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Application of Bayesian Vector Autoregressive Model to some Macroeconomic Variables in Nigeria and Global Factors

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Abstract-** Understanding the dynamic interplay between macroeconomic variables and external shocks is central to effective policy formulation especially in emerging economies such as Nigeria. Given Nigeria's heavy reliance on oil exports and increasing exposure to international trade, it becomes imperative to examine how fluctuations in commodity prices such as oil, copper and maize alongside broader global factors influence domestic macroeconomic indicators. These relationships are inherently complex and interdependent, requiring an empirical approach capable of capturing their dynamic interactions. This study applied a Bayesian vector autoregressive model (BVAR) approach which is the advance version of vector autoregressive model (VAR) to examine the dynamic interrelationships between some key Nigerian macroeconomic variables and Global factor, specifically commodity prices – copper, maize, and oil – along with the Common Commodity Price Factor (CCPF) and global factors (GF). The BVAR framework was chosen because of its robustness for handling multivariate systems with limited samples and its ability to incorporate prior information, improving predictive accuracy and structural interpretation. Monthly time series data from April 1971 to July 2024 were obtained from the Central Bank of Nigeria (www.cbn.org.ng). The results reveal strong autoregressive patterns, particularly in the CCPF, copper, oil prices, and global indices, indicating high price persistence and systemic inertia in Nigeria's macroeconomic environment. Granger causality tests highlight significant bidirectional links, particularly between the CCPF and both maize and global factors, as well as the central role of copper prices as a transmission channel influencing maize prices and the global factors." Impulse "response capabilities and model diagnostics confirm the reliability of the model with stable dynamics and superior forecast performance compared to traditional VAR models. Residual normality test shows deviations such as excessive kurtosis and skewness, validating the appropriateness of Bayesian techniques, which are not limited by classical distribution assumptions. The findings concluded and validated that Nigeria's economy is vulnerable to external shocks and the influence of global commodity trends on domestic economic indicators. The work in its conclusion recommended that government should build a fiscal buffers, adopt BVAR-based forecasts in macroeconomic planning, and diversify strategies to reduce commodity dependency, with particular emphasis on global factor and oil prices as early warning indicators of economic instability."

***Keywords:** Bayesian-VAR, Macroeconomic, Variables, Stationary, Multivariate, Time Series, Common Commodity Price Factor, Global Factor.*

I. INTRODUCTION

Traditional econometric models such as the unrestricted Vector Autoregressive (VAR) model have been widely applied in macroeconomic studies to capture the interrelationships among time series variables (Chukwukelo, V.N. 2025). However, the classical VAR model suffers from critical limitations when the number of parameters grows rapidly relative to the available data points, leading to issues of overparameterization, multicollinearity, and unstable forecasts (Olomola & Adejumo, 2006),. These limitations are particularly pronounced in developing countries where data may be limited, noisy, or prone to structural breaks. To overcome these challenges, the Bayesian Vector Autoregressive (BVAR) model has emerged as a superior alternative (Giannone *et al*., 2015)." By "incorporating prior distributions that shrink coefficients toward more plausible values, BVAR models improve estimation precision, mitigate overfitting, and enhance forecasting performance (Karlsson, 2013)

The motivation for this study stems from the growing relevance of Bayesian methods in macroeconomic modeling, especially in the context of volatile economies like Nigeria. While several studies have explored the use of VAR models to analyze Nigerian macroeconomic behavior (Adebiyi & Adenuga 2010) few have adopted a Bayesian framework that explicitly integrates global commodity price movements and common international shocks." Additionally, there has been limited focus on "incorporating the Common Commodity Price Factor (CCPF), which captures the co-movement among major commodity prices, offering a more comprehensive measure of external shocks. Similarly, many existing studies neglect the influence of global factors (GF), such as global monetary policy shifts, international interest rates, or global economic slowdowns, which have become increasingly significant in shaping domestic economic performance in an interconnected world (Mumtaz Surico, 2009).

Given these gaps, the present study aims to apply a Bayesian VAR model to investigate how Nigeria's macroeconomic variables respond to shocks in commodity prices—namely copper, maize, and oil—along with the CCPF and selected global factors. The BVAR model is particularly well-suited for this analysis as it accommodates the inclusion of a larger set of variables without sacrificing estimation efficiency." Moreover, the "Bayesian estimation procedure enables the researcher to incorporate prior knowledge and uncertainty about model parameters, thus providing a more robust and data-driven analysis of macroeconomic dynamics in the presence of external volatility.

Specifically, this study seeks to achieve the following objectives: First, to estimate the BVAR model using macroeconomic data for Nigeria and external variables including copper, maize, oil prices, the Common Commodity Price Factor, and global factors. Second, to examine the impulse response functions derived from the BVAR to assess the effect of commodity price and global shocks on Nigeria's key macroeconomic indicators (Chukwukelo V.N. 2025). Third, to conduct forecast error variance decomposition to determine the relative contribution of each external factor in explaining domestic economic fluctuations." Lastly, to evaluate the "model's predictive performance compared to conventional VAR models, with emphasis on the role of priors and shrinkage techniques in improving estimation accuracy. These objectives follow the sequential steps in estimating BVAR models, namely prior specification, posterior simulation, impulse response analysis, and variance decomposition—each of which provides a unique lens for understanding the impact of external shocks on the Nigerian economy. (Deebom, Z.D 2025)

By addressing the methodological limitations of previous studies and incorporating a broader set of relevant global and commodity-specific variables, this study contributes to the growing body of knowledge on macroeconomic modeling in emerging economies. It also provides policymakers with a more accurate and flexible framework for anticipating the effects of international economic developments on national economic outcomes.

II. LITERATURE REVIEW

Chukwukelo, V.N (2025) examined the interactions among selected Macroeconomic variables in Nigeria. His work made an attempt to study if there exist any interaction among prices of copper, maize and oil. Data on these selected variables ranging from April 1971 to July 2024 were carefully extracted from central bank of Nigeria's website (www.cbn.org.ng). Unlike other macroeconomic variables that exhibits interdependence among themselves, his study demonstrated price independence among copper, maize and oil. Result of his analysis showed stationary series, differencing was done to take care of stochastic disturbance term. Cointegration test was deemed not necessary. The work adopted lag 1 as suggested from the lag selection result output. Table 3 of his work displayed VAR estimates result that suggested weak interaction among the prices of copper, maize and oil. Their R-squared and adjusted R-squared show a very weak explanatory power among the three variables. For copper it is 13% and 12% respectively, for maize 7% and 6% respectively and for oil 4% and 3% respectively. These were convincing evidence to say that there was no interaction among the prices of the variables. Table 4 of his results showed VAR residual serial correlation. From the result output, all the probability values were greater than 0.05 at lag 1 so Ho was not rejected. He concluded that there was no serial correlation. Table 5 of his work showed VAR residual heteroskedasticity tests, from the result output he observed that the joint chi-square value of 253.2185 with 162 df and P=0.0000 showed significant evidence of homoscedastic. That is, there was no heteroskedasticity. Table 6 of same analysis showed VAR residual Normality tests, the jarque-Bera test value of 6896.977 was far way greater than the P-values (0.0000) indicating non rejection of Ho meaning that VAR residuals are normally distributed. Table 7 of the same analysis showed dynamic stability test of the VAR estimates, from the plot it was clearly seen that no root lies outside the unit circle. Hence, VAR satisfies the stability condition. His Table 8 showed granger causality/block exogeneity Wald tests. From the result output, there was clearly no variable that granger causes each other because their various chi-square values are greater than 0.05. Copper, maize and oil do not granger cause each other hence there will be no need for impulse response function (IRF) test. In conclusion, as important as these macroeconomic variables are, the researcher recommends that policy makers are to focus more on other variables such as interest rate, unemployment rate, crime rate, inflation, GDP etc to get macroeconomic variables with high interdependence for the purpose of policy formulation for the country.

Baltussen G. *et al.* (2021) examined global factor premiums. They examined 24 global factor premiums across equity, bond, commodity, and currency markets via replication and out-of-sample evidence between 1800 and 2016. Replication yielded ambiguous evidence within a unified testing framework that accounts for p-hacking. Out-of-sample tests revealed strong and robust presence of the large majority of global factor premiums, with limited out-of-sample decay of the premiums. They found out that global factor premiums were generally unrelated to market, downside, or macroeconomic risks in the 217 years of data. These results revealed significant global factor premiums that present a challenge to traditional asset pricing theories.

Sims, C.A. (1980) worked on macroeconomics and reality. In his paper he argued that existing strategies for econometric analysis related to macroeconomics are subject to a number of serious objections, some recently formulated, some old. These objections are summarized in his paper, and it is argued that taken together they make it unlikely that macroeconomic models are in fact over identified, as the existing statistical theory usually assumes. The implications of his conclusion were explored, and an example of econometric work in a non-standard style, taking account of the objections to the standard style, is presented. He furthermore criticized standard econometric time series modeling for being detached from economics and introduces the concept of Structural Vector Autoregression (SVAR) as a better approach for macroeconometric modeling. The paper argues that these models should be identified with economic theory rather than relying on naive or mechanically imposed restrictions, highlighting the need for methods that better capture the dynamics and causal relationships within the economy to provide meaningful insights."

Jinghao *et al.* (2021) Application of Bayesian Vector Autoregressive Model in Regional Economic Forecast." The Bayesian "vector autoregressive (BVAR) model introduces the statistical properties of variables as the prior distribution of the parameters into the traditional vector autoregressive (VAR) model, which can overcome the problem of too little freedom. The BVAR model established in this paper can overcome the problem of short time series data by using prior statistical information. In theory, it should have a good effect in China's regional economic forecasting. Most regional forecasting model literature lacks out of sample forecasting error evaluation research in the real sense, but their early forecasts of major economic indicators provided an excellent opportunity for their paper to evaluate the actual forecast errors of the BVAR model in detail. The analysis in their paper showed that the prediction error of the BVAR model was very small and the prediction ability was very satisfactory. At the same time, this article also analyzed and pointed out the direction of efforts to further improve the prediction accuracy of the BVAR model."

