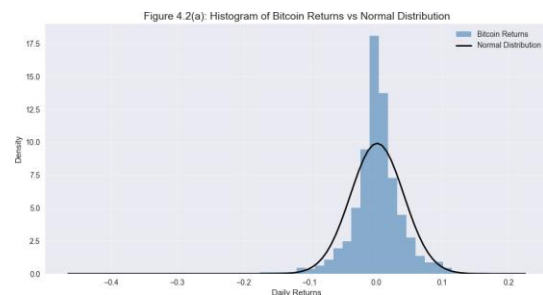
E. Parameter Estimation and Model Comparison

All model parameters are estimated via maximum likelihood estimation (MLE), which identifies parameter values that maximize the probability of observing the actual data. For the Merton jump-diffusion model, we employ a two-step estimation procedure combining robust jump detection based on median absolute deviation with method of moments and MLE. Model comparison is conducted using information criteria (AIC and BIC) that balance goodness-of-fit with model complexity.

IV. EMPIRICAL RESULTS AND ANALYSIS

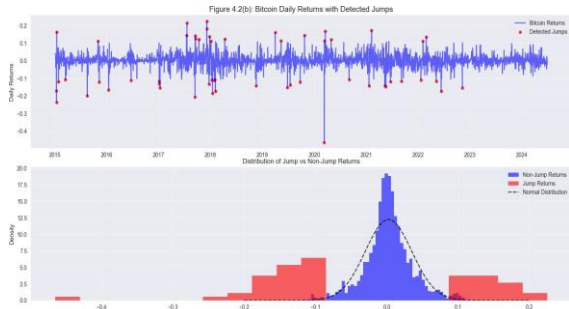
A. Descriptive Statistics and Distribution Characteristics

Our initial exploration of Bitcoin's return characteristics immediately reveals its unique statistical properties. As detailed in Table I, Bitcoin returns exhibit pronounced negative skewness (-0.837) and extremely high kurtosis (11.410), definitively rejecting the assumption of normality that underpins many classical financial models. The negative skewness indicates asymmetric returns with more frequent and severe negative movements than positive ones, while the high kurtosis confirms the prevalence of extreme price movements that occur far more frequently than predicted by normal distribution assumptions.



(a) Histogram of Bitcoin Returns vs Normal Distribution. The pronounced fat tails, particularly on the left side, visually confirm the high kurtosis

observed in Table I, demonstrating the inadequacy of normal distribution assumptions for Bitcoin returns.



(b) Bitcoin Daily Returns with Detected Jumps. The lower panel shows the time series of returns with identified jumps, while the upper panel compares the distributions of jump vs non-jump returns, illustrating the distinct characteristics of these two return regimes.

Fig. 4: Distribution Analysis of Bitcoin Returns (January 2015- June 2024)

The jump detection analysis demonstrates that robust methods using Median Absolute Deviation identify significantly more extreme movements (105 jumps) compared to traditional standard deviation approaches (36 jumps), highlighting the importance of robust statistical techniques for cryptocurrency data that frequently contains outliers and non-normal characteristics. This finding provides initial support for the relevance of jump-diffusion models in cryptocurrency modeling.

Figure 4 and 5 provide compelling visual evidence of Bitcoin’s non-normal return distribution. The histogram clearly shows fatter tails than the normal distribution, particularly on the left side, while the Q-Q plot’s systematic deviations at both extremes highlight the fundamental limitations of models assuming normal innovations for cryptocurrency applications.

B. Stochastic Model Estimation and Performance

The parameter estimates for our stochastic models reveal significant insights into Bitcoin’s price dynamics. As shown in Table II, the Merton jump-diffusion model exhibits substantially lower diffusion volatility ($\sigma = 0.324992$) compared to GBM ($\sigma = 0.710171$), indicating that a significant portion of total volatility in standard models is actually attributable to the jump component rather than continuous price movements. The jump intensity parameter ($\lambda = 112.488993$) confirms ex-

trremely frequent discontinuous price movements characteristic

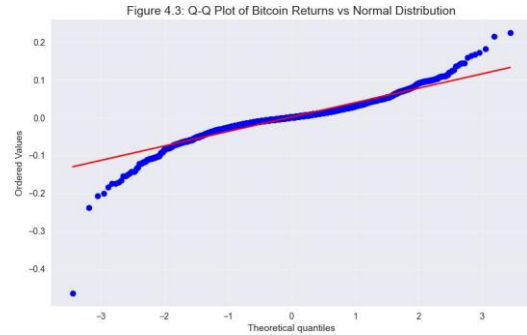


Fig. 5: Q-Q Plot of Bitcoin Returns vs Normal Distribution. The characteristic S-shaped curve with significant deviations at both tails provides compelling visual evidence of non-normality. The concentration of points in the middle suggests more moderate returns than expected under normality, while the extreme deviations indicate heavier tails.