Daewii *et al* (2023) studied the Application of Bayesian Vector Autoregressive in Modeling Nigerian Narrow Money and Quasi-Money. The study examined the application of Bayesian Vector Autoregressive model in modeling Nigerian narrow money and quasi money as a guide for monetary policy, using monthly data from 2015 - 2022. The objectives were to model and estimate the interaction between Nigerian narrow money and quasi money, determine the direction of causality, significance of the causality among the variables, and determine the fractions in each variable explained by the changes in the other variables. The data used for the study were narrow money and quasi money, extracted from the Central Bank of Nigeria online statistics bulletin. The model used in the study was Bayesian Vector Autoregressive models. The results of the descriptive statistics revealed that all the series were statistically significant at the 5 percent level of significance. Augmented Dickey Fuller (ADF) and Phillip Perron (PP) test were used to test for stationarity of the variables under investigation." The results of "Johansen Cointegration test showed that there was no cointegration or long-run equilibrium relationship between narrow money and quasi money at a 0.05 significance level. The Adjusted R-square value indicated that 97.7% variation in future narrow money values was explained by first and second per-determined value of narrow money itself and quasi money. The narrow money has a significant effect on quasi money during the studied period. The result of VAR model stability test (AR root circle) satisfied the stability condition, with all characteristic roots lying inside the circle. The result of the impulse response function revealed that narrow money responded positively to quasi money." It was found that narrow money granger caused quasi money. "This suggested that changes in the money supply have potential effect on economic activity through the narrow-money market, which may have implications for monetary policy decision. Therefore, it was recommended that there should be adequate monetary policy development measures to capture both short-run and long-run relationship between the study variables, including structural reforms to address issues related to shocks from one variable to the other

Alemho & Adenomon, (2022) Forecasted Some Selected Macroeconomic Variables with BVAR Models under Natural Conjugate Prior. Bayesian VARs are mostly used in computational analysis of macroeconomic variables of a nation. The natural conjugate prior when combined with the likelihood function gives a posterior distribution that belong to the same distributional family." The study "investigated the forecasting of some selected macroeconomic variables with BVAR Model using special types of natural conjugate prior called the symmetric and asymmetric natural conjugate prior. The data for the macroeconomic variables were obtained from the statistical bulletin of the Central Bank of Nigeria (CBN) ranging from 1986 to 2019. The forecasting assessment used was the Root Mean Square Factor Error (RMSFE). The RMSFE with small value indicated a better forecast performance. Forecasting the Macroeconomic variables with BVAR under the natural conjugate prior (symmetric and asymmetric natural conjugate prior), it was discovered from the study that asymmetric natural conjugate Prior was the best natural conjugate prior that should be used in forecasting macroeconomic variables of developing country like Nigeria. There is an inverse relationship between unemployment rate and other selected macroeconomic variables used in this study." Therefore, the policy makers should "endeavor to formulate policies that will reinvigorate the economy so that single–digit inflation rate can be achieved and unemployment rate will be reduced to the barest minimum. This will help to boost the GDP and economy at large.

Olayungbo, (2019) studied Bayesian Graphical Model Application for Monetary Policy and Macroeconomic Performance in Nigeria. The study applied Bayesian Graphical Networks (BGN) using Bayesian graphical vector autoregressive (BGVAR) model with efficient Markov chain Monte Carlo (MCMC) Metropolis-Hastings (M-H) sampling algorithm in a dynamic interaction among monetary policies and macroeconomic performances in Nigeria for the period of 1986Q1– 2017Q4." The motivation "stems from the instability in the movement of exchange rate, inflation rate and interest rate in Nigeria over the past years because of the structure of the economy. In this way, the monetary authority periodically applies the various policy instruments to stabilize the economy using reserve and money supply as at when due. This study adapts VAR and SVAR structure to examine the dynamic interaction among variables of interest, using BN, to provide a better understanding of the monetary policy dynamics and fit the changing structure of the Nigeria's economy as regards the dynamics in her economic structure. Their results showed that inflation was the strong predictor of interest rate in Nigeria. A monetary policy of broad inflation targeting was recommended for the country.

Kemisola *et al*. (2018), studied Interest Rate Channel and Real Economy in Nigeria: A Bayesian Vector Autoregression Approach." This study examined "the dynamic response of real economy to interest rate shocks using Bayesian vector autoregression model with Minnesota/ Litterman prior criterion. Impulse response functions showed that all the variables were consistent with the theory apart from investment whose response was counter intuitive. Forecast error variance decomposition confirmed theoretical interactions between monetary policy rate through interest rate and inflation. Interest rate channel under this framework is effective to bring the economy to stability by suppressing inflation rate and bring it to normalcy in Nigeria with adverse effect on growth rate of gross domestic product due to necessary and policy conflicts.

George *et al*. (2018) study Bayesian vector autoregressive models for modeling inflation rate in Nigeria. This study focused on the Bayesian approach to the estimation of Vector Autoregressive (VAR) model using data on inflation rates in Nigeria." This method allows a "combination of prior information and data information. The study provides a description of inflation rates in Nigeria using six different Bayesian VAR priors (diffuse prior, Minnesota prior, natural conjugate prior, independent normal Wishart prior, stochastic search variable selection prior-Wishart and stochastic search variable selection prior-VAR). The performance of various models was evaluated using root mean square error (RMSE). The stochastic search variable selection-Wishart outperforms other methods. The impulse response function was estimated. The study concluded that stochastic search variable selection prior-Wishart was the best of all the Bayesian VAR prior to model inflation rate in Nigeria.

III. METHODOLOGY

3.1: The monthly time series data were carefully extracted from the website of the central bank of Nigeria (CBN) www.cbn.org.ng. The data set ranged from April 1971 to July 2024. These macroeconomic data are for copper, maize and oil alongside common commodity price factors and global factor

3.2. VAR MODEL.

Vector autoregression (VAR) is a technique used by macroeconomists to illustrate the joint dynamic behaviour of a collection of variables without requiring strong restrictions as required in the identification of fundamental structural parameters Tuaneh (2018).

3.3. BVAR MODEL.

This is an improved version of VAR model (Giannone *et al*., 2015). It incorporates prior distributions that shrink coefficients toward more plausible values, BVAR models improve estimation precision, mitigate overfitting, and enhance forecasting performance (Karlsson, 2013)

In a univariate "autoregression, a stationary time-series variable y_t can often be modelled as depending on its own lagged values:

$$Y_t = a_0 + a_1 Y_{t-1} + a_2 Y_{t-2} + \cdots + a_{k-1} Y_{k-1} + \mathcal{E}_t \quad (1)$$

Where,

Y_t = is the series being considered

a_0 = denotes the mean

$a_1 Y_{t-1}$ = denotes AR polynomial of order 1

$a_2 Y_{t-2}$ = denotes AR polynomial of order 2

$a_{k-1} Y_{k-1}$ = denotes AR polynomial of order k

$\mathcal{E}_t$ = the white noise.

When multiple time series are analysed, the natural extension to the autoregressive model is the Vector Autoregressive or VAR, in which a vector of variables is modelled as depending on their own lags and the lags of every other variable in the vector. VAR model (vector autoregressive models) is used for multivariate time series." The structure is that each variable is a "linear function of past lags of itself and lags of the other variables. The model adopted in this work is the Vector Auto-Regressive Model (VAR) and a two-variables VAR (2) with lag could be specified as thus:

$$X_t = a_0 + a_1 Y_{t-1} + a_2 X_{t-1} + a_3 Z_{t-1} + a_4 M_{t-1} + \mathcal{E}_{1t} \quad (2)$$

$$Y_t = b_0 + b_1 Y_{t-1} + b_2 X_{t-1} + b_3 Z_{t-1} + b_4 M_{t-1} + \mathcal{E}_{2t} \quad (3)$$

Where X_t *and* Y_t represents variable 1 and variable 2 respectively. While the apriori expectation: $a_0, b_0 > 0$, these represent the intercept $a_1,$ *and* b_1= Short-run dynamic coefficients of the model's adjustment long-run equilibrium, $\mathcal{E}_{i.t}$ = Errors, impulses, shocks or innovations. Each variable is a linear function of the lag 1 values for all variables in the set. In a VAR (2) model, the lag 2 values for all variables are added to the right sides of the equations. Generally, for a VAR (p) model, the first p lags of each variable in the system would be used as regression predictors for each variable. VAR models are a specific case of more general VARMA models. VARMA models for multivariate time series include the VAR structure above along with moving average terms for each variable." Applied "macroeconomist use models of this form to both describe macroeconomic data, to perform causal inference and provide policy statement/and or advice.

$$\begin{bmatrix} y_t \\ x_t \\ z_t \end{bmatrix} = \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} \quad (4)$$

VAR is a multivariate autoregressive linear time series model of the form:

$$P_t = \alpha + \sum_{i=1}^{z} \alpha_i P_{t-i} + \varepsilon_t \quad (5)$$

Where; P_t is the set of series under consideration $P_t = (P_{t1}, P_{t2,...,} P_{nt})$ is a $n \times 1$ vector, α_i are full rank $m \times m$ matrix of coefficients, and i= 1, 2, 3, …z. $\varepsilon_t = (\varepsilon_{t1}, \varepsilon_{t2,...,} \varepsilon_{nt})$ is an unobservable i.i.d zero mean error term.

The three endogenous variables captured in this paper are copper, maize and oil while the exogenous variables are common commodity price factor and global factor.

3.4 Summary Descriptive Statistics

The summary descriptive statistics is usually done to determine whether a dataset is normally distributed. This test uses Jarque-Bera test statistics. According to Chinyere *et al*., (2015), the Jarque-Bera test statistics defines the joint test for skewness and kurtosis that satisfy normality," which practically "examine whether the data sets exhibit normal distribution characteristics.

The test statistics is defined as thus:

$$x^2 = \frac{N}{\sigma}\left[S^2 + \left(\frac{k-3}{4}\right)^2\right] \qquad 3.5$$

where;

x^2 - Chi-square

ó - standard deviation

S - Skewness statistics

K - Kurtosis

N - Sample size

The test statistics under the null hypothesis of normal distribution has a degree of freedom (df) 2 and the hypothesis is stated thus:

H_0: $a_1 = a_2 = ……………. = a_p = 0$, the variable is normally distributed as

χ^2 (P) > P – value (Standard Probability Value) against the alternative hypothesis

H_1: $a_1 \neq a_2 \neq ……………. \neq a_p \neq 0$ the variable is not normally distributed.

Also, the discussion for the test of the hypothesis will be considered, if the variable under investigation comes from a normal distribution process." If "Jarque-Bera (JB), asymptotically, have a chi-square distribution with two (2) degree of freedom then the null hypothesis would be accepted while the alternative be rejected conversely, if it is on the contrary then the alternative hypothesis will be accepted while the null hypothesis will be rejected. However, after testing the variables and they are not normally distributed then, we proceed to examine the stationary behaviour of the variables using the Unit root test.

3.5 Unit Root Test

The Unit Root Test for stationarity is tested using, the Augmented Dickey-Fuller (ADF) unit root test, which usually is employed in the analysis of random variables to determine the order of integration of a series." This is "considered very important in time series analysis and it is done using the Augmented Dickey-Fuller (ADF) and Philip Perron Test (PPT).

The test is based on the assumption that, a series y, is a random walk

$$Y_t = b_1 y_{t-1} + \mathcal{E}_t \qquad (3.6)$$

$$Y_t = b_0 + b_1 y_{t-1} + \mathcal{E}_t \qquad (3.7)$$

$$Y_t = b_0 + b_1 y_{t-1} b_2 t + \mathcal{E}_t \qquad (3.8)$$

However, to enhance stationarity we considered if y_{t-1} is subtracted from the Right-Hand Side (RHS) from each of the equation 3.6 – 3.8, we have

$Y_t - Y_{t-1} = b_1 Y_{t-1} - Y_{t-1} + \mathcal{E}_t$, $\quad \Delta Y_t = \vartheta Y_{t-1} + \mathcal{E}_t \quad$ (310)

$Y_t - Y_{t-1} = b_0 + b_1 Y_{t-1} - Y_{t-1} + \mathcal{E}_t$, $\quad \Delta Y_t = b_0 + \vartheta Y_{t-1} + \mathcal{E}_t \quad$ (3.11)

$Y_t - Y_{t-1} = b_0 + b_1 Y_{t-1} - Y_{t-1} + b_1 t + \mathcal{E}_t$, $\quad \Delta Y_t = b_0 + \vartheta Y_{t-1} + b_2 t + \mathcal{E}_t \quad$ (3.12)

Where $b_1 Y_{t-1} - Y_{t-1} = (b_1 - 1)\, Y_{t-1}$, let $(b_1 - 1) = \vartheta$, we have ϑY_{t-1} *and* $Y_t - Y_{t-1} = \Delta Y_t$

The null hypothesis is tested as thus:

For pure random walk, we have

$$\Delta Y_t = \vartheta Y_{t-1} + \sum_{i-1}^{p} \sigma_i \, \Delta Y_{t-1} + \mathcal{E}_i \qquad (3.13)$$

H_0:$\vartheta = 0$ and therefore r = 1 against the alternative that H_1: $\vartheta < 0$and r < 1

similarly, Random walk with drift we have

$$\Delta Y_t = b_0 + \vartheta Y_{t-1} + \sum_{i-1}^{p} \sigma_i \, \Delta Y_{t-1} + \mathcal{E}_i \qquad (3.14)$$

H_0:$\vartheta = 0$ and therefore r = 1 against the alternative that H_1: $\vartheta < 0$and r < 1

Also, Random walk with drift and trend

$$\Delta Y_t = b_0 + \vartheta Y_{t-1} + \sum_{i-1}^{p} \sigma_i \, \Delta Y_{t-1} + b_2 t + \mathcal{E}_i \qquad (3.15)$$

H_0:$\vartheta = 0$ and therefore r = 1 against the alternative that H_1: $\vartheta < 0$and r < 1

The decision that follows" will be "considered if '$\square_t$' is found to be more negative and statistically significant at least 5 percent level of significance, we compare the t-statistic value of the parameter, with the critical value tabulated (Mackinnon, 1996), we reject the null hypothesis and accept the alternative and conclude that the series does not have a unit root level. Conversely, we accept the null hypothesis and reject the proceed with the determination of lag length. Difference stationary refers to the situation where differencing is required to obtain stationarity. If the series is expressed as an AR process and the AR polynomial contains a unit root, that is if one root of the autoregressive polynomial lies on the unit circle, e.g. for an AR(1), α = 1, then differencing is necessary."

3.6 Cointegration Test
Cointegration is a statistical method used mainly for testing the correlation between two or more non-stationary time series in the long run or for a specific period. According to Syed (2008), the concept of co-integration between variables was developed by Granger and Engle in 1987. They explained the presence of a long-run relationship between two or more variables." In testing for co-integration, "there are several underlying assumptions and this include: all variables are said to be non-stationary; they are all integrated of the same order and where they are not integrated of the same order then will continue with co-integration analysis using multi co-integration.

3.7 Maximum Eigenvalue Test
The maximum eigenvalue test examines whether the largest eigenvalue is zero relative to the alternative that the next largest eigenvalue is zero. The first test is a test whether the rank of the matrix II is zero. The null hypothesis is that rank (II) = 0 and the alternative hypothesis is that rank (II) = 1. For further tests, the null hypothesis is that rank (II) 1, 2, ... and the alternative hypothesis is that rank (II) = 2, 3….

λ_{max} (r, r +1) = -T ln (1 - λ1) (3.16)

3.8 VAR Lag Order Selection Criteria.
The importance of Lag length determination is demonstrated by Braun and Mittnik (1993) who show that estimates of a VAR whose lag length differs from the actual lag length are inconsistent as are the Impulse response functions and variance decompositions derived from the estimated VAR." Also, over "fitting (selecting a higher order lag length then the actual lag length) causes an increase in the mean-square-forecast errors of the VAR and that underfitting the lag length often generates autocorrected errors.

According to Tuaneh (2018), VAR Lag Length Order is selected using some model selection criteria such as: Schwarz Information Criterion (SIC) or Gideon Schwarz Bayesian Information Criterion (SBIC), Final Prediction Error (FPE), Akaike Information Criterion (AIC) and Hannan-Quinn Information Criterion (HQ). however, the study adopted the Akaike information criterion since it chooses the value of the length that minimizes the model selection criteria (Lutkpohl, 2005). The general specification is to fit VAR models with order L = 0,1,2,….L max whereby models with too few lags could lead to systematic variation in the residual, too many lags is accompany with the penalty of fewer degree of freedom. Also, in the same way as in the univariate AR(p) models, Information Criteria (IC) can be used to choose the "right" number of lags in a VAR(p): that minimizes IC(p) for p = 1, …, P."

Some of the commonly used criteria are as follows:

a. Akaike Information Criterion (AIC)

$$\text{AIC (P)} = \text{In}\left|\sum_{I=1}^{N}\hat{u}\hat{u}(p)\right| + (k+pk^2)\frac{2}{r} \qquad (3.19)$$

b. Final Prediction Error (FPE)

$$\text{FPE(p)} \quad \left[\frac{T+kp+1}{T-kp-1}\right]^k \left|\sum_{I=1}^{N}\hat{u}\hat{u}(p)\right| \qquad (3.20)$$

c. Bayesian Criterion of Gideon Schwartz (SBC)

$$SC(P) = |\sum_{l=1}^{N} \hat{u}\hat{u}(p)|(k + pk^2)\frac{2In(In(T))}{T} \quad (3.21)$$

d. Hannan-Quinn Criterion (HQC)

$$\text{HQ} = In\left|\sum_{l=1}^{N} \hat{u}\hat{u}(p)\right|(k + pk^2)\frac{In(T)}{T} \quad (3.22)$$

Where $\hat{u}$ are denotes the residuals estimation from the model VAR (p), k number of dependent variables. T is number of observations and p is the length of model (Kirchgassner and Wolters, 2007).

Similar consistency results to the ones obtained in the univariate world are obtained in the multivariate world. The only difference is that as the number of variables gets bigger, it is more improbable that the AIC ends up over-parametrizing (Gonzalo and Pitarakis, 2002)."

Also, since adding more parameters to the model may increase the value of the maximum likelihood, the AIC balances this by penalizing for the number of parameters, hence searching for models with few parameters that properly fit into the data. Considering the models having lowest AIC is appropriate to selecting the best one! The lower the value is, the better the model is performing.

3.9 Stability Test for VAR Systems

A condition for stability for VAR(p) requires that all the eigenvalues of A (The AR Matrix of the comparison from Y_t) are smaller than one in modulus or all the roots larger than one. Consider the VAR(p) model

$$Y_t = c + \varphi_1 Y_{t-1} + \ldots + \varphi_p Y_{t-p} + \varepsilon_t \quad (3.23)$$

Substituting t=1 and t=2 we obtain respectively

$$Y_1 = c + \varphi_1 Y_o + \varepsilon_1$$
$$Y_2 = c + \varphi_1 Y_1 + \varepsilon_2$$

$$= c + \varphi_1(c + \varphi_1 Y_o + \varepsilon_1) + \varepsilon_2 = (I_k \varphi_1)c + \varphi_1{}^2 Y_o + \varphi_1 \varepsilon_1 + \varepsilon_2 \quad (3.24)$$

Therefore, if the eigenvalues are smaller than one in modulus then Y_t has the following representation

$$Y_t = (I - \varphi)^{-1} + \sum_{i=0}^{j} \varphi_1^i \varepsilon_{t-i} \quad (3.25)$$

Note that eigenvalues of φ satisfy det ($I_{kp} + \varphi_z$) $\neq 0$ $|z| \leq 1$, therefore VAR(p) is called stable if
det ($1_{kp} + \varphi_z$) = det ($I_k - \varphi_{tz} - \ldots - \varphi_{p^{z^p}}$) for $|z| \leq 1$ (3.26)

For a set of *n* time series variables $y_1 = (y_{1t}, y_{2t}, \ldots \ldots, y_m)$', a VAR model of order p(VAR(P)) can be written as:
$y_1 = A_1 y_{t-1} + A_2 y_{t-2} + \ldots . + A_p y_{t-p} + u_t$ (3.27)

Where the A_1 's are (nxm) coefficient matrices and u_1= ($u_{1t}, u_{2t}, \ldots \ldots, u_m$) is an unobservable i.i.d. zero mean error term." The stability of "the stationary VAR system, according to Halkos and Tsilika, (2012), the stability of a VAR can be examined by calculated the roofs of:

$(l_n - A_2 L - A_2 L^2 - \ldots .) y_t$ = A(L)y_t (3.28)

The characteristics polynomial is defined as $\prod(z) = (l_n - A_1 z - A_2 z^2 - \cdots)$

The roots of $|\prod(z)|$= 0 will give the necessary information about the stationarity or non-stationarity of the process. The necessary and sufficient condition for stability is that all characteristics roots lie inside circle. Then π is of full rank and all variables are stationary. In this section, we assume this is the case.

Later we allow for less than full rank matrices. Following the calculation of the eigenvalues and eigenvectors, we assumed an (nxn) square matrix A, whereby we are looking for a scalar λ and vector c ≠ 0 such that Ac = λ then λ is an eigenvalue, which will give up to n linearly independent associated eigenvectors.

For there to be a nontrivial solution, the matrix [A- λl] must be singular. Then λ must be such that

$$|z| \leq 1$$

Where, Z is the series being considered.