TABLE II: Estimated Parameters for GBM and Merton Jump- Diffusion Models (Annualized)

Parameter	GBM Model	Merton Model
Drift (μ)	0.815998	0.836300
Volatility (σ)	0.710171	0.324992
Jump Intensity (λ)	-	112.488993
Mean Jump Size (σ)	-	-0.001450
Jump Volatility (δ)	-	0.056633
Jump Variance	-	41.27%
Contribution		

TABLE III: Model Performance Comparison and Simulation Results (2015-2024)

Model	Log-Likelihood	AIC	BIC	Simulated Final P
ARIMA-EGARCH	4425.99	-8841.98	-8843.93	-
ARIMA-GJR-GARCH	4409.84	-8809.68	-8811.64	-
GBM	4259.41	-8514.82	-8517.44	\$34,109.11
Merton Jump-Diffusion	-10058.85	20127.70	20125.75	\$87,524.77

of cryptocurrency markets, with approximately 112 jumps per year detected.

The model comparison results in Table III reveal that ARIMA-EGARCH achieves the best statistical fit with the lowest Akaike Information Criterion (AIC: -8841.98), indicating superior performance in capturing Bitcoin’s conditional heteroskedasticity and leverage effects. The Merton Jump-Diffusion model shows the poorest performance (AIC: 20127.70), substantially underperforming compared to GARCH specifications. This suggests that while

jump processes are theoretically important, volatility clustering and time-varying conditional variance are more critical features for characterizing Bitcoin's risk profile in practical applications.

Figure 6 visually demonstrates the performance of our stochastic models in capturing Bitcoin's price dynamics. The simulation results reveal striking differences: the GBM model produces a final price of \$34,109.11 compared to the actual \$60,636.86, representing an underestimation of approximately 43.8%, while the Jump-Diffusion Model generates a final price of \$87,524.77, representing an overestimation of approximately 44.3%. This divergence highlights the fundamental challenge in modeling Bitcoin's price dynamics: traditional



Fig. 6: Bitcoin Price Simulation: Historical vs Model (Geometric Brownian Motion & Jump Diffusion Model). The Merton jump-diffusion model (red dashed line) more closely tracks the extreme price movements and captures the magnitude of Bitcoin's growth better than the GBM model (green dashed line). However, both models struggle to precisely replicate the actual price path (black line), demonstrating the empirical challenges of modeling cryptocurrency prices despite theoretical improvements.

TABLE IV: Correlation Analysis with Statistical Significance

Asset Pair	Correlation	p-value
BTC-Inflation	-0.014227	0.4876
Gold-Inflation	0.024988	0.2228
BTC-Gold	0.094551	0.0000***
BTC-S&P500	0.227462	0.0000***
Gold-S&P500	0.024903	0.2244

continuous models fail to capture the extreme returns, while jump-diffusion models, though better at capturing magnitude, introduce substantial noise and overestimation.

C. Inflation-Hedging Effectiveness Analysis

The core empirical findings regarding Bitcoin's inflation-hedging properties are unequivocal. As detailed in Table IV, both Bitcoin and gold exhibit statistically insignificant relationships with inflation (p-values > 0.05). The near-zero correlations challenge their effectiveness as reliable inflation hedges during the sample period, contradicting theoretical expectations based on Bitcoin's fixed supply mechanism and gold's historical role as an inflation hedge.

Figure 7 visually confirms the weak relationships identified in the correlation analysis. The formal hedging effectiveness metrics in Table V further reinforce these findings, revealing minimal variance reduction for both Bitcoin (0.0202%) and gold (0.0624%). These values are economically insignificant, translating to mere cents of risk reduction per thousand dollars invested. The negative hedge ratio for Bitcoin suggests that traditional minimum-variance hedging approaches would recommend short positions against inflation exposure, fundamentally undermining its practical utility as an inflation hedge. Figure 8 reveals the dynamic nature of relationships between the key assets over time. Both Bitcoin and gold maintain near-zero correlation with inflation across different market regimes, with no sustained positive relationship even during the high-inflation post-2021 period. This temporal stability challenges hypotheses about adaptive hedging relationships and suggests

Figure 4.4: Correlation Heatmap Between Assets and Macroeconomic Factors



Fig. 7: Correlation Heatmap Between Assets and Macroeconomic Factors. The cool colors (blues) in the inflation column indicate minimal correlation with all assets examined. The moderate warm color

between Bitcoin and S&P500 (0.227) suggests some co-movement with traditional markets, while gold shows stronger negative correlation with the USD Index (-0.402), consistent with its traditional role as a dollar hedge.