3.10 Granger Causality Test

Granger causality tests the null hypothesis that the coefficients of past values in the regression equation are zero." in simple terms, "the past values of time series (X) do not cause the other series (Y). So, if the p-value obtained from the test is lesser than the significance level of 0.05, then, you can reject the null hypothesis. if a given p-value is < significance level (0.05), then, the corresponding X series (column) causes the Y (row). the test for granger-causality given as: Assume a lag length of p.

$$X_t = C_1 + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots \alpha_p X_{t-p} + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \cdots \beta_p Y_{t-p} + \alpha_t \qquad (3.29)$$

Estimate by OLS and test for the following hypothesis

$H_0 : \beta_1 = \beta_2 = \ldots\ldots = \beta_p = 0$ (Y_t does not Granger-cause X_t)

H_1 : any $\beta_i \neq 0$

Unrestricted sum of squared residuals

$$RSS_t = \sum_t^n \hat{a}_t^2$$

Unrestricted sum of squared residuals

$$RSS_1 = \sum_{t=1}^{n} \hat{a}_1^2$$

Unrestricted sum of squared residuals

$$RSS_2 = \sum_{t=2}^{n} \hat{a}_2^2$$

Therefore,

$$F = \frac{(RSS_2 - RSS_1)}{RSS_1 / (T-2P-1)} \qquad (3.30)$$

3.11 Impulse Response Functions

The impulse response functions or IRFs are used in studying the dynamic behaviour of variables in a system." Its shows how one variable react "to a sudden changes of another variable. It traces out one unit or one standard deviation shock to an endogenous variable and its effects on all the endogenous variables in a VAR, keeping all other variables and shocks constant. The main objective of the impulse response function is to examine the reaction of the system to a shock.

$$Y_t = c + \Phi_1 Y_{t-1} + \Phi_1 Y_{t-2} + \ldots + \Phi_p Y_{t-p} + \alpha_t \qquad (3.31)$$

If the system is stable

$$Y_t = \mu + \psi(L)\alpha_1 = \mu + \alpha_{t-1} + \psi_2\alpha_{t-2} + \ldots\psi(L) = [\Phi(L)]^{-1} \qquad (3.32)$$

Redating at time t + s:

$$Y_{t+s} = \mu + \alpha_{t+s} + \psi_1\alpha_{t+s-1} + \psi_2\alpha_{t+s-2} + \ldots + \psi_s\alpha_t + \psi_{s+1}\alpha_{t-1} + \ldots$$

According to J. A. Petersen and V. Kumar (2012), impulse response function traces the incremental effect of a 1-unit (or one standard deviation) shock in one of the variables on the future values of the other endogenous variables. Consider the VAR(p) model.

$$Y_t = A_0 + A_1 X_{t-1} + e_t \qquad (3.33)$$

Vector error can be written as:

$$\begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix} = \frac{1}{det(A_1)} \sum_{i=1}^{\infty} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{bmatrix}^t \text{ x adj}(A_1) \text{ x } \begin{bmatrix} e_{1t-i} \\ e_{2t-i} \\ e_{3t-i} \end{bmatrix} \qquad (3.34)$$

Det(A_1) is a determinant value of A_1 and adj(A_1) is adjoint matrix of A_1, therefore:

$$\begin{bmatrix} x_1 \\ y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} + \frac{1}{det(A_1)} \sum_{i=1}^{\infty} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{bmatrix}^t \text{ x adj}(A_1) \text{ x} \begin{bmatrix} e_{1t-i} \\ e_{2t-i} \\ e_{3t-i} \end{bmatrix} \quad (3.35)$$

With φmatrix:

$$\begin{bmatrix} x_1 \\ y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} \sum_{i=1}^{\infty} \begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) & \phi_{13}(i) \\ \phi_{21}(i) & \phi_{22}(i) & \phi_{23}(i) \\ \phi_{31}(i) & \phi_{23}(i) & \phi_{33}(i) \end{bmatrix}^t \begin{bmatrix} e_{xt-i} \\ e_{yt-i} \\ e_{zt-i} \end{bmatrix} \quad (3.36)$$

With element $\varphi_{jk}(i)$:

$$\varphi_i = \frac{1}{det(A_1)} \sum_{i=1}^{\infty} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{bmatrix}^t \text{ x adj}(A_1) \quad (3.37)$$

That can also be written as:

$$Z_t = \mu + \sum_{i=1}^{\infty} \varphi_i å_{t-1} \quad (3.38)$$

The coefficient $ö_{jk}(i)$ is called Impulse Response Function (IRF). $ö_{jk}(i)$ plot is the best way to visualize the response toward the shocks."

IV. RESULTS

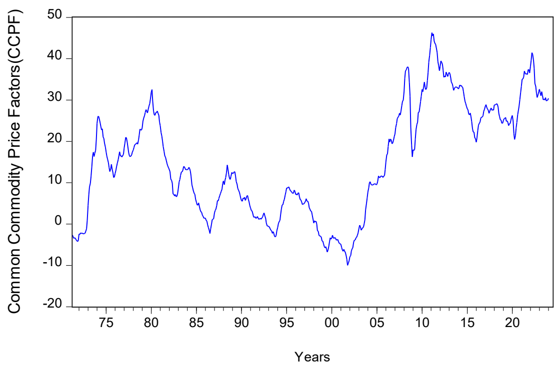


Figure 4.1: Time Plot of Raw Data on Common Commodity Price Factor.

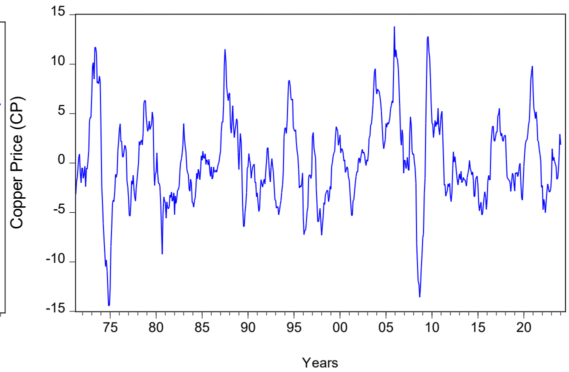


Figure 4.2: Time Plot of Raw Data on Copper Price

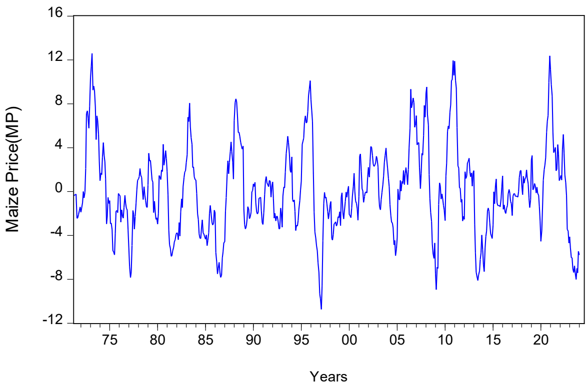


Figure 4.3: Time Plot of Raw Data on Maize Price.

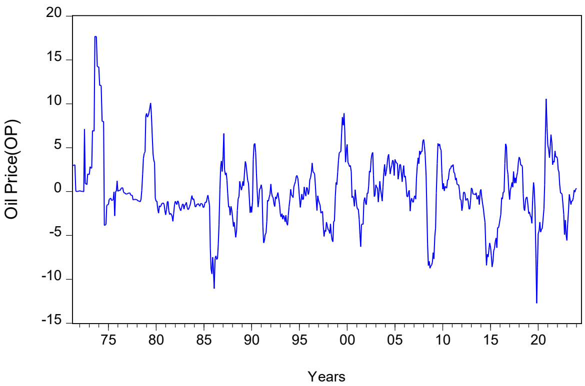


Figure 4.4: Time Plot of Raw Data on Oil Price

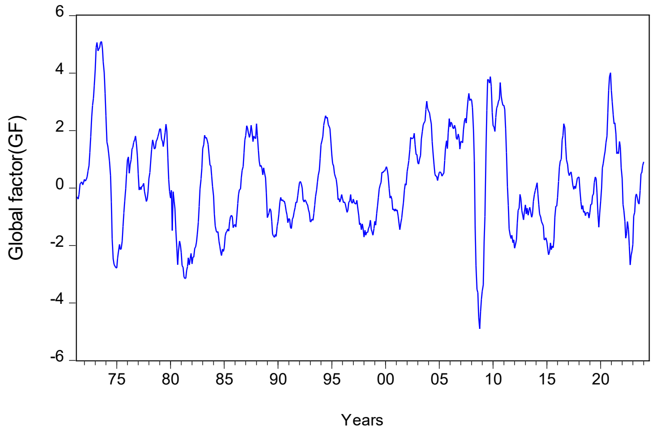


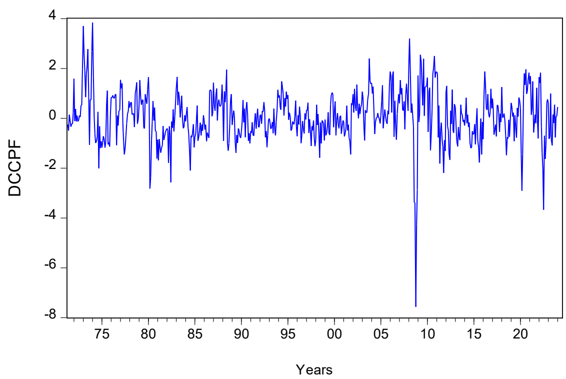
Figure 4.5: Time Plot of Raw Data on Global factor."

Table 4.1: "Descriptive statistics of CCPF, CP, GF, OP, MP

	CCPF	CP	GF	OP	MP
Mean	15.25279	0.106893	0.165394	-0.010457	0.060331
Median	13.44000	-0.465000	-0.025000	-0.400000	-0.335000
Maximum	46.29000	13.81000	5.100000	17.70000	12.60000
Minimum	-9.990000	-14.45000	-4.900000	-12.73000	-10.73000
Std. Dev.	13.56518	4.363748	1.690954	3.839605	4.096950
Skewness	0.161535	0.157677	0.281513	0.735470	0.447017
Kurtosis	1.916816	3.841890	2.961408	5.929299	3.272180
Jarque-Bera	33.75155	21.35064	8.413355	283.8329	23.07176
Probability	0.000000	0.000023	0.014896	0.000000	0.000010
Sum	9670.270	67.77000	104.8600	-6.630000	38.25000
Sum Sq. Dev.	116481.0	12053.78	1809.952	9332.047	10624.90
Observations	634	634	634	634	634"

Table 4.2: "Unit Root Test of the joint series.

Variables	ADFT		RMK	PPT		RMK	KPSST		RMk
CCPF	-1.988	0.292	1(1)	-1.860	0.351	1(1)	1.088	0.014	1(1)
	-12.469	0.000		-12.485	0.000				
GF	-5.542	0.000	1(0)	-4.866	0.000	1(0)	0.073	0.0013	1(1)
OP	-5.147	0.000	1(0)	-5.866	0.000	1(0)	0.178	0.0103	
CP	-5.080	0.000	1(0)	-5.544	0.000	1(0)	0.090	0.009	1(1)
MP	-4.837	0.000	1(0)	-5.482	0.000	1(0)	0.050	0.019	1(1)



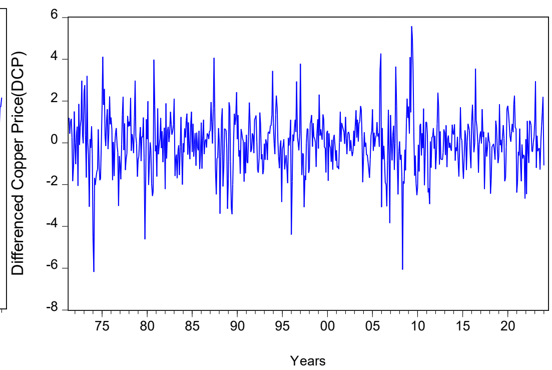


Figure 4.6: Time Plot of the Differenced Data on Common Commodity Price Factor.
Figure 4.7: Time Plot of the Differenced Data on Copper Price

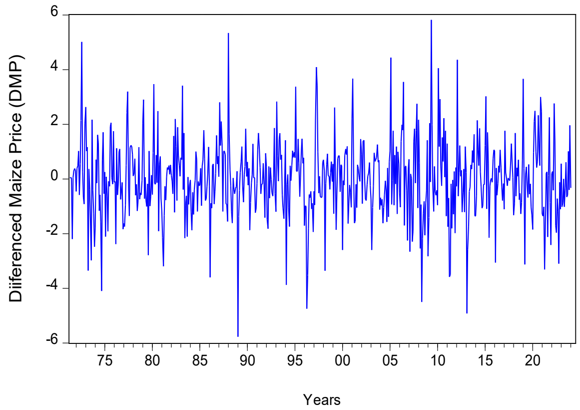


Figure 4.8: Time Plot of the Differenced Data on Maize Price.

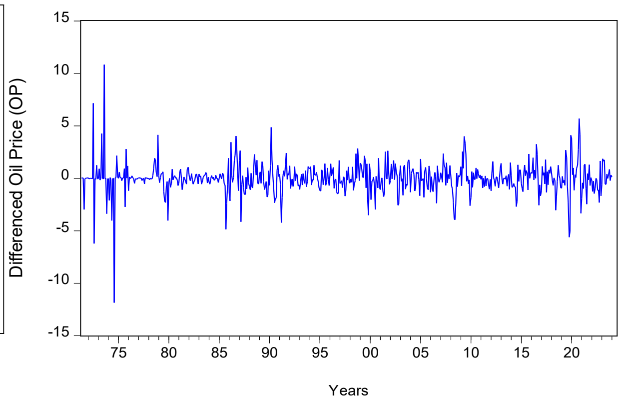


Figure 4.9: Time Plot of Differenced Data on Oil Price

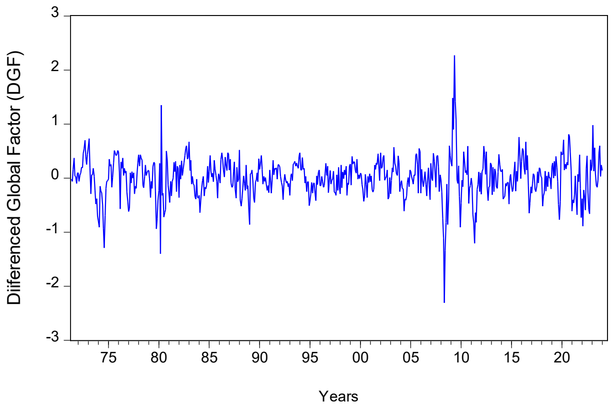


Figure 4.10: Time Plot of the Differenced Data on Global factor

Table 4.3: "Cointegration Test

Unrestricted Cointegration Rank Test (Trace)					Unrestricted Cointegration Rank Test (Max. Eigenvalue)			
Hypothesized No. of CE(s)	Trace Eigenvalue	Statistic	0.05 Critical Value	Prob.**	0.234	167.8880	33.877	0.000
None *	0.234	298.544	69.819	0.000	0.081	53.093	27.584	0.000
At most 1 *	0.081	130.656	47.856	0.000	0.064	41.840	21.132	0.000
At most 2 *	0.064	77.563	29.797	0.000	0.050	32.289	14.265	0.000
At most 3 *	0.050	35.723	15.495	0.000	0.005	3.434	3.841	0.064
At most 4	0.005	3.434	3.841	0.064	0.234	167.889	33.877	0.000
Trace test indicates 4 cointegratingeqn(s) at the 0.05 level					Trace test indicates 4 cointegratingeqn(s) at the 0.05 level			
* Denotes rejection of the hypothesis at the 0.05 level					* Denotes rejection of the hypothesis at the 0.05 level			
**MacKinnon-Haug-Michelis (1999) p-values					**MacKinnon-Haug-Michelis (1999) p-values"			

Table 4.4: "VAR Lag Order Selection Criteria

VAR Lag Order Selection Criteria
Endogenous variables: CCPF CP GF MP OP
Exogenous variables: C
Date: 04/17/25 Time: 03:49
Sample: 1971M04 2024M07

Included observations: 626

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-8420.924	NA	337898.3	26.91989	26.95534	26.93366
1	-3978.031	8800.619	0.250531	12.80521	13.01796	12.88787
2	-3777.429	394.1538	0.142961	12.24418	12.63422	12.39573
3	-3751.133	51.24756	0.142376	12.24004	12.80737	12.46047
4	-3709.686	80.11290	0.135097	12.18750	12.93212	12.47681
5	-3677.297	62.08793	0.131960	12.16389	13.08580	12.52209
6	-3512.591	313.0997	0.084463	11.71754	12.81674	12.14462
7	-3445.852	125.8015	0.073936	11.58419	12.86068	12.08016
8	-3278.532	312.7238*	0.046937*	11.12949*	12.58328*	11.69434*

* Indicates lag order selected by the criterion
LR: sequential modified LR test statistics (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion"
HQ: "Hannan-Quinn information criterion

Table 4.5.1: "Bayesian VAR Estimates

	CCPF	CP	GF	MP	OP
CCPF (-1)	1.045	0.017	0.021	0.068	0.077
	(0.031)	(0.047)	(0.012)	(0.052)	(0.054)
	[33.238]	[0.362]	[1.787]	[1.310]	[1.414]
CCPF (-2)	-0.068	0.009	-0.002	-0.050	-0.016
	(0.037)	(0.055)	(0.013)	(0.059)	(0.063)
	[-1.866]	[0.155]	[-0.114]	[-0.827]	[-0.247]
CCPF(-3)	-0.066	-0.003	-0.005	0.003	-0.013
	(0.027)	(0.039)	(0.010)	(0.043)	(0.046)
	[-2.497]	[-0.066]	[-0.534]	[0.079]	[-0.292]
CCPF (-4)	-0.015	-0.006	-0.009	-0.020	-0.023
	(0.021)	(0.031)	(0.008)	(0.034)	(0.036)
	[-0.707]	[-0.177]	[-1.117]	[-0.587]	[-0.635]
CCPF (-5)	0.016	-0.020	-0.008	-0.016	-0.021
	(0.017)	(0.026)	(0.006)	(0.028)	(0.029)
	[0.958]	[-0.762]	[-1.227]	[-0.586]	[-0.714]
CCPF (-6)	0.033	-0.013	-0.004	0.0003	-0.008
	(0.014)	(0.022)	(0.005)	(0.023)	(0.025)
	[2.324]	[-0.580]	[-0.691]	[0.012]	[-0.310]
CCPF (-7)	0.053	0.010	0.004	0.013	0.002
	(0.012)	(0.018)	(0.004)	(0.020)	(0.021)
	[4.387]	[0.552]	[0.847]	[0.632]	[0.079]
CP(-1)	0.021	0.950	0.032	-0.027	0.025
	(0.027)	(0.035)	(0.008)	(0.037)	(0.039)
	[0.932]	[27.479]	[3.812]	[-0.712]	[0.636]
CP(-2)	-0.0167	-0.064	-0.017	0.031	-0.010
	(0.023)	(0.036)	(0.009)	(0.039)	(0.041)
	[-0.710]	[-1.785]	[-1.932]	[0.809]	[-0.241]"
"CP (-3)	-0.012	-0.019	-0.010	-0.005	0.014
	(0.017)	(0.026)	(0.006)	(0.029)	(0.030)

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Table 4.5.2	Bayesian	VAR	Estimates	continues	
	[-0.704]	[-0.709]	[-1.633]	[-0.166]	[0.474]
CP(-4)	0.013	0.012	0.001	0.005	-0.007
	(0.014)	(0.021)	(0.005)	(0.023)	(0.024)
	[0.971]	[0.578]	[0.245]	[0.206]	[-0.235]
CP(-5)	0.020	0.006	-0.0001	-0.004	-0.001
	(0.011)	(0.017)	(0.004)	(0.019)	(0.020)
	[1.738]	[0.353]	[-0.032]	[-0.193]	[-0.062]
CP(-6)	-0.013	-0.012	-0.002	0.0138	-0.004
	(0.010)	(0.015)	(0.003)	(0.016)	(0.017)
	[-1.338]	[-0.811]	[-0.564]	[0.875	[-0.258]
CP(-7)	-0.013	-0.015	-0.001	0.013	-0.007
	(0.008)	(0.013)	(0.003)	(0.014)	(0.014)
	[-1.580]	[-1.167]	[-0.451]	[0.949]	[-0.499]
GF(-1)	0.262	0.621	1.078	0.354	0.686
	(0.095)	(0.144)	(0.035)	(0.156)	(0.164)
	[2.772]	[4.321]	[30.766]	[2.271]	[4.185]
GF(-2)	0.072	-0.270	-0.0230	-0.106	-0.360
	(0.101)	(0.153)	(0.037)	(0.166)	(0.175)
	[0.719]	[-1.768]	[-0.614]	[-0.637]	[-2.060]
GF(-3)	0.105	-0.056	-0.056	-0.151	-0.178
	(0.073)	(0.111	(0.027)	(0.121)	(0.127)
	[1.434]	[-0.507]	[-2.036]	[-1.251]	[-1.402]
GF(-4)	0.116	-0.041	-0.022	-0.047	-0.059
	(0.057)	(0.087)	(0.021)	(0.095)	(0.100)
	[2.023]	[-0.476]	[-1.012]	[-0.498]	[-0.590]
GF(-5)	0.084	-0.041	-0.015	-0.008	-0.015
	(0.047)	(0.071)	(0.017)	(0.077)	(0.081)
	[1.762]	[-0.569]	[-0.849]	[-0.108]	[-0.181]
GF(-6)	-0.122	-0.040	-0.015	0.049	0.007”
	(0.039)	(0.059)	(0.015)	(0.064)	(0.068)
	[-3.132]	[-0.671]	[-1.036]	[0.769]	[0.099]