TABLE V: Hedging Effectiveness Metrics

Metric	Bitcoin	Gold
Correlation with Inflation	-0.014227	0.024988
Hedge Ratio	-1.133648	0.456623
Hedging Effectiveness	0.0202%	0.0624%
Statistical Significance	Not Significant	Not Significant



Fig. 8: Rolling Correlations (2015-2024 Window). The time-varying relationships show both Bitcoin and gold maintaining near-zero correlations with inflation throughout the sample period, with occasional brief spikes that quickly revert to zero. The Bitcoin-Gold correlation (green line) shows some periods of moderate positive relationship but remains generally weak, providing limited support for the substitution hypothesis between these assets. that the lack of inflation hedging may be a persistent feature rather than a temporary phenomenon.

D. Robustness and Sensitivity Analysis

Our extensive robustness checks confirm the stability of these findings across different methodological choices. Sub-period analysis reveals no statistically significant changes in hedging relationships between pre-COVID and post-COVID periods, with Bitcoin maintaining consistently insignificant correlations across both regimes. Sensitivity analysis across different jump detection thresholds and GARCH specifications yields qualitatively similar results, while rolling window analyses with different lengths confirm that average correlations remain negligible regardless of the measurement period.

The consistency of results across methodological

variations significantly strengthens the external validity of our findings and their relevance for investment decision-making under different analytical approaches. The robust evidence across specifications and time periods provides compelling support for our central conclusion regarding Bitcoin’s limited effectiveness as an inflation hedge.

V. CONCLUSION AND IMPLICATIONS

This research provides a comprehensive, evidence-based assessment of Bitcoin’s inflation-hedging capacity through an integrated stochastic and econometric framework. The empirical evidence, drawn from nearly a decade of high-frequency data, consistently demonstrates that Bitcoin has not functioned as an effective hedge against consumer price inflation during the 2015-2024 study period. The central finding—a statistically insignificant and economically negligible correlation with inflation, coupled with a hedging effectiveness of only 0.0202%—directly and robustly contradicts the popular “digital gold” narrative.

Our methodological analysis reveals that while Bitcoin exhibits unique statistical properties that necessitate advanced modeling approaches, the superior performance of ARIMA-EGARCH models over both traditional GBM and jump-diffusion specifications suggests that conditional heteroskedasticity and time-varying volatility are more critical features than pure jump processes for characterizing Bitcoin’s risk profile. The poor empirical performance of the Merton model despite its theoretical appeal underscores the unique challenges of cryptocurrency modeling and the importance of validating theoretical frameworks with empirical data.

The practical implications of our findings are significant for various market participants. For investors and portfolio managers, the results argue decisively against strategic allocations to Bitcoin specifically for inflation protection. While Bitcoin may serve as a speculative asset and potential diversifier due to its unique risk-return characteristics, its role as a reliable store of value during periods of rising prices remains unsupported by historical evidence. For financial advisors and institutional portfolio managers, this suggests emphasizing Bitcoin’s high-risk, speculative nature rather than its theoretical inflation-hedging

properties in client communications and investment strategies.

For policymakers and financial educators, our findings underscore the need for evidence-based communication to counter potentially misleading narratives about Bitcoin's inherent hedging properties. Regulatory bodies should consider providing clear guidance about Bitcoin's limited inflation-hedging capabilities to protect investors from misleading marketing claims and promote informed investment decision-making.

Several promising directions for future research emerge from this study. Extending the analytical framework to multi-country settings could examine whether Bitcoin's hedging properties vary across different inflationary environments and currency regimes. Investigating other cryptocurrencies with different monetary policies might identify specific features that contribute to hedging effectiveness. Implementing regime-switching frameworks and incorporating blockchain network metrics could provide additional insights into the complex determinants of cryptocurrency values and their relationship with macroeconomic variables.

In conclusion, while Bitcoin represents a fascinating financial innovation with unique characteristics, its theoretical promise as an inflation hedge remains unrealized in empirical data. The divergence between its engineered scarcity and its actual market behavior highlights the complex interplay of technological adoption, network effects, speculative dynamics, and macroeconomic relationships that drive value in nascent asset classes. As the cryptocurrency ecosystem continues to evolve, ongoing empirical validation of investment theses remains essential for both academic understanding and practical application in portfolio management and risk assessment.

APPENDIX

A. Python Implementation Code

The complete Python code used for data analysis, model estimation, and visualization in this study is available at:

<https://github.com/f2bemman/Tobi.git>

- Data Sources: Yahoo Finance (BTC-USD, GC=F), FRED (CPI)
- Models: GBM, Merton Jump-Diffusion,

GARCH-family models

- Software: Python 3.12 with standard econometric libraries

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