Table 4.5.3

	Bayesian	VAR	Estimates	Continues	
GF(-7)	-0.103	-0.021	-0.009	0.040	0.024
	(0.033)	(0.049)	(0.012)	(0.054)	(0.056)
	[-3.165]	[-0.428]	[-0.713]	[0.749]	[0.426]
MP(-1)	0.0143	-0.019	0.011	0.950	-0.060
	(0.019)	(0.029)	(0.007)	(0.032)	(0.034)
	[0.739]	[-0.632]	[1.554]	[29.637]	[-1.792]
MP(-2)	-0.007	0.005	-0.007	-0.016	0.031
	(0.021)	(0.032)	(0.008)	(0.035)	(0.037)
	[-0.316]	[0.144]	[-0.836]	[-0.455]	[0.839]
MP(-3)	-0.001	-0.011	-0.008	-0.024	0.003
	(0.016)	(0.024)	(0.006)	(0.026)	(0.027)
	[-0.063]	[-0.443]	[-1.396]	[-0.922]	[0.122]

MP(-4)	0.002	-0.006	-0.002	-0.018	0.009
	(0.013)	(0.019)	(0.005)	(0.021)	(0.022)
	[0.139]	[-0.302]	[-0.463]	[-0.866]	[0.419]
MP(-5)	0.005	0.005	0.002	-0.014	0.019
	(0.010)	(0.016)	(0.004)	(0.017)	(0.018)
	[0.469]	[0.332]	[0.515]	[-0.786]	[1.048]
MP(-6)	-0.007	-0.003	-0.001	-0.012	0.010
	(0.009)	(0.013)	(0.003)	(0.014)	(0.015)
	[-0.819]	[-0.215]	[-0.258]	[-0.807]	[0.645]
MP(-7)	-0.002	0.001	0.001	-0.012	0.007
	(0.007)	(0.011)	(0.003)	(0.012)	(0.013)
	[-0.294]	[0.093]	[0.198]	[-0.982]	[0.561]
OP(-1)	-0.010	0.029	0.010	0.038	0.847
	(0.019)	(0.030)	(0.007)	(0.032)	(0.034)
	[-0.498]	[0.966]	[1.329]	[1.200]	[25.047]
OP(-2)	-0.005	-0.047	-0.014	-0.026	0.006
	(0.020)	(0.030)	(0.007)	(0.033)	(0.035)
	[-0.271]	[-1.557]	[-1.839]	[-0.772]	[0.177]

Table 4.5.4	Bayesian	VAR	Estimates	Continues	
OP(-3)	0.009	0.001	-0.007	-0.021	0.010
	(0.015)	(0.023)	(0.006)	(0.025)	(0.026)
	[0.590]	[0.025]	[-1.198]	[-0.856]	[0.375]
OP(-4)	0.013	-0.007	-0.005	-0.001	-0.006
	(0.012)	(0.018)	(0.004)	(0.020)	(0.021)
	[1.074]	[-0.376]	[-1.047]	[-0.050]	[-0.279]
OP(-5)	0.008	-0.008	-0.001	-0.004	-0.004
	(0.010)	(0.015)	(0.004)	(0.016)	(0.017)
	[0.831]	[-0.538]	[-0.142]	[-0.262]	[-0.235]
OP(-6)	-0.008	-0.004	-0.0003	-0.002	-0.005
	(0.008)	(0.013)	(0.003)	(0.014)	(0.015)
	[-0.940]	[-0.277]	[-0.103]	[-0.172]	[-0.338]
OP(-7)	-0.008	0.001	0.0004	0.006	-0.006
	(0.007)	(0.011)	(0.003)	(0.012)	(0.013)
	[-1.120]	[0.084]	[0.145]	[0.488]	[-0.465]
C	0.024	0.059	0.040	0.009	-0.011
	(0.051)	(0.078)	(0.0189)	(0.085)	(0.089)
	[0.473]	[0.753]	[2.095]	[0.105]	[-0.120]”
“R-squared	0.997544	0.920349	0.969091	0.897427	0.873982
Adj. R-squared	0.997399	0.915632	0.967261	0.891353	0.866519
Sum sq. resids	280.0495	958.5045	55.91384	1088.015	1171.408
S.E. equation	0.688373	1.273513	0.307586	1.356825	1.407863
F-statistic	6858.331	195.1110	529.4260	147.7364	117.1086
Mean dependent	15.46069	0.118549	0.168405	0.074019	-0.029872

S.D. dependent	13.49627	4.384447	1.699938	4.116372	3.853456

4.1: Dynamic Stability Test on BVAR Estimates

This tests for the adequacy of the BVAR model estimated above. The test revealed that the model is adequate

Root	Modulus
0.989248	0.989248
0.919635 + 0.119599i	0.927380
0.919635 - 0.119599i	0.927380
0.865084	0.865084
0.832477 + 0.142380i	0.844565
0.832477 - 0.142380i	0.844565
0.578786 + 0.514721i	0.774552
0.578786 - 0.514721i	0.774552
0.607489 + 0.119856i	0.619200
0.607489 - 0.119856i	0.619200
0.164553 + 0.515508i	0.541134
0.164553 - 0.515508i	0.541134
0.540454	0.540454
0.467930 + 0.254296i	0.532564
0.467930 - 0.254296i	0.532564
0.100539 - 0.522332i	0.531920
0.100539 + 0.522332i	0.531920
0.209524 + 0.479300i	0.523096
0.209524 - 0.479300i	0.523096
0.256344 + 0.402478i	0.477180
0.256344 - 0.402478i	0.477180
0.462943	0.462943
0.016701 + 0.462124i	0.462426
0.016701 - 0.462124i	0.462426
0.137062 - 0.439428i	0.460308
0.137062 + 0.439428i	0.460308
0.283488 + 0.335738i	0.439414
0.283488 - 0.335738i	0.439414
0.389017 + 0.191077i	0.433411
0.389017 - 0.191077i	0.433411

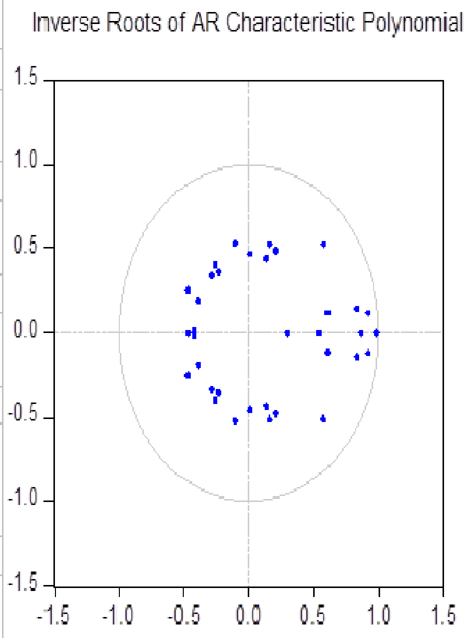

Figure 4.11: Dynamic Stability Test on BVAR Estimates

Table 4.6.1: "Impulse Response Function of the Bayesian VAR Estimates

Response of CCPF: Period	CCPF	CP	GF	MP	OP
1	0.673	0.000	0.000	0.000	0.000
2	0.713	0.072	0.064	0.020	-0.012
3	0.714	0.153	0.155	0.036	-0.026
4	0.678	0.248	0.282	0.049	-0.026
5	0.640	0.388	0.442	0.062	-0.013
6	0.626	0.578	0.627	0.075	0.006
7	0.640	0.734	0.771	0.078	0.004

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8	0.690	0.835	0.864	0.075	-0.027
9	0.736	0.906	0.928	0.068	-0.074
10	0.767	0.955	0.972	0.059	-0.131

Response of CP:

Period	CCPF	CP	GF	MP	OP
1	0.087	1.241	0.000	0.000	0.000
2	0.115	1.292	0.156	-0.029	0.036
3	0.143	1.244	0.239	-0.034	0.011
4	0.169	1.166	0.295	-0.046	-0.015
5	0.186	1.094	0.322	-0.064	-0.054

Table 4.6.2: Impulse Response Functions Continues

6	0.180	1.026	0.328	-0.071	-0.104
7	0.153	0.936	0.317	-0.079	-0.153
8	0.123	0.822	0.293	-0.083	-0.194
9	0.093	0.705	0.258	-0.086	-0.228
10	0.065	0.590	0.216	-0.089	-0.255”

”Response of GF:

Period	CCPF	CP	GF	MP	OP
1	0.030	0.175	0.242	0.000	0.000
2	0.051	0.232	0.270	0.0123	0.012
3	0.071	0.269	0.293	0.0181	0.008
4	0.085	0.282	0.299	0.016	-0.007
5	0.091	0.286	0.295	0.011	-0.028
6	0.086	0.284	0.285	0.009	-0.050
7	0.075	0.274	0.269	0.005	-0.069
8	0.063	0.257	0.250	0.001	-0.087
9	0.051	0.235	0.228	-0.003	-0.102
10	0.040	0.209	0.202	-0.007	-0.115”

Response of MP:

Period	CCPF	CP	GF	MP	OP
1	0.046	0.109	0.361	1.270	0.000
2	0.101	0.140	0.446	1.199	0.0489
3	0.127	0.211	0.497	1.121	0.058

Table 4.6.3: Impulse Response Functions Continues

4	0.150	0.251	0.498	1.023	0.036445
5	0.151	0.275	0.476	0.909	0.010448
6	0.131	0.280	0.440	0.786	-0.022929
7	0.106	0.301	0.408	0.661	-0.057249
8	0.089	0.338	0.382	0.534	-0.079572
9	0.077	0.365	0.356	0.421	-0.099939
10	0.069	0.379	0.328	0.324	-0.118973”

Response of OP:

Period	CCPF	CP	GF	MP	OP

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1	0.066	0.184305	0.448315	-0.187773	1.271959
2	0.128	0.300905	0.524299	-0.235508	1.077507
3	0.174	0.372656	0.537905	-0.224489	0.926016
4	0.208	0.428169	0.524641	-0.209963	0.800680
5	0.216	0.449149	0.503601	-0.187954	0.675911
6	0.201	0.457727	0.486164	-0.145482	0.556201
7	0.176	0.459836	0.475714	-0.094984	0.446662
8	0.151	0.449395	0.467742	-0.042885	0.345372
9	0.130	0.427649	0.450222	0.000459	0.257647
10	0.114	0.395207	0.422626	0.033543	0.180258
Cholesky Ordering: CCPF CP GF MP OP”					

Table 4.7: “Granger Causality Test of the estimates of the Bayesian VAR model

Pairwise Granger Causality Tests
Date: 04/16/25 Time: 08:22
Sample: 1971M04 2024M07
Lags: 5

Null Hypothesis:	Obs	F-Statistic	Prob.
CP does not Granger Cause CCPF	629	16.5947	2.E-15
CCPF does not Granger Cause CP		1.78083	0.1147
GF does not Granger Cause CCPF	629	38.0900	4.E-34
CCPF does not Granger Cause GF		3.27354	0.0063
OP does not Granger Cause CCPF	629	6.03735	2.E-05
CCPF does not Granger Cause OP		4.75970	0.0003
MP does not Granger Cause CCPF	629	4.13104	0.0011
CCPF does not Granger Cause MP		4.28080	0.0008
GF does not Granger Cause CP	629	4.93157	0.0002
CP does not Granger Cause GF		1.30985	0.2580
OP does not Granger Cause CP	629	3.80486	0.0021
CP does not Granger Cause OP		4.80777	0.0003
MP does not Granger Cause CP	629	0.95298	0.4461
CP does not Granger Cause MP		3.55634	0.0035
OP does not Granger Cause GF	629	3.06652	0.0096
GF does not Granger Cause OP		7.75668	4.E-07
MP does not Granger Cause GF	629	0.95443	0.4452
GF does not Granger Cause MP		5.09425	0.0001
MP does not Granger Cause OP	629	1.29177	0.2657
OP does not Granger Cause MP		2.78448	0.0169”

Table 4.8: “VAR Residual Normality Tests

VAR Residual Normality Tests
Orthogonalization: Cholesky (Lutkepohl)
Null Hypothesis: Residuals are multivariate normal
Date: 04/18/25 Time: 04:43
Sample: 1971M04 2024M07
Included observations: 627

Component	Skewness	Chi-sq	df	Prob.*
1	-0.336343	11.82175	1	0.0006
2	0.024636	0.063426	1	0.8012
3	0.225036	5.292008	1	0.0214
4	0.177628	3.297151	1	0.0694
5	0.140301	2.057007	1	0.1515

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Joint		22.53134	5	0.0004
Component	Kurtosis	Chi-sq	df	Prob.
1	6.985097	414.8910	1	0.0000
2	4.542991	62.19892	1	0.0000
3	6.854975	388.2393	1	0.0000
4	4.085889	30.80540	1	0.0000
5	11.23135	1770.102	1	0.0000
Joint		2666.237	5	0.0000”
“Component	Jarque-Bera	df	Prob.	
1	426.7128	2	0.0000	
2	62.26235	2	0.0000	
3	393.5313	2	0.0000	
4	34.10255	2	0.0000	
5	1772.159	2	0.0000	
Joint	2688.768	10	0.0000	

*Approximate p-values do not account for coefficient Estimation”

Table 4.9: “VAR Residual Heteroskedasticity Tests (Levels and Squares)

VAR Residual Heteroskedasticity Tests (Levels and Squares)
Date: 04/18/25 Time: 04:51
Sample: 1971M04 2024M07
Included observations: 627

Joint test:

Chi-sq	df	Prob.
1726.058	1050	0.0000

Individual components:

Dependent	R-squared	F(70,556)	Prob.	Chi-sq(70)	Prob.
res1*res1	0.357430	4.418216	0.0000	224.1085	0.0000
res2*res2	0.173438	1.666655	0.0010	108.7457	0.0021
res3*res3	0.244099	2.564950	0.0000	153.0504	0.0000
res4*res4	0.125687	1.141826	0.2125	78.80571	0.2204
res5*res5	0.172842	1.659726	0.0011	108.3717	0.0022
res2*res1	0.245443	2.583655	0.0000	153.8925	0.0000
res3*res1	0.210825	2.121908	0.0000	132.1875	0.0000
res3*res2	0.208400	2.091071	0.0000	130.6668	0.0000
res4*res1	0.248528	2.626877	0.0000	155.8272	0.0000
res4*res2	0.217155	2.203291	0.0000	136.1564	0.0000
res4*res3	0.230254	2.375947	0.0000	144.3693	0.0000
res5*res1	0.186422	1.820013	0.0001	116.8865	0.0004
res5*res2	0.196232	1.939173	0.0000	123.0376	0.0001
res5*res3	0.170823	1.636350	0.0015	107.1061	0.0029
res5*res4	0.180283	1.746894	0.0004	113.0372	0.0009”

V. CONCLUSION

Figures (4.1) – (4.5) are the time plots of the raw data. By visual inspection, Figure (4.1) shows high volatility and clear upward and downward swings indicating periods of commodity price booms and burst. Figure (4.2) shows a sharp price spike, likely due to geopolitical shocks or demand shifts in industrial sectors. Figure (4.3) shows seasonality and there are evidences of strong fluctuations due to global economic cycles, oil supply disruptions, and OPEC decisions. Figure (4.4) and Figure (4.5) display smoother trends but still exhibit cycles of economic expansion and contraction. These findings align with Hamilton (2009), who noted that oil prices are prone to frequent volatility due to geopolitical and market factors. Those type of plots show non-stationary behavior which by implication calls for differencing. Also, Cuddington and Jerrett (2008) reported similar cyclical patterns in commodity prices, which is in consistent with these our observations.

Table (4.1) contains descriptive statistics with a high standard deviation value of (13.56) indicating substantial volatility, Skewness values were slightly positive, which means right-tailed distributions, Kurtosis of Oil Price (5.93) shows leptokurtic behavior—fatter tails than normal distribution and Jarque-Bera test p-values all less than (0.05) confirming that all series deviate from normality. This supports the findings of Pindyck and Rotemberg (1990), who emphasized non-normality in commodity returns. Alquist and Gervais (2013) further noted that skewness and kurtosis are inherent in commodity returns due to demand shocks and storage constraints.

Table (4.2) shows the unit root tests analysis using the Augmented Dickey Fuller (ADF), Phillip Perron (PP), and KPSS. The result collectively confirm that the Common Commodity Price Factor is I(1), meaning non-stationary at levels but stationary after first differencing while Copper price, Maize price, Oil price, and Global Factor are I(0) meaning stationary at levels. These stationarity findings are in line with Elder and Serletis (2008) who found mixed stationarity across different commodities while some contradicts Baffes (2007) who reported that most commodity prices are I (1), which may depend on data period and structural breaks.

Figures (4.6) – (4.10) are the time plots of the differenced data. All plots show series fluctuating around a mean of zero with no visible trend—confirming stationarity after differencing (particularly for Common Commodity Price Factor). Visual confirmation aligns with statistical evidence in unit root test table shown in Table (4.2) of chapter 4 and this is methodologically supported by Granger and Newbold (1974), who emphasized visual inspection alongside unit root test for confirming stationarity.

Cointegration Test result which usually examines correlation between two or more non-stationary time series in the long run or for a specific period were presented in Table (4.3) and it shows that both trace and max eigenvalue tests indicate 4 cointegrating vectors at the 5% level. The interpretation is that despite short-term volatility, Common Commodity Price Factor, Copper Price, Global Factor, Maize Price and Oil Price move together in the long run. This finding confirm earlier work by Johansen (1991) on cointegration in financial time series and also supported by Chen *et al.* (2010), who found that commodity prices and global economic indicators are often cointegrated.

Table (4.4) shows the results of all the VAR Lag selection criteria (AIC, SC, HQ, FPE, LR). Optimal lag was observed at 8 when we opted for VAR Lag Order Selection. This high lag structure accounts for the slow dynamics and persistence in commodity markets. Similar lag selections are found in Kilian and Murphy (2014) where high-frequency commodity data necessitated longer lags to capture delayed responses and feedback loops

The Bayesian VAR estimates results were presented in Table (4.5). Each result output – Common Commodity Price Factor (CCPF), Copper (CP), Global Factor (GF), Maize Price (MP) and Oil Price (OP) is interpreted below according to the significance of their delayed variables (using t-statistics > |1.96| as the threshold for significance at the 5% level) and overall alignment (judged by the magnitude and persistence of own lag coefficients). The Bayesian VAR estimates for the five variables – Common Commodity Price Factor (CCPF), Copper (CP), Maize (MP), Oil Price (OP) and Global Factors (GF) – revealed distinctive patterns of interdependence and persistence. Below is the interpretation of each result output, considering the statistical significance of the parameters and the general fit, followed by a comparative discussion with previous studies. The result output for CCPF is characterized by strong autoregressive dynamics. The coefficient for the first layer of CCPF is very significant and large in size (1.045, t = 33.238), meaning high persistence and strong intrinsic layer effects. Delayed coefficients at position 3 and 7 are also significant, with negative and positive effects, respectively. The global factor of delay 1, 4 and 5 significantly affects the CCPF, which means that the global dynamics have a delayed but notable impact.

The R-square value of (0.997544) and the adjusted R-square of (0.997399) indicate a near-perfect model fit, showing that the chosen regressors account for almost all variations in the CCPF. This strong autoregressive behavior is consistent with the findings of Bańbura *et al.* (2010), which observed sustained dynamics of macro-financial indicators using Bayesian VARs. The CP result output indicates moderate persistence, as seen from the very significant coefficient of the eigen layer coefficient of CP (-1) (0.950, t=27.479) where the R-squared and adjusted R-square values of (0.920349) and (0.915632) respectively confirm a strong explanatory power. The global factor (GF) of delay 1 is also statistically significant (0.621, t=4.321) implying a notable influence of global trends on copper prices. The remaining delays of CP and other variables are mostly negligible, implying a predominant short-term feedback mechanism. This supports the findings in Giannone *et al.* (2015), where Bayesian VAR models captured high-frequency market reactions with sharp initial impacts that subside rapidly. The GF equation reveals a dominant autoregressive structure in which the GF (-1) has a large and significant coefficient (1.078, t=30.766) highlighting strong internal persistence. CCPF (-1) and CP (-1) also significantly affect GF, implying spillover from commodity-specific factors to global dynamics. The R-square value of 0.969091 and the adjusted R-square value of 0.967261 confirm that the model captures global factor dynamics effectively. In particular, the coefficient of CP (-1) (0.032, t=3.812) confirms the transmission of copper price shocks to the global variable. These patterns are consistent with Canova (2007), which highlighted the ability of BVAR models to capture global connections and interactions across commodity and financial markets. For the MP result output, the coefficient of MP (-1) is highly significant and almost unity (0.950, t=29.637) signaling high inertia in corn prices. In addition, GF (-1) affects MP significantly (0.354, t=2.271) implying that global conditions are transferred to domestic agricultural markets. The explanatory power is high with R-squared and adjusted R-square values of 0.897427 and 0.891353 respectively. This supports the evidence presented by Koop and Korobilis (2010), who observed that Bayesian VARs are particularly effective in explaining the price dynamics of agricultural commodities when global shocks are factored in. The OP result output also shows high explanatory power with an R-square of 0.873982 and an adjusted R-square of 0.866519. The most dominant variable is OP (-1), which is very significant and large (0.847, t=25.047), indicating a high degree of price persistence in the oil market. GF (-1) also has a significant impact (0.686, t=4.185), further supporting the evidence of global macroeconomic influence on oil prices. This is consistent with the findings of Baumeister and Hamilton (2015), which used Bayesian methods to show sustained oil price behavior influenced by global economic activity. The Bayesian VAR estimates strongly confirm the presence of persistence in commodity prices and interrelationships between the variables, especially through the global factor. CCPF, GF and MP show the strongest reliance on their own layer, while copper and oil prices are also reacting significantly to global and commodity-specific shocks. The model's excellent fit across all five equations confirms its robustness. In a separate development,

Figure (4.11) shows the Dynamic Stability Test on BVAR Estimates. The result output shows the Inverse Roots of the AR Characteristic Polynomial. The plot of the inverse roots of AR characteristic polynomial shows the inverse roots (or eigenvalues) of the estimated BVAR model in the complex plane. The unit circle (circle with radius 1) is displayed in the plot. All the blue dots representing the inverse roots are located strictly within the unit circle. The modulus values which represent the magnitude of the roots ranges from approximately 0.433 to 0.989 are all less than 1. The fact that no root has a modulus equal to or greater than 1 confirms dynamic stability. The BVAR model satisfies the stability condition, as all inverse roots lie inside the unit circle. This implies that the BVAR model is stationary, the estimated relationships among variables are stable over time and the impulse response functions derived from this model are valid for interpretation.

Table (4.6) displays the Impulse Response Function (IRF) derived from the Bayesian VAR estimates. This is used to analyze the dynamic responses of macroeconomic variables to structural shocks. The impulse response function (IRF) of the Bayesian VAR estimates provides insights into the dynamic interactions among the Common Commodity Price Factor (CCPF), Copper Price (CP), Maize Price (MP), Oil Price (OP), and Global Factors (GF). The response of CCPF to its own shock is persistently strong beginning at 0.673 and increases gradually to 0.767 by the tenth period. This indicates a significant internal momentum in commodity price movements.

Over time, CCPF responds increasingly to shocks from CP and GF, implying that fluctuations in these variables transmit into broader commodity price patterns. This finding supports earlier empirical work that underscores the systemic influence of key industrial commodities like copper and macro-level global drivers on commodity indices. Notably, OP has a slight negative effect from period 2 to 10; this is a counterintuitive pattern that might point to hedging behavior or structural divergences between oil prices and broader commodity movements. In the case of Copper (CP), the variable exhibits a strong and immediate reaction to its own shock with the value of 1.241 in the first period. This self-effect gradually diminishes but remains above unity until the fifth period, underscoring the persistent inertia in the copper market. CCPF's effect on CP increases initially and peaks in the fifth period before tapering, revealing that overall commodity momentum eventually feeds into copper pricing. Interestingly, global factors exert a growing influence from period 2 to 5 before stabilizing, which is consistent with studies emphasizing copper's sensitivity to macroeconomic cycles. The effects of MP and OP on CP are relatively minor and increasingly negative, indicating minimal spillovers or possibly substitution effects in commodity portfolios. The response of the Global Factor (GF) shows moderate sensitivity to shocks from CP and CCPF. The initial self-shock effect of 0.242 is modest and peaks at 0.299 in period 4 before declining. CP contributes significantly to GF across all periods, implying that industrial commodity markets like copper can serve as proxies for broader global economic expectations. The minimal and declining responses from MP and OP suggest that global macroeconomic sentiment is less affected by agricultural and energy commodities in the short to medium term, echoing previous literature that identifies base metals as more reflective of economic cycles than soft commodities or oil. Maize Price (MP) shows a sharp response to its own shock in the first period (1.270) which gradually decay to (0.324) by the tenth period. This highlights the transitory but notable influence of internal factors in the maize market. MP is highly responsive to GF and CP, especially in the early periods, with the impact of GF peaking at (0.498) by the third and fourth periods. This aligns with studies suggesting that agricultural prices are becoming increasingly linked to broader global market sentiments and industrial commodity pricing. The effect of OP is initially positive but turns increasingly negative, potentially indicating energy cost pressures or policy shifts affecting agricultural production costs over time. The dynamic influence of CP on MP is consistent with prior findings that point to copper serving as a barometer of global demand affecting even soft commodity markets. Oil Price (OP) reacts sharply to its own innovation with a starting value of (1.271) and a decline to (0.180) by the tenth period. This strong autoregressive pattern is typical of oil markets where pricing is influenced by geopolitical tensions, OPEC decisions, and supply-demand fundamentals. GF and CP exhibit moderate and persistent positive effects, suggesting that oil price dynamics are partially shaped by macroeconomic expectations and industrial activity. The initial negative effect of MP, especially in the early periods, is likely linked to substitution or cost-related adjustments in agricultural production. CCPF's influence on OP increases initially but gradually wanes, which implies that while broad commodity trends affect oil prices, their impact diminish over time. This is in line with previous studies that position oil as both a leading and lagging indicator in global commodity interactions. The results affirm much of the existing empirical literature on Bayesian VAR modeling and impulse response behavior in commodity markets. The findings emphasize the interconnectedness of industrial commodities like copper, global macroeconomic expectations, and broader commodity trends represented by the CCPF. The behavior of maize and oil prices, while also interconnected, shows more volatility and distinct response paths, highlighting their sensitivity to specific supply-side and geopolitical shocks.

Table (4.7) shows Granger Causality Test. This test is used to determine the direction of causality and interdependence among the variables. Granger Causality Test for the Bayesian VAR model estimated in Table (4.5) determines whether lagged values of one variable contain information that helps predict another variable. The major findings are CP → CCPF exhibits strong unidirectional causality from Copper Price (CP) to Common Commodity Price Factor (CCPF) ($F=16.59$, $p<0.00001$), Also, GF → CCPF and CCPF → GF shows bidirectional causality; Global Factor (GF) and CCPF Granger cause each other, with stronger influence from GF to CCPF. Similarly, OP ↔ CCPF and MP ↔ CCPF: Bidirectional causality exists, indicating mutual dynamic interactions. Similarly, CP ↔ OP: shows Bidirectional causality suggesting copper and oil prices are mutually influential. Also, CP → MP: CP Granger causes Maize Price (MP), but not vice versa while GF → MP, but MP does not so GF is unidirectional and OP → MP shows Weak unidirectional causality from oil to maize. The Implications of the results is that CCPF is significantly influenced by individual commodity prices (CP, OP, MP) and global factors (GF), affirming its integrative nature. Interdependencies between key commodities (oil, copper, maize) suggest that shocks in one market may spill over into others and Global factors have dominant predictive power, especially for CCPF and oil prices.

Table (4.8) shows VAR Residual Normality Test. This test assesses whether the residuals of the VAR model follow a normal distribution. This is an important assumption for valid inference. The summary of the results show that the skewness for component 1 is significantly skewed (p=0.0006) while others are near symmetrical. All components exhibit excess kurtosis (i.e., heavy tails) with component 5 having an extremely high values (11.23). For the Jarque-Bera (JB) Tests, all components significantly deviate from normality (p<0.0001) and Joint JB Statistic of (2688.768), (p=0.0000) shows strong evidence of multivariate non-normality. The residuals are not multivariate normally distributed, violating classical VAR assumptions. This supports the Bayesian VAR approach used in your analysis, as it is more robust to non-normality and fat-tailed distributions than classical frequentist VAR.

Table (4.9) shows VAR Residual Heteroskedasticity for Levels and Squares. The test examines whether the residuals exhibit constant variance (homoskedasticity), which is crucial for ensuring robust and efficient model estimation. From the result output we notice a Joint Chi-Square with (1726.06, df = 1050, p = 0.0000) showing significant evidence of heteroskedasticity in the residuals. Individual components show that significant heteroskedasticity exists in most individual residuals and their squares (especially res1, res2, res3, res5). Only res4 (res4*res4) shows weak evidence of heteroscedasticity (p=0.2204). The presence of heteroskedasticity suggests time-varying volatility in commodity price relationships. Indicates the possible need for GARCH-type modeling or Bayesian frameworks that handle variance instability well. Also justifies the use of BVAR and possibly Multivariate GARCH extensions (like Diagonal BEKK or VECH), which accommodate volatility clustering. Residuals are non-normal and heteroskedastic, justifying the use of Bayesian VAR over traditional VAR due to its robustness to such violations.

This study successfully extended the work of Chukwukelo, V.N who worked on VAR modelling of selected macroeconomic variables in Nigeria by applying a Bayesian vector autoregression (BVAR) framework to model and estimate the interrelationships between key macroeconomic variables in Nigeria, especially commodity prices (copper, maize, oil), common commodity price factor (CCPF), and global factors (GF). The BVAR estimates reveal strong autoregressive dynamics, especially in the CCPF, GF, and commodity-specific variables such as the price of copper and oil. These findings suggest a high degree of inertia in price behaviour, underscoring a consistent trend in Nigeria's macroeconomic indicators. Granger causality tests confirm significant interdependence and bidirectional causality between multiple variables. In particular, the mutual influence between CPOP ↔, MP ↔ CCPF and GF CCPF ↔ indicates that macroeconomic shocks can spread rapidly across different sectors. The price of copper emerges as a key driver that affects both the price of maize and the overall commodity index. The BVAR model exhibits high explanatory power, strong predictive accuracy (documented by high R-square values), and dynamic consistency (inverted root within the unit circle). Impulse response analysis further validates the model's ability to capture dynamic adjustments over time, outperforming traditional VAR models in structure and performance. Global factors significantly influence all of the macroeconomic variables examined, confirming the open nature of Nigeria's economy and its sensitivity to international trends. Shocks in the copper and oil markets have far-reaching implications on other macroeconomic indicators. Normality tests of the residual reveal significant deviations from normality (excessive kurtosis and skewness), which emphasize the suitability of the use of the Bayesian method which is not heavily dependent on classical assumptions.

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